Leonardo and Gaffurio on Harmony and the Pulse of Music

Leonardo, in his famous comparison of the arts, seeks to vindicate for painting the same elevated status enjoyed by music because of its position in the Quadrivium. While he cannot deny the mathematical foundation that justifies music’s placement with arithmetic, geometry, and astronomy, he seeks to undermine music’s prestige by comparing it unfavourably with painting on a number of points. The main argument takes place in his Paragone, but scattered remarks are found elsewhere; these show that Leonardo has thought much more deeply about the nature of music, especially polyphonic music, than most music theorists, if we can judge from their treatises—which, of course, we cannot really do, but that is the evidence we have. For all their treatment of consonance and dissonance and the intricacies of mensural notation, we find very little theoretical discussion about the most fundamental aspect of all: how music operates in time. It is the element of motion that Leonardo returns to again and again, and it is the concept of time that plays a crucial role in the redefinition of harmony at the end of the fifteenth century. Leonardo frequently appeals to two concepts that have a technical musical meaning: harmonic proportionality and harmonic tempo. In this article I investigate what those concepts meant to him, and whether his understanding of them agrees with contemporary music theory, especially as expounded by Franchino Gaffurio, his colleague in Milan.

Leonardo believed that painting was not numbered among the sciences for lack of writers on the subject; but since painting “is the sole imitator of all the manifest works of nature” and “nothing can be found in nature that is not part of science,” and furthermore, since it draws upon the lines and points of geometry, and perspective depends on arithmetic (discontinuous quantity) and geometry (continuous quantity), painting can rightly be considered a science. In the Paragone Leonardo compared the arts of poetry, painting, and music, Because the eye, “which is said to be the window of the soul,” is superior to the ear, painting is superior to poetry and to music. The poet is limited because his descriptive words “are separated from one another by time, which leaves voids between them and dismembers the proportions.” Music, by contrast, “composes harmony from the conjunction of her proportional parts sounded simultaneously,” but is “constrained to arise and die in one or more harmonic beats (tempi armonici).” Thus painting “excels and is superior in rank to music, because it does not perish immediately after its creation”; moreover, the eye can grasp the whole simultaneously. Therefore, if music is among the liberal arts, “either you should place painting there or remove music.” Statements such as these show that Leonardo was thinking in terms of musical performance: the sound of music is continuous—unlike poetry, where there are gaps between words in recitation—but it cannot be grasped as a whole, and when the piece finishes, the music is gone. Some writers on music were concerned with this phenomenon, but from a somewhat different point of view: is music evanescent and lost after performance, or is there such a thing as a work of art that remains? Tinctoris makes a distinction between music that arises in the process of singing super librum, “on the book,” that is, improvising a melodic line over a written melody taken from chant or polyphony, which he calls counterpart, and music that is composed according to rules regulating the relations between all the voices, which he calls res facta, a “made thing,” and which I have proposed to call “harmonic composition.” A res facta is fixed in writing; later authors called it an opus perfectum et absolutum. After performance it did not vanish, although it existed in another dimension. A res facta was also a visual object, and often a very beautiful one, but in that form it is not music. (On Leonardo’s admiration of a res facta as a work of art, see the Appendix.)

Leonardo and Gaffurio on Harmony and the Pulse of Music

3. Ibid., 34, but Kemp and Walker translate opera nel medesimo tempo as “which make their effect instantaneously” and armonici temp as “harmonic intervals” (Urb 156: “compone armonie con le componizioni delle sue parti proporzionali per eseguirle nel medesimo tempo costruite a mascer e morrino uno o più tempi armonici”).
6. Leonardo anticipated this riposte from the musician, but answers not quite to the point: “If you should say that music lasts for ever by being written down, we are doing the same here with letters”; Kemp and Walker, 95 (Urb 171: “et se tu dicessi la musica s’entra con lo scrivela il medesimo faciati noi qui con le lettere”).
Let us consider the words Leonardo used to describe music: it composes harmony from the conjunction of proportional parts in one or more harmonic temp. Just what did Leonardo mean by harmony, the conjunction of proportional parts, and \( \text{temp} \) armonici? There is no easy answer to this question, because the definition of harmony was a matter of considerable controversy in his time. In the 1470s Tintorius could define armonia as “a certain pleasantness caused by a combining of sound” and, given that general definition, he could also claim that melody is the same as harmony.

This same general meaning is the one we use today when we speak of something being harmonious because we think the parts fit together well, sometimes appealing to proportion, sometimes to symmetry. But at a certain point harmony began to develop a specifically musical use. Did Leonardo use “harmony” in a general or a technical sense?

Our modern understanding of harmony—the relationship of tones sounded simultaneously—brings us up short when we are confronted by some fifteenth-century definitions. In 1482 Bartolomé Ramos stated that many people believed that harmony and music were the same thing. He disagreed, defining harmony as “the mixture of concordant voices,” but music as the intellectual investigation of these concords, recalling Boethius’ definition of the musices. It has been suggested that theorists of the late fifteenth century “reserved the term ‘harmony’ for a chord of three pitches; chords of two pitches were concords or discords.” This is true of Niccolò Burzio, who, in 1487, gives a definition of harmony that specifies music of three or four parts: “It is a modulation of the voice and a concord of many sounds, as is very evident in mensural music, especially when we sing in three or four concordant parts.”

Franchino Gaffurio differs: for him only the harmonic division of a consonance produces what is called harmony. This view is set forth most clearly in his Angelicum et divinum opus musicæ (Milan, 1508). In his Pratica musicæ (Milan, 1496) he notes that the octave is the first of the intervals that can be divided according to a harmonic ratio, and this “harmony” imparts a more pleasing movement (modulatio) to all musical compositions. The mediated fifth (a triad) and mediated sixth (a sixth or first-inversion chord) achieve the same effect, but he hastens to say that they are produced by almost harmonic divisions. The fifth mediated to produce a major and minor triad “brings about a sweeter concord of the extremes as if it adhered to the harmonic mean by a kind of imitation”; the mediated sixth makes a concord “as if it were neighbor and partner of the harmonic mean.” But in the De harmonia musicorum instrumentorum opus (completed by 1500 but not published till 1518) Gaffurio appears to relax his strict understanding of the term “harmony”: a marginal note in chapter 10 of Book III summarizes: “What harmony is. Harmony differs from consonance: consonance consists of two sounds, harmony of three.” The text explains that a consonance forms only one proportion, but a harmony has at least two. Therefore every harmony is a consonance, but not every consonance a harmony. In the next chapter, however, we discover that Gaffurio is using both a loose and a strict definition of harmony: “Having disposed three tones according to a harmonic division . . . a melody is thus produced that we properly call harmony.” Having conceded this much, he feels bound to invent a term for other pleasant three-voice sonorities that are not “proper” harmonies. His solution is to characterize them as consonances divided by a “sonorous mean,” which accounts between the consonant and the less pleasant dissonant (cogito sunt consonantia et harmoniam idem esse posuerunt. nam quandoque harmonia consonantia est: omnis tamen consonantia non facit harmoniam. Consonantia non quod ex acuto et gravi generatur: HARMONIAM un ex acuto et grave conjunctum atque mediolum” (F. 880). An English translation is available in Franchinus Gaffurius, De Harmonia Musicorum Instrumentorum Opus, trans. Clement A. Miller (Musicological Studies and Documents, 20; n.p.: American Institute of Musicology, 1958).

“Postice musicæ, sig. ccg.”: “Quinta autem quam dis- positio integra tribus s. tonus sin minore sonantius ducta sesquialtera dimensione productum: medium obiectum concordum chordam eam extremis. Compositum enim ex duas primis simplicibus s. tertro minori atque tertro maioris concordis medietate servata. Inde succursum dictum extirpationis concordum quasi quin quire invasiones harmonice adserenda medium. ” Habemus secatam solam chordam medium et concinnam quae s. tertro est ad gravitatem et diatessaron subsonat ad acutam. Diatessaron enim consonantia et ali simplicis ducta dissona sit: constructa tamen conditi concussioni concordem effect curium extimus medietatem: quasi harmonice mediatum prorsum et partem . . .”


for the not quite harmonic division of major and minor sixths and tenths. Similarly, Ptolemy's adjustment of the intervals to superparticular ratios is deemed to produce a pleasing but not a harmonic division.

Gaffurio does not have a word for "chord"; the concept certainly exists for him, as we can see from his remarks, but it has been ferreted out from circumlocutions. (It could trap the unwary reader that he uses the term "chorda" to describe the mediating notes; here, however, it means "string" and goes back to the Greek nomenclature for the notes of the Greater Perfect System, deriving ultimately from the strings of the lyre. Leonardo, incidentally, uses the word "chorda" to mean narrow, tension, or ligament.)

There has been much speculation on Leonardo's acquaintance with Gaffurio, which is likely but not supported by direct evidence. Given Leonardo's intense interest in music, as demonstrated by scattered remarks in his notebooks, his acoustic experiences, which in fact is mainly that Leonardo and Gaffurio dealt with musical instruments; in his notes on anatomy discussing pitch in relation to the lengths and widths of a pipe or tube, written ca. 1508-80. Leonardo remarks "And I do not go into this at greater length because it is fully treated in the book about harmonical instruments." As Edward MacCurdy, *The Notebooks of Leonardo da Vinci, Arranged, Rendered into English and Introductory, 2 vols. (2d ed., London: Jonathan Cape. 1956), 1:176. Earlier it had been thought that Leonardo was referring to a book written by himself, "ne ho trattato," but the wording is actually "ne [n'è] trattato," as MacCurdy has translated it, according to Carlo Pedretti, *The Literary Works of Leonardo da Vinci, Commentary, 2 vols. (Oxford: Phaidon, 1977), 1207, who commented: "This is probably a reference to Franchino Gaffurio's *De harmonica musicae instrumentorum opus quadruplistum,* published in 1508 [sic], rather than a reference to a book 'delli strumenti armonici' written by Leonardo himself." It is a common misunderstanding of Gaffurio's title to believe that the book deals with musical instruments; in fact it refers to the harmonics of the universe and the harmonious relations of the human mind and body. (There is, however, a brief passage on the pitch of organ pipes at the end of the last chapter of Bk. II [ff. 69r-70r], but it does not appear in the Anglican opus.)

I agree with Winternitz (p. 8) that "the Portrait of a Musician" in the Pinacoteca Ambrosiana, once ascribed to Ambrogio de Predis but now accepted as by Leonardo, cannot be of Gaffurio; it is much more likely to be of one of the Sforza court musicians. (In *Leonardo and Music,* 1972, written earlier, he had suggested that Gaffurio was "probably the subject of Leonardo's painting").

ments, and his sketches of the mechanics of musical instruments—in addition to his known ability as a performer on the lira da braccio—it would seem fruitful to investigate his acquaintance with music theory of the time. Emanuel Winternitz, in his wide-ranging book on Leonardo and music, barely mentioned Gaffurio's theoretical writings as of possible interest to Leonardo; instead he concentrated on acoustics, improvisation, and musical instruments, apart from the discussion of the *Pantagruel.* This omission was noted with regret by Martin Kemp in his review of Winternitz's book. In particular, Kemp wondered about the basis of Gaffurio's musical thought and the association between harmonic systems and the science of nature, and whether Leonardo's ideas had any impact on Gaffurio. This, of course, would require a very wide-ranging investigation; in the present article I shall limit myself to Leonardo's notions of harmony and "tempi armonici," especially as they relate to polyphonic music.

Leonardo frequently uses the term "proportionalità armonica" in his writings on painting. "Proportionality" is a technical mathematical term; as Boethius and, following him, numerous music theorists explain, a proportion consists of two terms (e.g. the octave, c,d), but a proportionality consists of three (e.g. a fifth-octave chord, c,d,6). Simple proportions become proportionality when divided by an arithmetic, geometric, or harmonic mean. The terms in harmonic proportion if the greatest is to the least as the difference between the greatest and the mean is to the difference between the mean and the least, i.e. if the greatest term be $a$, and mean $b$, and the least $c$, then $a : c = a - b : b - c$.

The harmonic mean may be found by the following formula:

$$b = \frac{2ac}{a + c}$$

Leonardo is aware of the technical meaning, as can be seen in the following statement:

The eye is the true intermediary between the objects and the impressiva, which immediately transmits with the highest fidelity the true surfaces and shapes of whatever is in front of it. And from these is born the proportionality called harmony, which delights the sense with sweet concord, no differently from the proportionality made by different musical notes to the sense of hearing.

Nevertheless, he seems to use "proportionalità armonica" in a looser sense, more equivalent to "to a harmonious proportion." Indeed it would not be possible to compose a "dolce concerto" using only harmonic proportionality: in Gaffurio's strict definition, based on Pythagorean intonation, one could not have any triads.

20. See e.g. Kemp and Walker, 21, 24, 26, 57, mostly translated as "proportional harmonies."
21. Kemp and Walker, 24 (Urb 11): "'Occhio vero mezzo infer di obbietto & la impressione il quale immedi-
Harmonic proportionality is applied to painting in the same loose sense: music and painting are sister disciplines because they both make use of it. The great failure of poetry is that it lacks proportionality, since words are spoken successively. Leonardo relates an anecdote of King Matthias’ dispute with a poet to illustrate this point. The king says:

Do you not know that our soul is composed of harmony, and that harmony cannot be generated other than when the proportions of the form [le proporzionalita della obbietti] are seen and heard instantaneously? Can you not see that in your science, proportionality is not created in an instant, but each part is born successively after the other, and the succeeding one is not born if the previous one has not died? From this I judge that your invention is markedly inferior to that of the painter, solely because it cannot compose a proportional harmony [non compone proporzionalita armonica].

The same criticism applies to monophonic music, or voice parts sung singly.

The poet may be regarded as equivalent to a musician who sings by himself a song composed for four choristers, singing first the soprano, then the tenor, and following with the contralto and then the bass. Such singing cannot result in that grace of harmonic proportionality which is contained within harmonic beats [tempi armonici], ... Yet music, in its harmonic beat [tempo armonico], makes its suave melodies, which are composed from varied notes. The poet is deprived of this harmonic option, and although poetry enters the seat of judgement through the sense of hearing, like music, the poet is unable to describe the harmony of music [Armonia della musica], because he has not the power to say different things at the same time. However, the harmonic proportionality of painting is composed simultaneously from various components, the sweetness of which may be judged instantaneously.

In the comparison of painting with music, the argument from harmonic proportionality draws these two disciplines together, likening optical to harmonic space. Indeed, Leonardo seems in both cases to be using harmonic proportionality to indicate depth or volume, whereas poetry can produce no more than surface. It should be emphasized that Leonardo is not talking about music per se but specifically about polyphonic music. Music “composes harmony from the conjunction of her proportional parts sounded simultaneously.”

We now come to the more difficult passage in the Pangoro where Leonardo uses the expression “tempi armonici.” It comes directly after the above passage in the following context:

Music is not to be regarded as other than the sister of painting, in as much as she is dependent on hearing, second sense behind that of sight. She composes harmony from the conjunction of her proportional parts sounded simultaneously, constrained to arise and die in one or more tempi armonici. These tempi surround the proportionality of the component parts of which such harmony is composed no differently from the linear contours of the limbs from which human beauty is generated.

Here Leonardo turns from the aspect shared by painting and music—harmonic proportionality—to the aspect that separates them, the dimension of time. A painting can be grasped as a whole; music unfolds in time and is evanescent. This is a point he returns to several times because it is the linchpin of his argument for the superiority of painting; music perishes immediately after its creation, painting endures.

Emanuel Winternitz translated tempi armonici as “harmonic sections” and interpreted the passage not as chords moving in time but as the proportional relationships between sections of a musical composition, though he saw the text as “obscure or at least inconsistent” (Leonardo, 211). Thus he credits Leonardo with having “applied the concept of proportion to the relation between successive portions of Music and thus established the notion of a quasi-spatial structure of portions balanced against one another” (ibid.). In another section Winternitz translates in turn tempi armonici as “in as many sections of musical time” (p. 225), and elsewhere as “moments of harmony,” interpreted as chords, and tempe armonici as “harmonious flow” (ibid. 217). Leonardo certainly was acquainted with the proportionality of sections of a composition (the subject is covered thoroughly in Gaffurio’s Poetica music, Bk. IV, and Tinciani’s Proportionale), since he discusses the proportionality of time in the Codex Arundel (the passage is cited by Winternitz on p. 221), but I do not believe that this is what he means by tempi armonici. Winternitz did not take into account Leonardo’s use of the term in other contexts that do not refer to musical compositions.
Tempi armonici is a term Leonardo uses many times, also in other contexts. Often tempo armonico is used to indicate a regular beat, which can be employed to measure the velocity of moving objects since, Leonardo says, it is more reliable than the pulse. He uses it to ascertain how far water travels in an hour:

This is done by means of harmonic time [tempo armonico], and it could be done by a pulse if the time of its beat were uniform; but musical time is more reliable in such a case, for by means of it it is possible to calculate the distance that an object carried by this water travels in ten or twelve of these beats of time; and by this means it is possible to make a general rule for every level canal.29

As a musician himself, Leonardo was familiar with the concept of a regular musical beat. Since harmony or harmonic proportionality is not relevant in this context, tempo armonico means no more than musical beat; in the early Codex B he in fact says "tempo di musica." How fast is this beat? In Codex Arundel one hour is said to contain 1080 tempi, based on human respiration, making one tempo equal to 3.3 to 3.5 seconds, or 8 per minute.30

This is quite slow; thus it is likely that Leonardo is using the word tempo in its technical musical sense: tempus = breve.31 In tempus perfectum 1080 tempi per hour would come to 54 semibreves per minute.

Placing the musical beat on the breve reflects early fifteenth-century thought: in his treatise of 1434, Giorgio Anselmi, one of the earliest authors to discuss the musical beat, states:

And this notated mensur is called one tempus. Still, this mensura is not fixed, not exceeding limits, but according to the judgment of the singer [15] here more broad and now more strict ... the mensura is near enough to a moderate tempo in which the singer, not much accelerating the song or extending the note-lengths, stamps the front part of the foot, keeping the heel still, or clasps one hand to the other or the back of the student as regularly as possible.32

By Leonardo's time, however, the musical beat, called mensura, battuta, or (in Germany) tactus, was considered to fall on the semibreve. Like Leonardo, both Bartolomeo Ramis and Gaffurio like the musical beat to the beat of the pulse.33 Ramis considers the mensura to be the interval between the diastole and the systole;34 Gaffurio, however, says that the mensura comprises both the diastole and systole.35 (The discrepancy will be explained below.) When discussing the permissible length of dissonances in composition, Gaffurio states that a dissonance cannot last the length of a semibreve, calculated as the full mensura of time "in modum scilicet pulsus acue respirantis."36 There has been some disagreement as to how this phrase is to be understood: is Gaffurio equating the length of the semibreve with the time between beats of a normal pulse, allowing thereby an approximate indication of the tempo at his time, or is he simply likening a regular musical beat to a regular pulse? Clement Miller translated this passage as "For a semibreve, equal to a complete measurement of time [a tactus], like the pulse of a man breathing evenly, cannot be given to a dissonance," noting that "acue respirantis" had frequently been mistranslated as "quietly breathing."37 Irwin Young translated it as "a normal semibreve occupying a full measure of time, in the manner of a pulse throbbing evenly, cannot support a discord."38 Dale Bonge, pointing out that the pulse and respiration were considered to be linked in contemporary medical thought, believes that Gaffurio intended no more than the analogy of the regularity of musical beat and pulse. He would therefore translate it: "For a regular semibreve equalling a full measure of time, namely, in the manner of a pulse dilating and contracting evenly, cannot lie under a dissonance in counterpoint."39

It would indeed seem that Gaffurio's statement emphasizes regularity of beat, not the particular length of the semibreve. However, the restatement of this passage in his Italian treatise, the Angelicum ac divinum opus music, of 1508, clearly makes the equation of tempo between semibreve and pulse:

They are writing about the pulse of music, not the music of pulse. Since antiquity physicians had sought to discern musical proportions in the uneven rhythms of the pulse as a diagnostic tool. The discussions were still very much alive in the Middle Ages. See Nancy G. Siraj. The Music of Pulse in the Writings of Itenian Classical Physicians (Fourteenth and Fifteenth Centuries).2

"... the time between beats of a normal pulse, allowing thereby an approximate indication of the tempo at his time, or is he simply likening a regular musical beat to a regular pulse? Clement Miller translated this passage as "For a semibreve, equal to a complete measurement of time [a tactus], like the pulse of a man breathing evenly, cannot be given to a dissonance," noting that "acue respirantis" had frequently been mistranslated as "quietly breathing." Irwin Young translated it as "a normal semibreve occupying a full measure of time, in the manner of a pulse throbbing evenly, cannot support a discord." Dale Bonge, pointing out that the pulse and respiration were considered to be linked in contemporary medical thought, believes that Gaffurio intended no more than the analogy of the regularity of musical beat and pulse. He would therefore translate it: "For a regular semibreve equalling a full measure of time, namely, in the manner of a pulse dilating and contracting evenly, cannot lie under a dissonance in counterpoint."

It would indeed seem that Gaffurio's statement emphasizes regularity of beat, not the particular length of the semibreve. However, the restatement of this passage in his Italian treatise, the Angelicum ac divinum opus music of 1508, clearly makes the equation of tempo between semibreve and pulse:
For just as the measure of the human pulse is considered to be one tempo divided into two motions, that is in one ascending and the other descending, which physicians call systole and diastole, and musicians arsis and thesis, so have the scholars of later ages ascribed the measure of a sonorous tempo to the semibreve equal to the tempo of the pulse; and it is divided into two equal motions of tempo that are dedicated and applied to two minims. 40

The Angelicum ac divinum opus musica, a compendium derived from all of Gaffurio’s treatises, including the as yet unpublished De harmonia musicorum instrumentorum opus, was written in Italian for the benefit of practical musicians (and nuns) who were not able to read Latin, or who found the sometimes ornate and obscure style (such as Gaffurio’s own humanistic Latin) difficult to understand. 41 The decision to write in the vernacular must reflect Gaffurio’s experience in speaking with many musicians in Milan, and quite possibly Leonardo himself, whose Latin was rudimentary at best.

Leonardo clearly does give a length to the tempo armonico. This was necessary for his purpose; he needed to calculate how fast an object was moving, and therefore the measurement had to be based on exact units of time. He calculated the length not from the pulse, rejected for its irregularity, but from human respiration: “1080 are those [tempi armonici] that man universally passes in breathing in and out.” As mentioned above, this tempo is quite slow and must correspond to the breve, not the semibreve. At some later point Leonardo changed the calculation of the tempo armonico so that an hour was equal to 3000 tempi. 42 Augusto Marini wondered that the reason for the substitution was that 3.33 seconds per beat was too large for measurement, leaving too many fractions; at 3000 tempi per hour the unit is 1.2 seconds. He also believed that just as the previous measurement had had its origin in a natural rhythm (respiration), so must the new one, which he took to be the rhythm of the pulse, but a rather slow one, 50 beats per minute (Leonardo does not clarify his change from 1080 to 3000). Marini wondered how Leonardo became accustomed to the new tempo, and posited the need for an instrument to measure time. 43

[Further text about Leonardo’s calculation of the pulse, including references to Marinoni’s work and the influence of the metronome.]

Leonardo and Gaffurio on Harmony and the Pulse of Music

40 “Nam secundo de la mensura del pulso humano se considera en uno tempo diviso en dos movimientos: uno ascendente et el otro descendente; quedo diviso en dos minis (Tempo per minutae) que son divisi et aplicati a do minima” (Tr. III, chap. 1, sig. F7). Here Gaffurio uses the word tempo to mean unit of time, not note.

41 The book begins: “Perche molti illirati fano professione di musica: et con grande difficoltà per vengono alla vera cognizione di li precepiti harmonici per non intenderle lo oper e nostro et de altri degni accadutori latini quale son scrizie con qualche ornato et alquanto obscuro stilio; havemo considerato subvenire non solamente a lei vostri et desiderii ma anch’a la devozione de molto dove religiose intende ad uaudere la eterno Dio con tutta la corte celeste” (sig. B4). There is a facsimile edition (Antiquae musicae italicae scriptores, 8; Bologna; A.M.I.S., 1977), but no translation.

42 Gaffurio had already published, under the name of his pupil Francesco Caza, a short Italian version of his treatise on notation, which eventually became Book II of the Principe music. See Francesco Caza, Tratato volgare del canto figurato [Milan, 1492], facs. and trans. by Johannes Wolf (Veröffentlichungen der Musik-Bibliothek Paul Hirsch; Berlin: Martin Beilauer, 1961).

43 In the Codex Arundel, on f. 97v, he first wrote “escludo un’ ora 1080 temp.” then crossed out 1080, writing 3000 above it. See Marini, p. 47.
ing the unequal movements of the pulse to unequal musical proportions. Gaffurio, however, considers the diastole and systole to be equal in length (except in a fevered state), and therefore he equates the length of the semibreve with the whole pulse, which conveniently makes the arsis and thesis on the minimum equivalent to the systole and diastole. Thus, even though both theorists place the mensura on the semibreve, the regular unvaried unit for Ramos is the breve and for Gaffurio the minim.

Leonardo’s use of *tempo armonico* as a unit of measurement stresses the aspect of *tempo*; when he uses it in a musical context more weight is given to *armonico*. “Harmonic time” is a unit of time encompassing a harmonic simultaneity: music “composes harmony from the conjunction of her proportional parts sounded simultaneously, constrained to arise and die in one or more *tempi armonici*. So much is clear. The continuation, however, is somewhat obscure: “These *tempi* surround the proportionality of the component parts of which such harmony is composed no differently from the linear contours of the limbs from which human beauty is generated.” Here Leonardo seems to be attempting a visual image of an auditory phenomenon. Time envelops sound just as a line may be drawn around the members of a human body. Does he have in mind here the famous image of the Vitruvian man, whose outstretched limbs fit exactly within a circle? Or does he mean the outline of a human figure? The proportions of the human body greatly occupied Leonardo, as did outlines, contours, and boundaries, not only of the human figure but also objects of nature, including landscape. Outlines define the wholeness of a figure, the conjunction of proportional parts, which might be likened to a chord in music.

By attaching *tempo* so firmly to *armonia* and stressing the successive nature of musical sounds, Leonardo underlines another aspect of music that has a counterpart in painting: motion. The painter, of course, has much more difficult task: he must convey the appearance of motion, not motion itself. Time is frozen in painting, and yet everything that happens in that frozen moment can be conveyed at once. Music, by contrast, is continually in motion and can never be grasped as a whole.

Motion plays a very important role in the dispute concerning harmony in the late fifteenth century. While all theorists agreed that two-note chords formed either consonances or dissonances, they did not agree on the classification of three-voice chords. Gaffurio, as we saw above, limits the strict use of the term “harmony” to consonances mediated by a harmonic division, although he also uses it in a looser sense to describe other divisions that produce a pleasant concord. Burzio was willing to describe any three- or four-voice chord made of up consonant parts as a harmony, and he probably reflects the attitude of contemporary musicians as well as other theorists who did not have as great a stake in ancient Greek theory as did Gaffurio. But there was one theorist who accepted neither of these definitions of harmony, Giovanni Spatarno. In his polemical answer of 1491 to Burzio’s equally polemical treatise against Spatarno’s teacher, Bartolomeo Ramos, he was scathing about Burzio’s knowledge in general and his discussion of harmony in particular. For Spatarno, harmony was something quite different. He agreed with Burzio that “consonance is only the consideration of the interval between a low and a high note and vice versa,” but insisted that the addition of one or more voices does not turn consonance into harmony. To have harmony, movement is necessary: “it is called harmony when considering the process they make by concording together (i.e., procedere che fanno insieme concordando), because if they do not move (se non si movono), even if there are four voices, it is not called harmony, but consonances. . . . Let harmony be defined as the mixture of consonances and dissonances in a composition, because it is quite true that good composers exert themselves to make dissonances marvellously consonant in harmony.”

Here, then, is that elusive view of music as a process that unfolds in time, a process sometimes called by another term that has a different meaning today: modulation. Now, modulation indicates a change of key area; originally it simply meant measurement of any kind: measured or rhythmical in respect of music, or singing or playing, whether melody alone or a whole composition. Burzio had used “modulation” in his definition of harmony, calling it a *modulatio vocis*, a modulation or movement of the voice. Leonardo understood this very well: for him, harmony is a conjoining of proportional parts, and this conjoining is described as arising and dying in harmonic *tempus*. We have here a description of the process chords make in musical time.

Musical notions and musical terminology have a central position in Leonardo’s discussion of painting. In fact, if the subject is not stated, one would sometimes be hard put to know whether Leonardo is talking about painting or music: “thus x simultaneously conveys the proportional harmony of which the parts of the whole are com-
posed, and delights the senses." The subject here is painting. Indeed, the parts of a painting can have the same "proportione armonica" as music, and when viewed together, they can make a "armonico concetto." "Concerto" could be interpreted in the general sense of sounding together, but it is also a technical term at this time for a polyphonic composition; Gaffurio calls his musical examples concerti.

It has been suggested with some plausibility that Leonardo based the Last Supper on musical proportions, even that the four groups of disciples resemble the four vocal ranges. To be a truly armonico concerto, however, the parts of this painting would have to move in harmonic time, that is, harmony in Spataro's definition: dissonance resolving into consonance, which in modern parlance could be termed "functional harmony." I would submit that this happens in the Last Supper. Leonardo has captured the moment where the disciples react with harsh gestures to Jesus' words, "One of you will betray me," as if he had portrayed a dissonant suspension. The painting is alive with harmonic movement, resolving in the calm, central figure of Jesus.

Quite possibly Gaffurio saw Leonardo at work on the Last Supper in the refectory of Santa Maria delle Grazie. Did he remark on its musicality? So far I have considered "Consonant numbers offer much to other arts, but you will see that both the measurements of bodies and the mixtures of colours, and thus the beauties of painting, have been determined according to numbers and symmetries, and that it is thus that the beauties of the paintings have been arranged, and that in turn it is through numbers that the art itself imitates primary nature. For whatever proportion has created beauty in natural bodies, such proportion has also ensued in the measurements of shapes and the comparisons of colours; for which reason, by colours, form, and shape painters themselves mean character and life to be understood."

What a generous compliment to contemporary artists, one thinks! But in fact, these are not Gaffurio's words at all. He has taken the whole passage, verbatim and unacknowledged, from a Latin translation made for him in 1494 of the Peri mouisou of Aristides Quintilianus, a writer of the late third or early fourth century AD. It was Gaffurio's habit—and not only his—to incorporate passages from many ancient sources without specifically crediting them. We need to be aware of this in evaluating theoretical statements, even those that have a contemporary ring. The habit was surely common among teachers, who taught by explication texts.

That Gaffurio viewed himself above all as a teacher is made clear in the woodcut that graces his two last treatises, where he is shown expounding music ex cathedra, as holder of the chair in music at the University of Milan, to twelve disciples (see Plate 1). From his mouth issues the famous saying, "Harmonia est discordia concors," "harmony is discord concord," or harmony is concord brought forth from disparate parts. This is not merely a play on words, or even only a statement about the nature of music; it places music at the heart of the universe. The phrase discordia concors comes from the Astronomica of Manilius, an author of Augustus' and Tiberius' time who after centuries of neglect had been discovered during the Council of Constance and copied many times over in the
course of the Quattrocento; there are six or seven incunable editions (including one published by Dulcinius at Milan in 1489 with an enthusiastic preface on the rebirth of letters).

Reviewing the various competing theories of the universe, Manilius comes to Empedocles' view that it is the product of four elements in the two relations of love and strife:

aut neque terra patrem novit nec flamma nec aer
aut maris, faciuntque deum per quattuor artus
et mundi struere globum prohibente requiri
ultra se quicquam, cum per se cuncta creariat,

frigida nec calidis desint aut umida siccis,
spiritus aut solidis, sitque haec discordia concors
que nexus habitus et opus generabile fingit
acque omnis partus elementa capacia reddit. 60

Seneca similarly declares (Naturales quaestiones 7. 27. 4): "tota haec mundi concordia ex discordibus constat," this entire universal concord is formed out of discordant elements. The same notion is expressed by Horace (Epistles 1. 12. 19), with explicit mention of Empedocles, as "rerum concordia discors," the discordant concord of things, and by Ovid, Metamorphoses 1. 430–43:

quippe ubi temperiem sumpsere umorque calore,
conspicit, et ab hi orientur cuncta duobus;
cumque sit ignis aquae pugnas, vapor umidis omnis
res creavit, et discors concordia fetibus apta est. 61

Gaffurio acknowledges Ovid as his source and quotes these four lines in Bk. I, chap. 1 of the De harmonia, though he has them indirectly, from Lactantius, Divinae institutiones ("2. 10" = 2. 9. 20).

In the woodcut this concord is illustrated in the harmonic division of the octave, 34:6, shown in string lengths on the right, and in organ pipes on the left. A pair of compasses underlines the geometrical aspects of music as continuous quantity. An hourglass is set at Gaffurio's left elbow: this might have been used to time his lectures, but it is also a reminder of the temporal aspect of music. His listeners are, on the left, monks and clergy; in the center, young laymen who wish to learn about music; and on the right, adolescents who are probably choirboys. Not included here are men and women in the wider social world of Milan, especially those associated with the court, with whom Gaffurio must have come in contact. "Harmonia discordia concors" is a concept that would have had particular resonance for one of them, Leonardo.
Appendix

A Musical Silhouette at the Sforza Court

"The first picture was merely a line, drawn round the shadow of a man cast by the sun upon a wall."64 As commonplace as this notion was, for Leonardo it had particular meaning because of his continuing involvement with pictorial boundaries, outlines, and contours. Thus one can understand his astonishment and delight at a demonstration of the art of paper-cutting at the court of Milan in 1499. In his youth, the Spanish chronicler Gonzalo Fernández de Oviedo visited Milan and met Leonardo. In his memoirs, written some fifty years later, he relates the following story:

In 1499 in Milan I cut a polyphonic motet for four voices with the arms of the Duke, who was at that time Ludovico Sforza, also called El Moro, who, astonished at the subtlety of that work, wanted to see me cut, and in his presence I cut everything he wished to give me. Marveling at what he saw, he asked his great painter and sculptor named Leonardo da Avinse, whose art, as some said, was unique in Italy, what he thought about what I was doing. And Leonardo said: "Your Excellency may believe that of all the things I have seen in the world, this is the one that has impressed me the most, and if I myself had not seen him cut, I would not believe that a man could do something so subtle with scissors alone, without any drawing, but moving the hands solely by memory." Then the Duke said: "If this Spanish Lived had at the time of the ancient Romans, he would have been crowned God of the Scissors."65

I know of no motet on the arms of Ludovico, and it is a puzzle how they might have been shown in polyphony. Ghiselin Danckerts' Tua est potentia was written using a representation of the arms of Paul III in the tenor part; the arrangement of the six lilies superimposed on a staff can be read as the six initial notes of the Da paen.66 Perhaps there was some such device in the tenor of the motet.67

Like many men who became famous in later life, Fernández de Oviedo regretted the time wasted in his youth on frivolities. He is known today as the great chronicler of the Indies (he was officially appointed in 1512), author of Historia general y natural de las Indias, the first part of which was published in Seville in 1535, the second part in 1557. It was based on five trips to the New World, between 1514 and 1546.68 He was born in Madrid in 1478, and at about the age of 12 became a page in the household of the Duke of Villahermosa, nephew of Ferdinand the Catholic. Three years later he became a mozo de cámara in the household of Prince John, only son of Ferdinand and Isabella, who at the time of his early death in 1497 was married to Margaret of Austria. In 1499 he went to Italy, serving various patrons until his return to Spain in 1501. Details of his career are vague until the point where he became deputy to Lope Conchillos, Secretary for Indian Affairs, and left for the New World in 1514. From the time he published a novel on chivalry in 1519 he continued to write and to translate from Italian.69

Obviously Fernández de Oviedo had some musical knowledge. What little we know of the musical establishment of Prince John is based on his Libro de la Camara Real del Príncipe Don Juan y oficios de su casa e servicio ordinario. In it he recalls Prince John's inclination to music, his instruments, his musicians, and the master of his chapel, Johannes de Anchieta.70

Fernández de Oviedo's memoirs reveal that his skill with scissors was not unique: there were others who had learned the art, both in Spain and outside it, but he held that none surpassed him. He disapproved of those who drew their patterns first, with rulers

64. The cryptic version was in the lost Treviño Ms. 10, for a sketch of the tenor and an explanation see Bonnie J. Blackburn, Music for Venerable Cathedrals in the Late Sixteenth Century: A Rerum description of the Lost Manuscripts 25 and 30 (Royal Musical Association Monographs, 1; London: Royal Musical Association, 1987), 112 and 40-45. The motet was printed by Kriestein in 1540 with the tenor resolved. Other compositions on coats of arms by Isaac (Pallia) and Cortazar Porta (Musica duale) are discussed ibid. See also Mitchell R. Brown's interpretation of the two cantus firmi in Carle's seven-voice motet Vidi Dominum in the Manuscript Verona, Accademia Filarmonica, B. 218 and its Polifonic Motet, Studi musicali 16 (1987): 5-112.
65. The Sforza arms are a crowned eagle quartered with the Visconti hoeve. Jeffrey Deal has suggested to me that the arms might have been separate from the music, since there was no sopranino. If so, there is a good candidate for the composition (though the piece is rather long): Gaffurio wrote a motet addressing Ludovico, Sveus deus, found in Milan, Archivio della Veneranda Fabbrica del Duomo, Sezione Musicale, Libro i (olim 2256). ff. 8a-8b, facsimile edited by Howard Mayer Brown (Renaissance Music in Facsimile, 112; New York: Garland Publishing, 1982). There is a modern edition of the piece in Franchinio Gaffurio, Motetti; ed. Luciano Migliavacca (Archivio musicale metropolitanum mediceo, 5; Milan: Veneranda Fabbrica del Duomo di Milano, 1959), 69-74.
67. Ibid., pp. x-xv.
68. See Figueroa Anchéz, La música en la Corte de los Reyes Católicos, I (Monografías de la Máusica Española, 13; Madrid: Consejo Superior de Investigaciones Científicas, 1963), 74-75, and Mary Kay Dougan, "Queen Joana and Her Musicians," Musica disciplina 50 (1996): 75-93.
69. The Libro de la camara real was edited by J. M. Escudero de la Peña (Sociedad de Bibliófilos Españoles, 27; Madrid: Vidasa y hijos de Galano, 1870).
and gouges, and used gouges and tools in addition to scissors (making the technique more similar to wood-cutting). He recalls cutting the arms of Prince John and Margaret, some in size no larger than a small coin. With such a skill he must have found an easy entry into the Italian courts.

The history of the art of paper-cutting is badly in need of study. Although related to the silhouette, paper-cuts are much more ornate since they involve cutting not only around the edges to create an outline but cutting inside; the result can be reminiscent of filigree technique. The art has not died out; I have seen extremely elaborate modern Chinese paper-cuts. The English term "silhouette" derives from the name of Étienne de Silhouette (1709–67), and the technique became extremely popular in the later eighteenth century, but it is not at all the same as a paper-cut.

Because the name and the technique of the silhouette are so well known, scholars have been misled into believing that the paper-cut that is pasted to one of the opening folios of Bologna, Museo Civico Bibliografico Musicale, Ms. Q 19, a stag chained to a tree, is of eighteenth-century origin. Rainer Heyink, in his 1994 study of the manuscript, was more inclined to see it as close to the date of the manuscript (one piece is dated 1518), but without producing new evidence on the history of the technique. Instead, he proposed that the emblem itself was that of Lucrezia Gonzaga, a member of the Bozzolo branch of the Gonzaga family, born in 1522. Leeman Perkins first drew attention to the bird perched in the tree as a possible Gonzaga emblem.

Fernández de Oviedo's biographical remarks permit us to discard the theory of the eighteenth-century origin of the paper-cut, as it should now be called, in Bologna Q 19. Indeed, one is even tempted to suggest that it is one of his own works, and therefore made before, not after, the preparation of the manuscript. After his visit to Milan, Fernández de Oviedo went to Mantua, where his art impressed not only Isabella d'Este but also Andrea Mantegna ("another Leonardo da Vinci"). Joining the entourage of Car-

dinal Juan Borgia, nephew of Alexander VI and archbishop of Valencia, and newly appointed legate a latere to the French king, Fernández traveled to Milan, where he saw the entrance of Louis XII, then to Turin, Ferrara, Bologna, Urbino, Rome, and finally Naples, returning to Spain in 1501. In all these courts he demonstrated his art. Quite possibly there are still examples of his artistic creations, tucked away in books or manuscripts of the early sixteenth century or later, preserved because of their beauty.