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Criticism of trepidation models and advocacy of uniform precession in medieval Latin astronomy

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Abstract A characteristic hallmark of medieval astronomy is the replacement of Ptolemy's linear precession with so-called models of trepidation, which were deemed necessary to account for divergences between parameters and data transmitted by Ptolemy and those found by later astronomers. Trepidation is commonly thought to have dominated European astronomy from the twelfth century to the Copernican Revolution, meeting its demise only in the last quarter of the sixteenth century thanks to the observational work of Tycho Brahe. The present article seeks to challenge this picture by surveying the extent to which Latin astronomers of the late Middle Ages expressed criticisms of trepidation models or rejected their validity in favour of linear precession. It argues that a readiness to abandon trepidation was more widespread prior to Brahe than hitherto realized and that it frequently came as the result of empirical considerations. This critical attitude towards trepidation reached an early culmination point with the work of Agostino Ricci (*De motu octavae spherae*, 1513), who demonstrated the theory's redundancy with a penetrating analysis of the role of observational error in Ptolemy's *Almagest*.

1 Introduction

Our main source for the knowledge of precession in antiquity is Ptolemy's *Almagest*, where the phenomenon is treated as a gradual change in alignment between the equinoctial and solstitial points and the sphere of the fixed stars, a change caused

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by the latter's rotation around an axis that passes through the poles of the ecliptic.[1] Although it is nowadays regarded as one of the most important scientific discoveries of the Hellenistic period, traceable back to Hipparchus in the second century BC, the concept of precession appears to have found relatively few adherents during antiquity, with some astrologers outright rejecting its validity.[2] It is thus hardly surprising that we find no clear references to it in medieval Latin sources before the twelfth century, when Europe received a fresh infusion of astronomical knowledge through translations from Arabic, Hebrew, and Greek. During subsequent centuries, Ptolemy's views of a uniform precession competed with alternative theories, devised by Arabic astronomers, of a variable and even bidirectional motion of the sphere of the fixed stars. Historians of astronomy have become used to referring to this nonlinear motion as the 'trepidation' of either the equinoxes or the fixed stars, although it is worth pointing out that no such term appears in medieval sources prior to the mid-fifteenth century.[3]

The development and reception of trepidation models has played a relatively marginal role in standard accounts of the history of medieval astronomy, which is understandable in light of the fact that the phenomenon they were designed to explain was a fictitious one, which occupied the minds of astronomers only due to their trust in Ptolemy's oftentimes problematic observational data. An important role in overcoming this uncritical reliance on ancient authority is usually accorded to Tycho Brahe, who was allegedly the first to recognize that '[t]he variation in the rate of precession, widely accepted from the ninth to the sixteenth century, was only due to errors of observation'.[4] This picture of Brahe's critical discernment has been recently revised by Noel Swerdlow, who demonstrates that the Danish astronomer took Ptolemy's data and the existing evidence for long-term variations in the relevant parameters very seriously indeed and that his assistants Christian Longomontanus and Johannes Kepler maintained this stance in their later writings.[5] The present article seeks to complicate the received picture even further by offering a survey of instances where Latin

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[1] Ptolemy, *Almagest* 7.2–3 (trans. Toomer 1998, 327–28). For discussion, see Pannekoek (1955), Swerdlow (1979–80): 300–6, Graßhoff (1990): 73–91, Goldstein and Bowen (1991): 111–14, Evans (1998): 259–62, Pedersen (2011): 236–60. The term 'precession' of the equinoxes, to the east, was introduced by Copernicus for what was considered a motion of the sphere of the fixed stars, to the west, but it is now common to apply the term precession to both.

[2] The evidence is discussed in Jones (2010). See also Duhem (1913–59) : II, 190–204, Evans (1998): 262. Precession (with an implicit rate of ca. 1°/78 y) was accepted by the author of the second-century astronomical text in Papyrus Fouad inv. 267 A, edited and discussed by Fournet and Tihon (2014).

[3] The term *motus trepidationis* makes its first known appearance in Nicolaus Comes de Comitibus's *Tractatus de accesu et recessu motus octave spere*, written in 1450. It survives in MSS Arras, Bibliothèque municipale, 688 (748), fols. 66r–69r; Florence, Biblioteca Medicea Laurenziana, Ashburnham 208, pp. 407–17; Vatican City, Biblioteca Apostolica Vaticana, lat. 3379, fols. 1r–4v. For general orientation on the history of medieval trepidation models, see Duhem (1913–59): II, 223–66, Goldstein (1964–65), Dobrzycki (1965/2010); North (1967, 1976): III, 155–58, 238–70, Mercier (1976–77, 1996), Di Bono (1990): 23–31, Samsó (1992): 219–40, Ragep (1993): II, 400–8, Ragep (1996), Pantin (1995): 435–43, Evans (1998): 274–80.

[4] Evans (1998): 281. See most recently (Lerner et al. 2015): III, 242: 'C'est Tycho Brahe qui, le premier, a reconnu que toute cette théorie ne résultait que des défauts des observations'. Significant earlier accounts include Dreyer (1953): 276–79, 329–30, 371, Pannekoek (1961): 166, 169, 197, 215, Hartner (1971).

[5] Swerdlow (2010).

astronomers of the later Middle Ages espoused linear instead of variable precession or openly attacked existing trepidation models, thereby putting themselves in opposition to the standard computational practice of their day. In the first part of the article, I will focus on criticisms expressed in Latin sources before 1320, when discussions of this sort were still guided by the *Toledan Tables* and the theory of 'access and recess' generally, if falsely, ascribed to Thābit ibn Qura (d. 901). After 1320, the dominant computational framework changed to the *Alfonsine Tables*, which are notorious for the way they combined a form of trepidation with linear precession. The focus of the second part will rest on critical reactions to this combined model in fourteenth-century sources. I will conclude the article by briefly dealing with the work *De motu octavae spherae* (1513) by Agostino Ricci, who partook in a wider tradition of Jewish astronomers critical of nonlinear precession models and who enriched the discussion with his penetrating analysis of the causes of error in Ptolemy's *Almagest*.

2 Toledan trepidation

During the Latin Middle Ages, the central term of art used when speaking about precession models that involved a bidirectional change in stellar longitude was *accessus et recessus* (sometimes also *accessio et recessio*) *octavae spherae*—'the access and recess of the eighth sphere'. The term became entrenched mainly due to the treatise *De motu octave spere*, which was translated in the twelfth century by Gerard of Cremona from a lost Arabic text.[6] Rather than having the sphere of the fixed stars move around an axis passing through the poles of the ecliptic, the model for the access and recess of the eight sphere, shown in part in Fig. 1, uses motion in small circles located at the equinoxes. The fixed ecliptic intersects the celestial equator at the fixed or mean equinoxes, Aries and Libra, here shown for Aries as $\overline{♈}$, at the mean obliquity of the ecliptic $\bar{\varepsilon}$. The mean equinox is the centre of a small circle of radius r on which move sidereal heads of Aries and Libra of the moveable ecliptic, shown for Aries as ♈* where sidereal longitude $\lambda^* = 0°$. As these heads are carried around the small circle through the 'motion of access and recess', the argument ϑ, they take with them the moveable ecliptic and the entire sphere of the fixed stars with the sidereally fixed apsidal lines of the planets. The result of the motion around the circle is that the moveable ecliptic is shifted above and below the fixed ecliptic, as shown in the figure in two positions (below the fixed ecliptic dashed), and intersects the equator at the true, tropical equinoxes, shown for Aries as ♈ where tropical longitude $\lambda = 0°$, with the true obliquity ε, which is variable. In the direction of increasing longitude $+\lambda^*$, the moveable ecliptic meets the fixed ecliptic 90° from ♈* at ♋*, which slides along the fixed ecliptic on either side of the fixed $\overline{♋}$ by the amount of the radius r of the small circle. The perpendicular from ♈* to the equator shows the declination δ of ♈*, $+\delta$ above the equator, $-\delta$ below the equator. The 'access and recess' or 'equation of the eighth sphere' is the distance $\delta\lambda$ on the moveable ecliptic between ♈* and ♈. It

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[6] Editions of the Latin text appear in Carmody (1960): 102–13 (two versions); Millás Vallicrosa (1945)= Millás Vallicrosa (1943–50): 496–509 = Millás Vallicrosa (1960): 191–209. An English translation was published by Neugebauer (1962): 290–99.

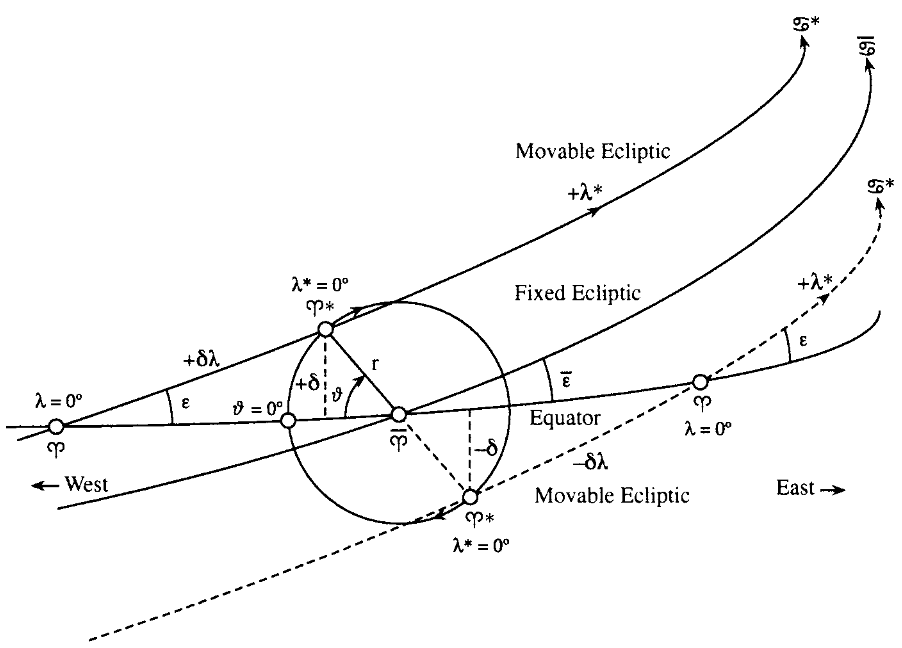

Fig. 1 (Courtesy of Noel Swerdlow)

is equivalent to the difference between sidereal longitude λ^* measured from sidereal Aries ♈* and tropical longitude λ measured from the true, tropical equinox ♈, that is, $\lambda = \lambda^* \pm \delta\lambda$; for $\vartheta < 180°$, $\lambda = \lambda^* + \delta\lambda$, for $\vartheta > 180°$, $\lambda = \lambda^* - \delta\lambda$.

The direction of motion of the true equinox ♈, determined by $\delta\lambda$, also determines the difference between the variable tropical year, return of the sun to ♈, and the constant sidereal year, return of the sun to ♈*. When $\vartheta = 0°$, the moveable ecliptic crosses the equator at ♈*, so ♈ coincides with ♈*, $\delta\lambda = 0°$, $\lambda = \lambda^*$, and the sidereal and tropical years are equal; the maximum of $\delta\lambda$ takes place at $\vartheta = 90°$. Now, considering each quadrant of ϑ: (1) For $\vartheta = (0°–90°)$, $+\delta\lambda$ increases, $\lambda > \lambda^*$, the equinox ♈ moves to the west, so the sun returns to ♈ sooner, and the tropical year is shorter than the sidereal year. (2) For $\vartheta = (90°–180°)$, $+\delta\lambda$ decreases, yet still $\lambda > \lambda^*$, the equinox ♈ moves to the east, the sun returns to ♈ later, and the tropical year is longer than the sidereal year. When $\vartheta = 180°$, the moveable ecliptic again crosses the equator at ♈*, so ♈ coincides with ♈*, $\delta\lambda = 0°$, $\lambda = \lambda^*$, and the sidereal and tropical years are again equal; the next maximum of $\delta\lambda$ takes place at $\vartheta = 270°$. (3) Continuing, for $\vartheta = (180°–270°)$, $-\delta\lambda$ increases, $\lambda < \lambda^*$, the equinox ♈ continues to move to the east and is ahead of the sidereal equinox ♈*, the sun returns to ♈ later, and the tropical year is longer than the sidereal year. (4) Finally, for $\vartheta = (270°–360°)$, $-\delta\lambda$ decreases, still $\lambda < \lambda^*$, the equinox ♈ again moves to the west but is still ahead of the sidereal equinox ♈*, the sun returns to ♈ sooner, and the tropical year is again shorter than the sidereal year. One peculiar consequence of this model is that for $\vartheta = (90°–270°)$, the equinox moves to the east, although observation has always shown it moving to the west with respect to the fixed stars, and likewise the tropical year is longer than the sidereal year, although observation has always shown it to be shorter.

The parameters of the model, which are not entirely consistent, are: mean obliquity of the ecliptic $\bar{\varepsilon} = 23;33°$, radius of the small circle $r = 4;18,43°$, maximum equation of the eighth sphere $\delta\lambda_m = 10;45°$. The argument ϑ is tabulated for epochs at intervals of 30 Arabic lunar years and for single years, from which it can be found that $\vartheta = 0;5,9,56,8^{\circ/y}$ and its period 4181;30,48... Arabic years $\approx$ 4056;55 Julian years. There are two tables of corrections for $\vartheta = (0°–90°)$ at 5° intervals, which can be used for all four quadrants: for the declination δ, called 'equation of the radius of the small circle', and for the 'access and recess' or 'equation of the eighth sphere', also called 'equation of the difference of the beginning of Aries and Libra from the equator'. The first is computed from the radius r of the small circle, not accurately, by $\delta = 4;18,43°$ $\sin\vartheta$ or $\delta = \sin^{-1}(\sin 4;18,43° \sin\vartheta)$. It can be used to find $\delta\lambda$ from $\delta\lambda = \delta/\sin\varepsilon$ or $\delta\lambda = \sin^{-1}(\sin\delta/\sin\varepsilon)$, using a table for declination from longitude inversely to find $\delta\lambda$. But this is insecure since ε is variable, any table for declination uses a single ε, and the δ just found is inaccurately computed in the table. However, it is not necessary to use this method at all since a second table gives $\delta\lambda$ directly, computed from $\delta\lambda = 10;45,0° \sin\vartheta$ or $\delta\lambda = \sin^{-1}(\sin 10;45,0° \sin\vartheta)$, although also rather inaccurately. Nevertheless, this is the only table that is necessary for the investigation of the motion of access and recess that follows.

Latin manuscripts and authors regularly attributed *De motu octave spere* and the theory espoused in it to the ninth-century Sabian astrologer-mathematician Thābit ibn Qurra, but it appears certain that the text, at least as we know it, originated in eleventh-century al-Andalus in close connection with the so-called *Toledan Tables*, which had been put together by a group of astronomers assembled by Qadi Ṣā‘id al-Andalusī (d. 1070). The *Toledan Tables* went on to enjoy their greatest dissemination in Latin Europe, where they set the standard for computational astronomy from their introduction in the twelfth to their abandonment in the early fourteenth century when they were succeeded and replaced by the *Alfonsine Tables*.[7] Among their principal hallmarks was the use of a sidereal longitude λ^*, which it made it necessary to add the 'equation of the eighth sphere' $\delta\lambda$ to find tropical longitudes, $\lambda = \lambda^* \pm \delta\lambda$. The tables for finding $\delta\lambda$ in the *Toledan Tables* are identical to those in *De motu octave spere*,[8] which explains why Latin writers associated the theory of access and recess not only with Thābit ibn Qurra, but also with the Andalusian astronomer Azarquiel, regarded as the principal author of the *Toledan Tables*.[9]

In what follows, however, I shall be less interested in the sources for this model or the history of its dissemination than in cases where Latin authors in the thirteenth and early fourteenth century commented on its failure to agree with observation. For a better understanding of the nature, scope, and quality of these criticisms, it is best to focus on the fluctuating value of $\delta\lambda$, which according to the Toledan model reached

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[7] See the monumental survey and edition in Pedersen (2002).

[8] Toomer (1968): 118–22, Pedersen (2002): IV, 1542–66. That the trepidation of model of pseudo-Thābit was designed to work with the *Toledan Tables* was conclusively demonstrated by Mercier (1996): 306–21.

[9] For two twelfth-century examples of this attribution, see Millás Vallicrosa (1936): 467–74; Magister Cunestabulus, *Computus*, c. 38 (ed. Lohr 2015: 110).

0° on 14 November AD 604. For it to reach its respective maximum or minimum of ±10;45°, ϑ must increase from 0° to ±90°, which will take 4057/4 ≈ 1014 Julian years. It follows that $\delta\lambda = -10;45°$ in 411 BC and $+10;45°$ in AD 1618. The key to this choice of parameters is hidden in the historical stellar longitudes medieval astronomers were able to find in two especially authoritative works of mathematical astronomy: Ptolemy's *Almagest* and al-Battānī's *Zīj al-Sabi'*, a Latin translation of which, called *De scientia stellarum*, was completed between 1134 and 1138 by Plato of Tivoli. Ptolemy had compared stellar longitudes transmitted by Hipparchus 265 years before his time with his own observations, which seemed to exhibit a uniform increase by 2;40°, at a rate of nearly 1° in 100 years. Al-Battānī, in AD 879/880, proceeded in a similar fashion, but instead of an equivalently slow change, he found that longitudes had increased by 11;50° using β Sco in the 782 years that separated him from the Roman mathematician Menelaus, whose observations were quoted in the *Almagest* (7.3), and by 11;30° using α Leo in the 741 years since Ptolemy. The approximate rate of precession supported by these observations was 1° in 66 years, a non-Ptolemaic value that had already been introduced into Arabic astronomy in the first half of the ninth century.[10] For a rough idea of the success of the Toledan access and recess model at describing this apparent speeding up of precession, it will suffice to compare the increase in longitude $\Delta\lambda$ of α Leo, as observed by Hipparchus (129/28 BC) and Ptolemy (AD 139),[11] and by Ptolemy and al-Battānī (AD 879/80), with the corresponding change in stellar longitude $\Delta\lambda$ using the differences in $\delta\lambda$ at the date of each observation computed from the *Toledan Tables* and *De motu octave spere*. From the table below, it can be seen that there is an extremely close agreement between observed and Toledan $\Delta\lambda$ for the intervals between Hipparchus, Ptolemy, and al-Battānī. This effectively means that the stellar observations recorded by these three astronomers could not be used to falsify or undermine the access and recess model.[12]

	α Leo	Observed $\Delta\lambda$	Toledan $\Delta\lambda$
Hipparchus	119;50°	–	–
Ptolemy	122;30°	2;40°	2;39°
al-Battānī	134°	11;30°	11;30°

In order to obtain evidence against trepidation, medieval astronomers needed to compare ancient observations with more recent ones, in particular with stellar longitudes verifiable for their own time. If the observed change in longitude conflicted, beyond a reasonable margin of error, with the Toledan $\Delta\lambda$ for the dates of the observations, the model could be regarded as invalidated. That this was much more easily said

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[10] al-Battānī, *Opus astronomicum*, c. 51 (ed. Nallino 1903: 124–26). The following chapter 52 contains an account of trepidation, which is translated and studied in Ragep (1996). Tables for stellar precession based on al-Battānī survive in MSS Cambridge, Fitzwilliam Museum, McClean 165, fol. 79v; Cambridge, Gonville & Caius College, 456, p. 139. The wider history of the 1°/66 year-precession parameter is discussed in Kurtik (1990), Mozaffari (2016): 303–7.

[11] For the observations by Hipparchus and Ptolemy, see Ptolemy, *Almagest* 7.2 (trans. Toomer 1998: 328).

[12] For the Toledan $\Delta\lambda$ cited in this table, I have used the $\delta\lambda$-values for 1 Jan 127, 1 Jan 139, and 1 Jan 880, as predicted by the simulation of the *Toledan Tables* in Raymond Mercier's software *Deviations*, which is available for download at http://www.raymondm.co.uk.

than done becomes clear if one takes into account that the most common way of establishing the ecliptic longitude of a given reference star, that of measuring its distance from the moon, would have required computing the current solar longitude. If such solar longitudes were derived from the *Toledan Tables*, by computing $\lambda = \lambda^* \pm \delta\lambda$, the measurement would include any error the access and recess model might entail at the time of the observation. Attempts to check the model for accuracy were hence threatened by circular reasoning.

A case that clearly speaks to the difficulty in refuting pseudo-Thābit in the mid-thirteenth century is offered by John of London, a mathematician and astronomer active in Oxford and Paris, who in 1246 created a list of observed longitudes and latitudes of 40 bright stars. John also alluded to these observations in letter addressed to his *amantissimus magister, R. de Guedingue*, who had previously sent him a mélange of questions on points of astrological interest.[13] In response to the first of these questions, John came to discuss the 'great deception' (*grande mendacium*) inherent in the theory of access and recess attributed to Thābit:

> For according to his opinion the fixed stars do not move forward beyond 10 degrees and 45 minutes and revert after [having reached] them. Yet according to what I have often observed, they have already progressed by roughly 16 degrees since Ptolemy's time and from the time of Hipparchus to the time of Ptolemy, as the latter says in the second and third chapter of the seventh book of the *Almagest*, they progressed by about 3 degrees, and so they progressed by roughly 19 degrees from the time of Hipparchus up to now. In this way it is manifestly demonstrated that Thābit's positions is contrary to the facts [*inconveniens*].[14]

Far from refuting the access and recess of the fixed stars, John's criticism reveals that he had not grasped pseudo-Thābit model's as well as he may have believed. Although it is true that the maximum equation $\delta\lambda$ in this model was 10;45°, John failed to realize that this difference can increase in both directions, for an amplitude of 21;30°. The observed $\Delta\lambda$ of 19° since Hipparchus's time was thus still within the boundaries allowed by the theory. From the Toledan Tables for the equation of the eighth sphere, one would have been able to obtain $\delta\lambda = -9;52°$ for 150 BC, the approximate time of Hipparchus's observations, and $\delta\lambda = +8;59°$ for the start of AD 1246. The resulting $\Delta\lambda$ of 18;51° comes close to the 19° John had found for the same period, which explains why he conceded that the 'tables of Thābit' had not yet begun to deviate

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[13] The letter is preserved in MSS Paris, Bibliothèque nationale de France, lat. 7413(II), fols. 19v–21ra; Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1340, fols. 84rb–85rb. A deeply deficient transcription of the text in the Paris MS appears in Fontès (1897–98): 146–60. On John, his letter, and the star table, see Poulle (1956): 313–16, Kunitzsch (1966): 39–46, Kunitzsch (1986): 51–57, Knorr (1990): 309–10, 320–21, 327.

[14] MS Paris, Bibliothèque nationale de France, lat. 7413(II), fol. 19va: '[S]ecundum opinionem eius non procedunt stelle fixe ultra 10 gradus et 45 minuta et post ea revertuntur. Sed secundum quod ego frequenter consideravi iam processerunt a tempore Ptholomei quasi 16 gradus et a tempore Abrachis usque ad tempore Ptholomei, sicut ipse dicit capitulo 2° et 3° septimi libri Almagesti, processerunt fere per 3 gradus et ita a tempore Abrachis usque modo processerunt quasi 19 gradus. Quo manifeste probatur quod positio Thesbith est inconveniens'. John's claim that the stars have progressed by 16° agrees in a roundabout way with his star list for AD 1246, which adds on average 15;17° to the longitudes recorded in the *Almagest*. See Poulle (1956): 315.

dramatically from reality. Still, 'after many years there will be a great falsity'. In order to find the correct rate of precession, which to his mind appears to have been linear, he recommended dividing the number of years between Hipparchus and AD 1246, the year of his stellar observations, but no concrete result are spelled out in the letter to *R. de Guedingue.*[15] (Accepting $\Delta\lambda = 19°$ and 150 BC as the approximate year of Hipparchus's observations, the average precession over the whole period would have been ca. 1°/73.5 years).

Although John's overall demonstration must be ranked as a failure, his method of testing the Toledan access and recess model was not without potential. If the observed change in stellar longitude since antiquity exceeded the maximum $\Delta\lambda$ allowed for by its parameters, the model could indeed be considered refuted. An added benefit of this approach was that it could offer support, not just for rejecting the specific model in the *Toledan Tables*, but for parting ways with theories of access and recess altogether. In short: the greater the distance in ecliptic longitude a star could be shown to have moved, the less plausible it became that it ever moved in the opposite direction. In order to make this approach work, however, astronomers had to compare observations made over a great interval of time, greater indeed than afforded by the data in the *Almagest*, which knew of no stellar observations earlier than those made by Timocharis and Aristyllus ca. 300 BC.

An effort to get to the bottom of things by comparing the available historical data was made in the second half of the thirteenth century by the Franciscan Bernard of Verdun, whose *Tractatus super totam astrologiam* listed the longitudes of α Leo (Regulus) and the solar apogee as recorded by Hipparchus, Ptolemy, Thābit, al-Battānī, and Azarquiel. For α Leo, the total change in longitude was only 18° (Hipparchus's 119;40° vs. Azarquiel's 137;40°) and hence within the limits permitted by the Toledan model, which according to Bernard allowed for a maximum $\Delta\lambda$ of 20° (rather than 21;30°). The solar apogee, which appeared to have moved 21;19° between the time of Hipparchus (65;30°) and Azarquiel (85;49°), exceeded 20°, but its larger $\Delta\lambda$ compared to α Leo also contradicted the notion, implicit in the Toledan model, that the solar apogee moved at the same rate as the fixed stars. While Bernard still accepted that the rate had varied over time, his remarks show that he had grown sceptical that the model associated with pseudo-Thābit could correctly account for these variations.[16] Another attack on this model was launched in the early 1290s by the Parisian astronomer John of Sicily, who wrote a lengthy commentary on the canons to the *Toledan Tables*. In John's view, the postulated access and recess of the eighth sphere violated the Aristotelian principle of uniform circular motion, which made him favour Ptolemy's description of precession as continuous and uniform. How swift this motion was, John did not

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[15] MS Paris, Bibliothèque nationale de France, lat. 7413(II), fol. 19v: 'Tamen, si consideremus motum octave spere per tabulas Thebith a tempore inicii annorum Arabum usque modo, non fiet inde magna diversitas, sed post multos annos erit falsitas sensibilis. [...] Dividatur numerus annorum a consideratione Abrachis usque ad considerationem meam, que fuit anno Christi 1246 Parisius, et iam poterit sciri in quot annis stelle fixe moveantur uno gradu. Et ita scietur numerus motus octave spere'.

[16] Bernard of Verdun, *Tractatus super totam astrologiam* 6.2–4 (ed. Hartmann 1961: 119–20), Duhem (1913–59): III, 458–59. Bernard's source for the solar apogee of Azarquiel appears to have been a lost Latin translation of Azarquiel's work on solar theory, in which he demonstrated the apogee's proper motion. See Toomer (1987).

dare to say, adding that one would need 'many and reliable experiences' to establish its true rate.[17]

It is probably not without significance that John and Bernard tackled the question of precession at a time when the Toledan access and recess model had begun to predict a notable decrease in the growth of $\delta\lambda$, which was going to lead to a reversal of direction by the early seventeenth century. The result was a widening predictive error, which manifested itself most clearly in the tropical solar longitudes λ the *Toledan Tables* yielded once $\delta\lambda$ was added to λ^*, the sidereal *motus* of the sun ($\lambda = \lambda^* + \delta\lambda$).[18] In contrast to the approaches mentioned thus far, these tropical solar longitudes could be tested without recourse to ancient observations. The time of the vernal equinox, for instance, could be measured independently by observing the sun's meridian altitude at dates before and/or after the expected equinox and then interpolating for the date and time at which the altitude was equal to the local colatitude. If the resulting $\lambda = 0°$ at the time in question was compared to the sun's sidereal longitude λ^*, as computed from the *Toledan Tables*, the difference $\lambda - \lambda^* = \delta\lambda$ could be used to test the value of $\delta\lambda$ predicted by pseudo-Thābit's model.[19]

A rough sketch of how to measure $\delta\lambda$ empirically in this way was included in John of Sicily's commentary as an addendum to his discussion of precession, where he mentions 41;12° as the equinoctial altitude to be used in the calculation.[20] A slightly different value of 41;10° for the colatitude of Paris was established around the same time by the philosopher and theologian Peter of Limoges (d. 1306), who reported on his astronomical activities in an autograph note contained in a copy of the third book of Giles of Lessines's *Summa de temporibus*.[21] Peter's handwritten testimony reveals that he had been engaged in solar observations since the early 1280s, partly in collaboration with a man named Franco. The latter may perhaps be identified with Franco of Poland (Franco de Polonia), who is otherwise known for his treatise on

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[17] *Scriptum Johannis de Sicilia super canones Azarchelis de tabulis Toletanis*, ed. Pedersen (1986): 256 (J542): 'De quantitate vero motus, utrum scilicet in 100 annis aut in maiori minorive tempore pertranseat gradum unum, non possumus per rationem asserere, sed oportet multis et certis experientiis hoc probare'. For further discussion, see Duhem (1913–59): IV, 6–10. Ptolemy's precession rate of 1°/100 years, without any hint of trepidation, is still endorsed in the *Liber theoreumacie*, written in Strasbourg in 1214. See, for example, MS Munich, Bayerische Staatsbibliothek, Clm 14684, fols. 52v–70r, at fol. 69r.

[18] The deficiencies of the *Toledan Tables* are analysed in Gingerich and Welther (1977). By the early fifteenth century, North African astronomers had experienced analogous problems with the trepidation model they had inherited from the school of Azarquiel. See Samsó (2001): 169–74, Comes (2002): 130–34.

[19] One flaw of this method is that it does not allow to discern if any mismatch between observation and prediction is due to an error in computed $\delta\lambda$, an error in computed λ^*, or a combination of both. Medieval authors, as we shall see, generally accepted the Toledan λ^* and instead imputed the error to $\delta\lambda$. This reflects the fact that it was the access and recess model, and not the sidereal parameters of the *Toledan Tables*, that provoked the greatest doubts.

[20] *Scriptum Johannis de Sicilia super canones Azarchelis de tabulis Toletanis*, ed. Pedersen (1986): 257 (J544). See also Pedersen (1984): 175.

[21] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fols. 51r–174v. The note, which appears on fol. 139r, was transcribed by Delorme (1936): 559. For the identification of the annotator's hand as that of Peter of Limoges (rather than William of Saint-Cloud), see Hamesse (1994): 192–93, Vescovini (1992): 402–3.

the turquet (ca. 1284).[22] Together, the two men measured the sun's declination at the solstices in order to obtain the value of the obliquity of the ecliptic, using as their principal tool a large quadrant with a scale divided at intervals of 5 minutes.[23] From the length of the noon shadow on Saturday, 11 March 1290, Peter of Limoges inferred a current solar altitude of 40;30°, which indicated that the true equinox was going to occur 40 hours later, since at the equinox the declination changes by 0;24° per day or 0;1° per hour, on 12 March, 16 hours after noon, with an implied $\delta\lambda$ of 10;13° (the computed sidereal longitude at the equinox being $\lambda^* = 349;47°$). Yet, rather than revealing whether this $\delta\lambda$ confirmed or contradicted $\delta\lambda$ computed by the Toledan trepidation model, Peter ended on a cautious note: 'I cannot draw any precise conclusions from this observation until I have made several further observations, not just of equinoxes, but also of solstices'.[24] Less reticence in this regard was displayed by Peter's contemporary William of Saint-Cloud, another astronomer active in Paris, whose observations, recorded in the introduction to his *Almanach planetarum*, made him find the exact same date and time of the vernal equinox in 1290 and, hence, the same value of $\delta\lambda = 10;13°$. William went on to calculate $\delta\lambda$ by the *Toledan Tables*, which he used in a version adapted to the meridian of Toulouse. It turned out that the Toledan $\delta\lambda = 9;23°$ was 0;50° below the observed value.[25]

William's reaction to this discrepancy is revealing. Rather than considering a modification of the Toledan sidereal longitude λ^*, which would have closed the gap, he identified $\delta\lambda$ of the access and recess model as the most obvious source of 'shrinkage or error'. His solution was to return to a simpler conception of precession, which described the eighth sphere as moving 'evenly from west to east in accordance with the motion of the planets'. When it came to the rate of this motion, William seemed ready to endorse not Ptolemy's 1°/100 years or al-Battānī's 1°/66 years, but rather a more accurate value of 1°/70 years upheld by the so-called *magistri probationum*, who according to the Parisian astronomer 'had a longer period of time at their disposal than Ptolemy had had, and so they were in a better position to assess the truth'.[26] His appeal to these *magistri probationum* reveals the influence of the eminent Jewish scholar Abraham Ibn Ezra, more specifically of his *Liber de rationibus tabularum*, which Ibn Ezra wrote in 1154 to supplement a set of astronomical tables he had com-

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[22] This identification was already suggested by Vescovini (1992): 403. Franco appears to have left further traces of his activity in MS Paris, Bibliothèque nationale de France, lat. 16211, fol. 10va, cit. in Pedersen (2002): I, 167. See also ibid.: II, 336, and Thorndike (1945).

[23] MS Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1191, fol. 139r: 'Hec fecimus cum magno quadrante cuius latera erant de ligno et limbus de latone divisus usque ad 5 minuta'.

[24] Ibid.: 'Nichil precise per hanc observationem possum asserere, quousque plures alias fecerim observationes, non solum in equinoctiis, set etiam in solstitiis'.

[25] See William of Saint-Cloud, *Almanach planetarum* 2–5 (ed. Pedersen 2014: 8–10). The pertinent passage is also printed in Delorme (1936): 560–61. See the comments in Duhem (1913–59): IV, 16–17, Millás Vallicrosa (1943–50): 393–94, Harper (1966): 41–43, Mercier (1976–77), pt. 1: 201–4, Chabás and Goldstein (2003b): 262–64, Hugonnard-Roche (1996): 296–97, Lejbowicz (1997): 209–10. One should note that William used different values for the sun's solstitial noon altitudes than Peter of Limoges and that he established the equinox directly from the sun's noon altitude on 12 March rather than from its noon shadow on 11 March. There is hence a strong likelihood that his result was arrived at independently.

[26] William of Saint-Cloud, *Almanach planetarum* 6 (trans. Pedersen 2014: 34).

puted for the meridian of Pisa.[27] At the base of these tables, which also existed in a version for the meridian of London, was the lost tenth-century *zīj* of al-Ṣūfī, which had been brought to Italy from Antioch before 1143.[28] In the context of the accompanying *Liber*, the expression *magistri probationum*, which is a literal translation of *aṣḥāb al-mumtaḥan* ('masters of examination'), served as a generic label for Arabic astronomers who constructed tables based on their own observations.[29]

The parameters Ibn Ezra attributes to these astronomers sometimes vary in rather confusing ways. Thus, he mentions a larger group of *magistri probationum* including al-Battānī, al-Ṣūfī, and Azarquiel, of which one part upheld a tropical year of $365\frac{1}{4} - \frac{1}{106}$ days, as was demonstrably the case with al-Battānī, whereas the remainder opted for the longer (and more accurate) $365\frac{1}{4} - \frac{1}{130}$ days.[30] In another passage, Ibn Ezra again mentions al-Ṣūfī and Azarquiel, as well as Ibn Yūnus, and asserts that these *magistri probationum* believed the tropical year to contain 365 days + a revolution of 87;21°.[31] This is equivalent to $365^d5;49,24^h$ or very nearly $365\frac{1}{4} - \frac{1}{136}$ days, which Ibn Ezra later goes on to cite as underlying the *Pisan Tables*.[32] The preserved Tables for Pisa and London appear to tell a slightly different story, in that they attribute to the mean sun 20 full revolutions +0;9° in 20 Julian years, which corresponds to a tropical year of very nearly $365\frac{1}{4} - \frac{1}{131}$ days or $365^d5;49^h$.[33] An anonymous mid-twelfth-century treatise closely connected to the *Pisan Tables* makes these values explicit, claiming that al-Ṣūfī's findings about the length of the tropical year were later confirmed by Azarquiel.[34]

There are some intriguing signs that the constant tropical year implicit in the *Pisan Tables* enjoyed a resurgence in Paris in the second half of the thirteenth century, when dissatisfaction with the *Toledan Tables* and their access and recess model was on the increase. Examples include the Dominican philosopher Giles of Lessines, who attributed the tropical year of $365\frac{1}{4} - \frac{1}{131}$ days to al-Ṣūfī, albeit without taking any particular stance on the theory of precession.[35] The same parameter was used by

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[27] See the edition in Millás Vallicrosa (1947), and the analysis in Samsó (2012).

[28] Mercier (2004), Burnett (2003): 36–42, Pedersen (2007).

[29] The original *aṣḥāb al-mumtaḥan* were credited with constructing the *zīj al-Ma'mūnī li'l Mumtaḥan* under the reign of Caliph al-Ma'mūn (ca. 830). See Vernet (1956), Samsó (2012): 193–95.

[30] Millás Vallicrosa (1947): 76.

[31] Ibid.: 83.

[32] Ibid.: 95–96. The edition (p. 95, ll. 13–14) has $365\frac{1}{4} - \frac{1}{106}$ days, in line with the wording in most manuscripts, but $365\frac{1}{4} - \frac{1}{136}$ days appears to be the authentic reading. See Toomer (1969): 317–19. The canons to the *Toledan Tables* in MS Paris, Bibliothèque nationale de France, lat. 7411, fol. 18v, attribute to 'Habraham Iudeus' a solar year of $365\frac{1}{4} - \frac{1}{126}$ days, which may be an error for $365\frac{1}{4} - \frac{1}{136}$ days.

[33] This value receives some support from the testimony of Ibn Yūnus, who attributes to al-Ṣūfī the view that the sun moves 11s 29;45,40,2° in 365d, implying a year of $365\frac{1}{4} - \frac{1}{131}$ days = $365^d5;49^h$. See Mercier (2004): 1, 14, 22.

[34] MS Cambridge, Fitzwilliam Museum, McClean 165, fols. 67r–76v, at fols. 68r–69r. The *annus praesens* is indicated to be 1145 on fol. 70r ('et incipe a quoto anno volueris, utpote a 1144 completis ante presentem annum').

[35] See Giles of Lessines, *Summa de temporibus*, bk. III, 1.3, 1.7, 1.12, 3.4 (ed. Steele 1926: 14, 32, 56, 159). Giles's views on precession are discussed in Nothaft (2014). One may also cite Roger Bacon, *Opus tertium*, c. 68 (ed. Brewer 1859: 275), for a tropical year of $365\frac{1}{4} - \frac{1}{130}$ days.

William of Saint-Cloud in a set of tables for finding the month, day, and hour of the sun's entry into each zodiacal sign for the bissextile cycle from 1296 to 1299. For years outside this range, a supplementary table taught that the times listed had to be decreased in steps that amounted to 11 hours in the course of 60 years, or an average of 11 minutes per year, implying a tropical year of $365^{d}\ 5;49^{h}$, or $365\frac{1}{4} - \frac{1}{131}$ days.[36] Precisely the same numbers are endorsed in a set of anonymous notes from the end of the thirteenth century, whose author prefaced them by noting his conclusion that 'the opinion of *Alzophi* on the length of the solar year is closest to the truth'.[37] Like William of Saint-Cloud, he evidently worked in Paris, for which he had established a local colatitude of 41;12° (same as John of Siciliy's) and corresponding latitude of 48;48°, based on an observed solstitial noon altitude of 64;46° and an obliquity of the eliptic of 23;34° (64;46°–23;34° = 41;12°). He also predicted that the sun was going to enter (tropical) Aries in 1293 on 12 March, 9 h after noon, which is roughly consistent with the observations carried out by William of Saint-Cloud and Peter of Limoges in 1290, if the slip of 11 min per year is applied to the vernal equinox on 12 March, 16 h after noon.[38]

Acceptance of a fixed parameter for the tropical year naturally also had implications for the preferred model and rate of precession. Ibn Ezra, for instance, claimed that the fixed stars perform a full revolution in 24,000 years, which corresponds to a rate of 54 seconds per year or 1° in 66 2/3 years.[39] Roughly the same value is implicit in a set of tables showing the longitude of the solar apogee for the meridian of London, which circulated alongside texts and tables connected to Ibn Ezra's *Pisan Tables*. The sun's apsidal motion is here indicated to be 0;18° for every 20 years, in line with an annual shift of 54 seconds.[40] This choice of parameters would seem to be in rough agreement with a star catalogue al-Ṣūfī had created in AD 964, which added 1° in stellar longitude for every 66 years since Ptolemy (as opposed to 1° in 66 2/3 years).[41] Rather than mentioning this precedent, Ibn Ezra linked al-Ṣūfī and other *magistri probationum* to

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[36] The tables originated in William of Saint-Cloud, *Kalendarium regine*, c. 11–12 (ed. Harper 1966: 124–34). A stand-alone version appears in MS Los Angeles, J. Paul Getty Museum, Ludwig XII.6, fols. 13v–14r. Only the supplementary table appears (together with a canon) in MS Cambridge, University Library, Gg.VI.3, fol. 129r.

[37] MS London, British Library, Harley 3735, fol. 73ra: 'Et puto quod opinio Alzophi de quantitate anni solaris sit propinquior veritati. Posuit autem quod annus contineat 365 dies et quartam diei minus 131a parte diei, ita quod secundum ipsum debent solsticia et equinoctia in kalendario signata in 131 annis retrocedere per unum diem integrum. Est autem 131a pars diei 11 minuta hore, ita quod annus secundum hoc continet 365 dies et 5 horas et 49 minuta hora'. See Thorndike (1959): 157–58. The manuscript is available online at http://www.bl.uk/manuscripts/Viewer.aspx?ref=harley_ms_3735_f073r.

[38] MS London, British Library, Harley 3735, fol. 73ra. Assuming a tropical year of $365^{d}5;49^{h}$, if the sun enters tropical Aries in 1290 on 12 March, 16:00 h, it will do so in 1293 on 12 March, 09:27 h (= 16:00–6:33 h). The manuscript also records entry times for Cancer, Libra, and Capricorn, which together with Aries collectively fall one hour later than those recorded for 1297 in William of Saint-Cloud's solar table (ed. Harper 1966: 129). This is again consistent with a subtraction of 11 min per year, followed by rounding.

[39] Millás Vallicrosa (1947): 94.

[40] MSS London, British Library, Arundel 377, fol. 9v; Oxford, Bodleian Library, Ashmole 361, fol. 27v; Oxford, Bodleian Library, Savile 21, fol. 82r; Paris, Bibliothèque nationale de France, lat. 16208, fol. 4v; Toledo, Archivo y Biblioteca Capitulares, 98–22, fol. 104v. See Pedersen (2007): 6.

[41] Samsó and Comes (1988).

a precession of 1°/70 years and a sidereal years of 365 days + a revolution of 92;30° $(= 365^{d}\ 6;10^{h})$,[42] which would come close to the tropical year of $365\frac{1}{4} - \frac{1}{131}$ days actually implied by the *Pisan Tables*. A precession parameter of 51 seconds per year or ca. 1°/70.6 years was accessible to Latin astronomers in the form of solar apogee tables, which can still be found in a handful of twelfth- and thirteenth-century manuscripts.[43] Among the known followers of this minority tradition was the author of a geometrical text known as *Artis cuiuslibet consummatio*, which teaches how to apply the precession of 51 seconds per year to the locations of the planetary apogees in the present year AD 1193.[44]

A likeminded preference for 1°/70 years is suggested by the remarks William of Saint-Cloud made on the motion of the eighth sphere a century later, but this is belied by the tabulated values of William's *Almanach planetarum*, which show an increase in $\delta\lambda$ from 10;13° in AD 1290 to 10;32° in AD 1311—0;19° in 21 years, which comes closer to al-Battānī's rate of 1°/66 years. William's principle of adding a minute of arc for nearly every year appears to have been followed by his contemporary Peter of Saint-Omer, whose treatise on the *Quadrans*, written in 1293, gives $\delta\lambda$ as 10;14°. The most plausible explanation for this state of affairs is that Peter picked up William's initial value of $\delta\lambda = 10;13°$, but misapplied it to 1292, the starting year of the *Almanach planetarum*, while adding 0;1° to adjust it to 1293.[45] An analogous explanation would suffice to account for a note found in a manuscript of Flemish origin, now at the British Library, which expressly gives the annual increase in $\delta\lambda$ as 0;1° and the value for AD 1296 as 10;17°, thus an increase of 0;4° over the four years since the start of the *Almanach planetarum*.[46]

Outside Paris, a slower and more accurate linear precession of 1°/70 years was endorsed by Walter Odington, monk of Evesham Abbey in Worcestershire, who in ca. 1301 created an almanac mentioning the coordinates of Evesham.[47] Its supplementary material included a table recording the coordinates of 35 stars together with a subtable instructing users to add a precessional increment of roughly 51 seconds per year. Odington justified this choice by citing various past longitudes of α Tau (Aldebaran) as well as his own determination for present times made 'via observation with the turquet and through computation'.[48] These observations and computations had convinced him

Odington, che scrive la *Declaratio motus octavae spherae*, è anche l'autore del *De speculatione musices*

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[42] Millás Vallicrosa (1947): 78, 83, Samsó (2012): 181.

[43] See, for example, MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 92v, and the documentation provided in Pedersen (2002): III, 1226–27.

[44] *Artis cuiuslibet consummatio* 2.26 (ed. Victor 1979: 282–85). Knorr (1990: 318–21) argues that the *Artis* was written by the astronomer John of London, making him read 1193 not as the year of composition, but only as the date of the parameters cited. Yet the table in MS Paris, Bibliothèque nationale de France, lat. 16208, fol. 92v (s. $\text{XII}^{\text{ex.}}$), shows that the 51″/year-value already circulated prior to AD 1246.

[45] See Petrus de S. Audomaro, *Quadrans 1* (ed. Pedersen (1983–84): II, 616, 618), as well as Pedersen (1983–84): I, 42–43, 489; II, 580–82, 666, 699, Poulle (1980): I, 209, 264–65.

[46] MS London, British Library, Harley 267, fol. 217v. See Pedersen (2002): I, 128.

[47] MS Cambridge, University Library, Ii.I.13, fols. 160r–70r (almanac), 170r–76r (calendar), 176v–81v (appended material). On Odington and the contents of this manuscript, see North (1976): III, 238–52; Nothaft (2016b).

[48] MS Cambridge, University Library, Ii.I.13, fol. 179v (star table), fols. 180r–v (text). See n. 51 below for the Latin quote.

that α Tau was currently located at 29° in the tropical sign of Taurus, 16;20° ahead of the longitude in the *Almagest* (12;40°). Over the ca. 1301–138 = 1163 years that separated him from Ptolemy's catalogue, $\Delta\lambda = 16;20°$ implied a change of 1°/71.2 years. This is in close, but not in perfect agreement with the parameter Odington used in his own star table, perhaps because he made his observations at an earlier date than 1301.[49] Although he failed to assign any dates to the other observations he cited, Odington's historical data clearly showed a continuous growth in the longitude of the reference star in question, which appeared to be traceable back as far as the time of the mythical Hermes, who supposedly observed α Tau at 25;17° Aries.[50] An acceptance of this datum urged the conclusion that the stellar longitudes had changed by at least 33° since earliest antiquity, which clearly contradicted the Toledan access and recess model. Odington, for his part, arrived at the belief that the motion of the fixed stars was 'always progressive and that they have no contrary motion', noting that 'observation by instruments' favoured the ancients over Thābit and Azarquiel.[51]

A second English astronomer likely to have taken such a stance is Richard of Wallingford, who added a star table for the year 1327 to his treatise on the *Albion*. Here, the increase relative to Ptolemy's longitudes is a full 17°, which is almost exactly 1° for every 70 years contained in the interval between AD 138 and 1327. The Toledan access and recess model, by contrast, would have justified no more than $\Delta\lambda = 16;45°$.[52] Considering what has been said before about the tropical year that contemporaries ascribed to al-Ṣūfī, it is also striking to find that one of Richard's earlier works, the *Exafrenon pronosticacionum temporis* (after 1316), included exactly the same set of solar tables for the years 1296–1299 devised by William of Saint-Cloud, even though they are here claimed to apply to the meridian of London.[53]

An even better constant of precession was found in 1306 by an investigator who may have been identical to Johannes de Luna, a German astrologer active in Bologna whom Robert de Mauvoisin, the later archbishop of Aix, consulted in ca. 1305 (as he himself admitted to the Inquisition in 1317).[54] The name *Magister Iohannes de Luna*

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[49] As already noted by North (1976): III, 240–41, at a rate of 1°/70 years, $\Delta\lambda = 16;20°$ would have been reached after 1143 1/3 years, pointing to ca. 1282.

[50] Odington here probably relied on the *Liber Hermetis de quindecim stellis ... et quindecim imaginibus* (ed. Delatte 1942: 246): '[C]um aspicerem Aldebaran, vidi eam in 15° gradu Arieties et 27 minutis et cum eodem capiebat medium caelum'. See Thorndike (1947): 224–27. None of the manuscripts inspected by me contains the 25;17° Odington ascribes to Hermes, although some come close, e.g. 25;7° in MS Vienna, Österreichische Nationalbibliothek, 5311, fol. 38ra; 25;27° in MS London, British Library, Royal 12.C.XVIII, fol. 31r; 25;37° in MS London, British Library, Royal 12.E.XXV, fol. 161r.

[51] MS Cambridge, University Library, Ii.I.13, fol. 180r: 'Et nos per observacionem turketi et per compotum invenimus eam habere 29 gradus Tauri complete. Unde motus fixarum et augium planetarum est in 70 annis unus gradus, ut patet in brevi tabula premissa. Unde patet quod motus fixarum est semper progressivus et quod non est aliquid motus earum reductivus, quapropter iudicia referenda sunt ad motum earum, hoc est ad progressionem. Quare non magis debemus inniti radicibus Tebit vel Arzachel, qui ponunt ad aliquot gradus sperem [!] reductionem, quam radicibus antiquorum, cum experiencia instrumentorum agat pro antiquis in progressione spere'.

[52] On this table, see North (1967): 83, North (1976): I, 398–99 (edition and translation of the *Tractatus Albonis*); II, 198, 242–44 (commentary); III, 87 (table).

[53] North (1976): I, 186–91 (edition of the *Exafrenon*); II, 100–2 (commentary).

[54] See Thorndike (1965), Shatzmiller (1999): 143–44, 175–76.

Theutonicus appears at the start of an intriguing record of solar and stellar observations carried out in Bologna, Montpellier and Genoa between 1305 and 1312.[55] Among the findings recorded here is one concerning the longitude of α Vir (Spica), which was 12;50° Libra in 1306, whereas Ptolemy, in around 140, had located the same star in 26;40° Virgo. The investigator took the implied $\Delta\lambda = 16;10°$ over the relevant time interval of 1305 − 140 = 1165 years to infer that the longitude since Ptolemy's day had grown at an average rate only a little above 1°/72 years. This, in turn, was tantamount to saying that the fixed stars moved 0;10° for every 12 years, 'if their motion is uniform'.[56] As the author of this passage realized, it was possible to support this contention by going back to Ptolemy's *Almagest*, which reported that Timocharis had found α Vir at 22;20° Virgo in the 36th year of the First Kallippic Cycle [= 294 BC], but at 22;30° 12 years later.[57] Not only were these ancient observations in apparent agreement with the findings made in AD 1306, but their joint implication was that the rate of precession had not changed since antiquity.

Such findings no doubt added weight to Walter Odington's view that precession was 'always progressive', but the question remained how this view could be reconciled with the data recorded by Ptolemy and al-Battānī, which seemed to document a speeding up. A semblance of an answer had been given by Abraham Ibn Ezra, who in his *Liber de rationibus tabularum* briefly pondered the role imperfect measurements might have played in producing recalcitrant data. In order to drive this point home, he used the example of the Banū Šākir, two brothers who had made simultaneous observations using the exact same type of astrolabe, but arrived at divergent results.[58] Similar thoughts were expressed one and a half centuries later by the aforementioned John of Sicily, who pointed out that the study of precession by necessity involved the use of observations made over several centuries, some of which might be tainted by human error. The likely culprit according to him was not Ptolemy, but the first observers on

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[55] See the record in MS Vienna, Österreichische Nationalbibliothek, 5311, fol. 137r, which was previously mentioned by Thorndike (1965): 207, and Goldstein (1994): 196–97 (n. 39). The first entry begins 'Magister Iohannes de Luna Theutonicus dixit se expertum', whereas all following observations are stated in the first person. This may point to a different author, although Johannes de Luna's name reappears immediately above the paragraph dealing with precession cited below. He is also mentioned on fol. 134v of the same manuscript, as having made corrections to the mean motion parameters in the *Toledan Tables*.

[56] MS Vienna, Österreichische Nationalbibliothek, 5311, fol. 137ra: 'Inveni itaque per rationem stellam Alaazel esse in Libra 12 gradibus et 50 minutis in annis Christi perfectis 1305. Subtraxi itaque de dictis gradibus et minutis locum eius tempore Ptolomei, scilicet 5 signa, 26 gradus et 40 minuta, et remansit motus eius a tempore Ptolomeus usque nunc, et sunt 16 gradus et 10 minuta. Et subtraxi a predicto tempore Christi quod nunc annos Christi tempore Ptolomei in quo fecit considerationes locorum et Alaazel et aliarum stellarum fixarum tabulatarum in Almagesti, qui erant 140, et remanent [...] anni 1165, qui anni ab illa consideratione Ptolomei usque nunc, quos si diviseris per 16 gradus et 30 [!] minuta, provenient 72 et parum plus qui sunt anni correspondentes motui unius gradus fere. Sed secundum hoc in omnibus 12 annis moventur stelle fixe 10 minutiis a tempore Ptolomei, si motus ipsarum est equalis'.

[57] Ibid., fol. 137rb: 'Et hoc videtur convenire cum duplici consideratione quam fecit Tymocarides de Spica, id est Alaazel, quia in anno 36 prime revolutionis Pheliz reperit eam in Virgine 22 gradu et 20 minutis et in anno 48 eiusdem revolutionis in 22 gradu et 30 minutis Virginis'. See Ptolemy, *Almagest* 7.3 (trans. Toomer 1998: 335–36).

[58] Millás Vallicrosa (1947): 81.

which Hipparchus had relied for his measurement of precession, since these 'were not very knowledgeable people, as is clear from several of their statements'.[59]

Further signs of Ibn Ezra's influence are manifest in the treatise *De motu octavae spherae* completed in 1310 by the Paduan astrologer-physician Peter of Abano, who translated several of the Ibn Ezra's works from Hebrew into Latin.[60] Of Peter it is known that he spent some of the years around 1300 in Paris, which may explain why he fell in line with William of Saint-Cloud in pitting the value of $\delta\lambda$ predicted by the Toledan Tables (9;27,16°) against the observations of recent astronomers, which showed that $\delta\lambda$ had already grown to 10;30,10°.[61] In another passage of *De motu*, he essentially repeated Odington's argument when claiming that 'Hermes Abydimon' had found α Tau at 27;27° Aries, whereas modern observations placed it at 0;10° Gemini. While this change in longitude clearly exceeded the maximum $\Delta\lambda$ of the Toledan access and recess model, Peter of Abano made the same mistake as John of London in assuming that this maximum was only 11° (rounded from 10;45°). Even Ptolemy's slow rate of 1°/100 years would have been enough to shatter this boundary during the 1434 years that separated Hipparchus from the present year AD 1310.[62]

Ready to accept the one-directional nature of precession, Peter of Abano was left to explain why Ptolemy's slow rate had been replaced with much swifter ones in the course of history, first by al-Battānī's 1°/66 years, later by the 1°/70 years favoured by al-Ṣūfī and other *magistri probationum*. Citing Ibn Ezra, he highlighted the role human fallibility and inadequate instruments played in producing observational error.[63] In the case of stellar precession, the problem was aggravated by the need to compare observations made over several centuries in order to perceive the effects caused by the very slow motion of the eighth sphere. Practically speaking, this required astronomers to rely on the data recorded by their predecessors, as Ptolemy did when he based himself on the work of Hipparchus. From the *Almagest*, it was known that Hipparchus had in turn drawn upon the even earlier observations made ca. 300 BC by Aristyllus and Timocharis even though he deemed them 'not trustworthy, having been made very crudely'.[64] The implications were worrying, especially if one took into account that observational errors in the works of Hipparchus and Ptolemy were likely to have

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[59] *Scriptum Johannis de Sicilia super canones Azarchelis de tabulis Toletanis*, ed. (Pedersen 1986: 253 (J538)): 'Si ergo primi consideratores ante Abrachis, qui non multum scientes homines extiterunt, sicut patet ex pluribus quae dixerunt, in aliquo contigerit erravisse, oportuit sequentes etiam oberrare'.

[60] Peter of Abano, *Tractatus de motu octavae spherae* (ed. Vescovini 1992: 347–65). This treatise expands upon the account of precession in Peter of Abano, *Lucidator dubitabilium astronomiae* 2.3 (ed. Vescovini 1992: 180–91). See the summaries and discussions in Vescovini (1984), Paschetto (1984): 287–94, de Callataÿ (1996): 174–81, Seller (2009): 124–95.

[61] Vescovini (1992): 191, 360. In MS Utrecht, Universiteitsbibliotheek, 725, fols. 201v–6r, the text of *De motu octavae spherae* is merged with William of Saint-Cloud's *Almanach planetarum*. The cited Toledan value of $\delta\lambda = 9;27,16^\circ$, which Peter of Abano attributed to Thābit and Azarquiel, would have been adequate for June AD 1300. A marginal note in MS Vatican City, Biblioteca Apostolica Vaticana, lat. 3118, fol. 53r, attributes to Peter's contemporary Cecco d'Ascoli an (observed?) *motus* of $\delta\lambda = 10;41,47^\circ$ compared to a Toledan $\delta\lambda = 9;33,26^\circ$, which points to August 1313.

[62] Vescovini (1992): 188–89, 357–58.

[63] Vescovini (1992): 189–90, 355–56. Both of these discussions feature verbatim quotes from Abraham Ibn Ezra, *Liber de rationibus tabularum* (ed. Millás Vallicrosa 1947: 81, 89).

[64] Ptolemy, *Almagest* 7.3 (trans. Toomer 1998: 329).

produced effects on later astronomers such as al-Battānī. If these later astronomer were indeed mistaken, it was practically inevitable 'that we are similarly deceived, since there cannot be an examination of our observation except through their observations'.[65]

3 Alfonsine trepidation

The examples discussed thus far make it transparent that by ca. 1310 the authority of the old access and recess model had eroded to a point where a growing number of astronomers felt motivated to opt for a simple, linear account of precession, a move that in at least some cases was supported by fresh observational data. Yet it has also been seen that this option came at a price, in that to deny the existence of long-term changes in the relevant parameters was tantamount to rejecting most of the solar and stellar observations Ptolemy had transmitted in his *Almagest*. As the example of Peter of Abano demonstrates, this step could be viewed as problematic for the way it cast a general doubt on the reliability of past astronomical data, a doubt that effectively eradicated the historical material necessary for a new, and better, analysis of the phenomenon of stellar precession. A desire to avoid either horn of this apparent dilemma may account in part for the massive popularity in late medieval Europe of the so-called *Alfonsine Tables*, which originated ca. 1272 at the court of King Alfonso X of Castile and León. Here is not the place to comment in any depth on the genesis and rationale behind these tables, which existed in two significantly different versions: a Castilian one using sidereal longitude with what appears to be the model of *De motu octave sphere* and the *Toledan Tables* to convert to tropical longitude, and a Latin one using tropical longitude directly. It is the latter version that greatly influenced European astronomy after its introduction in Paris ca. 1320.[66]

In the Latin-Parisian *Alfonsine Tables*, it is the tropical year that is uniform, and the access and recess component of the motion of the eighth sphere in the tradition of pseudo-Thābit is combined with a linear term so that it is the sidereal year, along with the motion of the fixed stars and apogees, that is nonuniform. The linear term is represented by a table for the 'mean motion of the apogees and fixed stars' with a period of 49,000 Julian years and a mean tropical motion of 360°/49,000 years $=$ 1°/136 years or $0;0,26,26\ldots^{\circ/\text{year}}$. Since the mean sidereal year is taken as a Julian year, $365;15^{\text{d}}$, the tropical year is (49,000/49,001) $365;15^{\text{d}} \approx 365;14,33,10^{\text{d}} = 365^{\text{d}}5;49,16^{\text{h}}$. The periodic term is obtained from a table of the 'mean motion of access and recess of the eighth sphere', with a period of 7000 years and a motion ϑ of $0;3,5\ldots^{\circ/\text{year}}$, together with a table of 'equations of motion of access and recess', reaching a maximum $\delta\lambda = \pm 9^{\circ}$ (instead of 10;45°), computed very nearly from $\delta\lambda = \sin^{-1}(\sin 9^{\circ} \sin \vartheta)$. If we call the mean tropical motion of the apogees and fixed stars in some period $\Delta\bar{\lambda}$, then the true motion $\Delta\lambda = \Delta\bar{\lambda} \pm \delta\lambda$. Because of the nonuniform motion of the solar apogee through $\Delta\lambda$, which slowly changes the

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[65] Vescovini (1992): 356: 'Et nos similiter necessario decipimur, cum observationis nostre nisi per eorum observationes esse possit consideratio'. Peter here tacitly quotes al-Battānī, *De motu stellarum*, ch. 52 = Al-Battānī 1537, fol. 81r.

[66] The key study is Chabás and Goldstein (2003b). See, in addition, the contrasting viewpoints expressed in North (1996), Swerdlow (2004) and Poulle (2005).

solar equation at each tropical longitude, there is a nonuniformity in the period of the true motion of the sun, its period of return to the equinox or to the same tropical longitude, but the effect is small and is not considered in the computation of the mean tropical motion of the sun, in which the tropical year is uniform. Another departure from pseudo-Thābit and the *Toledan Tables* concerns the epoch at which the motion of access and recess $\vartheta = 0^\circ$. The tables in *De motu octave spere* placed it in AD 604, whereas the *Alfonsine Tables* have it on 16 May AD 16, 1236 years before the beginning of Era Alfonso, 1 June 1252. For this period, the total precession computed from the *Alfonsine Tables* was 17;07,56°, implying an excellent mean rate of ca. 1°/72 years.[67]

Also in contrast to the *Toledan Tables*, which were supported by pseudo-Thābit's *De motu octave spere*, the *Alfonsine Tables* reached Latin Europe without any description of the underlying physical or geometrical models, leaving the theory of the motion of the apogees and fixed stars open to debate and interpretation.[68] The resulting uncertainty is already on display in a *Theorica planetarum* penned in 1335 by Jean de Lignères, who was among the first astronomers in Paris to employ and discuss the new tables in their writings.[69] When it came to the thorny subject of the motion of the eighth sphere, Jean de Lignères acknowledged the empirical need for a theory that made the precession of the fixed stars speed up and slow down, but at the same time he denied that the access and recess model his contemporaries attributed to Thābit was adequate to explaining this change over time. His objections to Thābit's theory were manifold, but not all of them revealed a correct understanding of the model he criticized. For instance, when Jean complained that the circular motion of the heads of Aries and Libra should have caused the fixed stars near these positions to undergo noticeable changes in ecliptic latitude, he overlooked that the fixed ecliptic mentioned in *De motu octave spere* existed only in the geometrical imagination and that the mobile ecliptic, in which the sun moved, was supposed to move along with the fixed stars.[70] More persuasive was Jean's reminder that the longitude of α Vir had increased by more than 21;40° between the time of Timocharis (22;20° Virgo) and the present year 1335, which obviously exceeded the $2 \cdot 10;45^\circ = 21;30^\circ$ allowed for

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[67] Useful discussions of precession in the *Alfonsine Tables* include Duhem (1913–59): II, 262–65, Dobrzycki (1965/2010): 34–40, Mercier (1976–77), pt. 2: 58–60, Samsó (1984): 87–109, Casanovas (1987), Samsó and Castelló (1988), North (1996): 459–69, Chabás and Goldstein (2003a): 165–68, Chabás and Goldstein (2003b): 256–66.

[68] See on this point Dobrzycki (1965/2010, 1983).

[69] Jean of Lignères's *Theorica planetarum*, which carries the incipit 'Spera concentrica vel circulus dicitur', survives in MSS Cambridge, University Library, Mm.III.11, fols. 76r–80v; Paris, Bibliothèque nationale de France, lat. 7281, fols. 163r–72r. See Duhem (1913–59): IV, 66–69, Poulle (2005): 41.

[70] MS Paris, Bibliothèque nationale de France, lat. 7281, fol. 166v: 'Eandem enim latitudinem a circulo signorum fixo servant stelle fixe modernis temporibus et ad eandem partem celi declinant sicut tempore Ptholomei et aliorum preteritorum temporum antiquorum et etiam tempore ipsius Thebit, ut estimo, nec etiam aliquam experientiam posuit ad oppositum. Et tamen secundum illam positionem apparent diversitates satis notabiles in motu stellarum fixarum in latitudine qua stelle fixe site in capite Arietis in celo stellato mutarent locum suum in latitudine ad septentrionem et meridiem a circulo signorum fixo aliquando plusquam per quatuor gradus declinantes ad septentrionem et postea ad consimilem distantiam in meridie revertentes et stelle iuxta puncta tropica site parum vel nichil mutarent situm suum in latitudine'.

by the Toledan model.[71] With *De motu octave spere* out the window, it remained an open question what description of the motion of the eighth sphere one should accept instead. Although Jean de Lignères had not found an adequate answer in any of the books known to him, he considered it likely that the *Alfonsine Tables* were correct in ascribing to the fixed stars and apogees two complementary motions:

> one motion [runs] from west to east and is uniform around the poles of the ecliptic; the other is a motion of apparent back-and-forth, which is referred to as 'equated'. It sometimes adds to the first motion, namely when it goes into the same direction, and sometimes takes away from the aforementioned motion, namely when it goes into a direction opposite to the first one. [...] Yet [concerning the question] around which poles this second motion, which is called the 'access and recess', goes, I have found nothing worthy of note except for the geometrical model [*ymaginationem*] of Thābit described above.[72]

Finding an adequate geometrical interpretation of King Alfonso's precession model was not made easier by the way the tables seemed to work with a fixed length of the tropical year, quite as if the 'access and recess' of the eighth sphere left the lower spheres, including that of the sun, entirely unaffected. The Parisian author of a critical *Expositio tabularum Alfonsii*, written in 1347, found this feature unacceptable in light of the historical evidence (provided by the *Almagest* and al-Battānī) that showed the year length to have changed over the centuries. His various complaints about the contradictory nature of the Alfonsine precession model also tackled the strange fact that the tabulated equation for the access and recess of the eighth sphere reaches zero close to the beginning of the Christian era. In theory, this zero-point in the first century should have been accompanied by the fastest possible rate of precession, whereas the historical record indicated that the greatest acceleration had only taken place between Ptolemy (second century) and al-Battānī (ninth century). The *Alfonsine Tables* thus failed to provide what models of access and recess had originally been created for, an explanation and correct computation of the apparent changes in precession recorded by past astronomers.[73] Another point he considered worthy of special attention was the way in which

> according to [these tables] the circles of the planets move with the motion of the fixed stars, but are nevertheless not moved by the motion of the access and

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[71] Ibid., fol. 166v: '...et in modernis temporibus, scilicet anno Christi 1335, mota est ad orientem a puncto equalitatis predicto per 14 gradus Libre et plus, sic quod a sito primo recessit ad orientem motu continuo per 21 gradus'. The present-day longitude was probably derived by taking the longitude of α Vir in the Alfonsine star catalogue (ed. Rico y Sinobas 1863–67: I, 7–145), which is 13;48° Libra, and adding $\Delta\lambda$ according to the two-part precession model, which would have been between 0;15° and 0;16°.

[72] MS Paris, Bibliothèque nationale de France, lat. 7281, fol. 167r: 'Quid autem tenendum sit de motu stellarum fixarum et augium a nullo auctore sufficienter scriptum invenio. Tamen estimo ponere duplicem motum earum: unum motum ab occidentem in orientem continuum et uniformem super polos orbium signorum, alium de motu apparente accedendo et recedendo, qui dicitur equatus, qui aliquando additur supra motum priorem, scilicet quando fit ad partem eandem, et aliquando minuitur de motu predicto, scilicet quando fit ad partem oppositam priori. [...] Sed super quos polos fit motus secundus, qui dicitur accessus et recessus, nichil invenimus notabilem preter ymaginationem Thebit superius recitatam'.

[73] See the *Expositio tabularum Alfonsii* of 1347, as discussed in Nothaft (2015): 81–86.

> recess of the heads of Aries and Libra, and this applies especially to the circle of the sun.[74]

At first glance, this statement was contradicted by the way users of the *Alfonsine Tables*, including the Parisian astronomer of 1347, normally took the periodic term into account when computing the position of the sun's apogee. Yet there were at least two considerations that spoke in favour of the author's conclusion. One was the uniform length of the tropical year, which could only be plausibly maintained if the sun's path was not subject to the trepidation of the sphere of the fixed stars. The other was the complete absence of any indication that the ecliptic ever changed its obliquity relative to the celestial equator. The Parisian commentator found this latter point worthy of special mention, as he realized that the access and recess of the eighth sphere should have made the sidereal heads of Aries and Libra move on small circles in a northerly or southerly direction. If the ecliptic was fixed in its relation to the celestial equator, as seemed to be demanded by the *Alfonsine Tables*, the predicted consequence of this motion should have been changes in stellar latitudes of a kind nobody had ever observed.[75]

A less sophisticated criticism of the Alfonsine treatment of precession has been preserved from the University of Oxford, where at least one commentator showed himself irritated by the way the new tables, with their respective headings, separated the 'mean motion of the apogees and fixed stars' from the 'access and recess of the eighth sphere'. Even though most understood that the access and recess had to be added to the linear motion to find the longitude of the apogees for any given epoch, a belief to the contrary is preserved in an obscure text on the motion of the eighth sphere, datable to 1350, the author of which complained that King Alfonso

> assumed that the motion of the apogees of the planets is uniform and that, in addition to that, the whole motion of the eighth sphere is non-uniform because of its access and recess. Yet this position entails a contradiction. For if the apogees of all the planets are moved together with the motion of the eighth sphere, it is a contradiction that the apogees of the planets would be moved uniformly and the eighth sphere non-uniformly. And for this reason many deem the *Alfonsine Tables* to be more affected by falsehood than the tables of Azarquiel. Still, both will grow false as time progresses. But so far the *Alfonsine Tables* are truer. Still, they will grow falser over a long period of time. The moderns accordingly hold that the eighth sphere keeps approaching the pole of the world at rate of one minute in five years and that the eighth sphere now, namely in the year of Christ

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[74] MS Berlin, Staatsbibliothek, lat. fol. 192, fol. 212va: 'Est et aliud in tabulis illis quod mirabile videtur: quomodo secundum eas circuli planetarum moveantur motu stellarum fixarum, non tamen moveantur motu accessus et recessus capitum Arietis et Libre, maxime de circulo solis'.

[75] Ibid.: 'Concedunt enim stellas fixas retinere semper eandem declinationem ab ecliptica, tota ergo ecliptica numquam situm mutabit. Signabo ergo in circulo solis punctum suppositum capiti Arietis incipiatque caput Arietis declinare versus septentrionem in circulo illo parvo: vel punctus signatus in circulo solis similiter declinabit vel manebit in loco suo. Si manebit, ergo non amplius manet sub ecliptica, quod est veritati dissonum. Si sequatur, quantum ergo caput Arietis distabit a sectione zodiaci cum equatore, tantum et punctus ille signatus distabit neccessario ab eadem. Movebitur ergo ex neccessitate isto motu, quod non ponunt'.

> 1350, moves eastwards, gaining one degree in 60 years, whereas at the time of Ptolemy it gained one degree in 100 years, such that both this eastward motion of the eighth sphere and the motion of access of its pole towards the pole of the world keep getting faster and faster. For the eighth sphere by its eastward motion from the time of Ptolemy to this year of Christ 1350 has gained 18 degrees and 10 minutes, as has been observed [*sicut expertum est*] at Oxford by many competent persons.[76]

The text just cited is extant in two manuscripts, O = Oxford, Bodleian Library, Digby 97, fol. 143r (s. $\text{XIV}^{\text{ex}}/\text{XV}^{\text{in}}$), and U = Utrecht, Universiteitsbibliotheek, 725, fol. 200r–v (s. XV).[77] In both cases, the mention of the year 1350 is followed by some further remarks on the changing obliquity of the ecliptic and a statement that Ptolemy made his observations 138 years after Christ.[78] From this point on, the two manuscripts diverge, although both make tacit use of the works of John Ashenden, an astrologer active at Merton College in the mid-fourteenth century. U appends a passage from Ashenden's gargantuan *Summa iudicialis de accidentibus mundi*, from which we learn that in 1340 'certain famous men' had shown through *magnas experiencias* an increase of $\Delta\lambda = 18°$ since Ptolemy's day.[79] O instead continues by mentioning an addition of $\Delta\lambda = 16;40°$ to Ptolemy's stellar longitudes made by the masters of Oxford in 1316.[80] This is then followed by a passage from John Ashenden's treatise on the Jupiter–Saturn conjunctions of 1357 and 1365, which reported $\Delta\lambda = 18;17°$ for the interval between Ptolemy and the current year 1357, again claiming that the eighth sphere 'now covers a minute in each year'. As the source for this information,

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[76] Oxford, Bodleian Library, Digby 97, fol. 143r: 'Posuit etiam motum augium planetarum esse uniformem, et propter [*lege*: preter] hoc totum motum octave spere difformem propter eius accesionem et recessionem. Sed ista positio contradictionem includit. Nam cum auges omnium plentarum moventur ad motum octave spere contradictio est quod auges planetarum moverentur uniformiter et spera octava difformiter. Et propter istam causam multi reputant tabulas Alfonsii magis falsas tabulis Arzachelis. Tamen utreque fient false per processum temporis. Sed adhuc tabule Alfonsi sunt veriores. Fient tamen magis false per longum processum temporis. Unde tenent moderni quod plus octave spere accedit continue polo mundi in quinque annis minuto uno et quod octava spera movetur versus orientem modo, scilicet anno Christi 1350, adquirendo in 60 annis gradum unum, cum tamen tempore Ptholomei in 100 annis quesivit gradum unum. Unde tam iste motus octave spere versus orientem quam motus accessus poli eius ad polum mundi continue est velocior et velocior. Nam 8 spera adquisivit per motum eius versus orientem a tempore Ptholomei usque in hunc annum Christi 1350 18 gradus et 10 minuta, sicut expertum est Oxonie per multos valentes'.

[77] The content of O, fol. 143r–v, was previously translated into English by Thorndike (1948) and transcribed by North (1976): III, 243–44. Both were unaware of the parallel attestation of part of the text in U, fol. 200r–v, the wording in which differs in a few places.

[78] O, fol. 143r: 'Et Ptholomeus fecit consideraciones suas per 138 annos post Christum'.

[79] U, fol. 200v: 'Et iam ponam tibi inicia et fines omnium ymaginum zodiaci in spera nona, scilicet quod fuerunt tempore Ptholomei, sed iam ipsa variantur et similiter omnes stelle fixe variantur a locis in quibus fuerunt tunc. Nota motum stellarum fixarum a tempore Ptholomei usque ad annum Christi 1340. Nam anno Christi 1340 verificate sunt per quosdam famosos secundum magnas experiencias et tunc addebantur super loca earum temporum Ptholomei 18 gradus quam quidem addicionem reputo multum appropinquare veritati'. This text, together with the following list of longitudes, was taken from John Ashenden's *Summa iudicialis de accidentibus mundi* (1.6.1), printed as Ashenden (1489), fols. 37vb–38ra. See the remarks in North (1976): III, 261.

[80] This appears to be a reference to the star table drawn up in 1316 by John Maudith, who calculated $\Delta\lambda$ using the Toledan access and recess model. See North (1967, 1976): III, 155–58, 239.

Ashenden here mentions 'Master Simon of Bredon', another fellow at Merton, 'who around the year of Christ 1340 equated the motion of the eighth sphere with the greatest diligence'.[81] Following the reference to Bredon, *O* appends the complete text of Walter Odington's discussion of precession, into which an otherwise unknown author named Thomas Werkwoth interpolated his observation of the longitude of α Tau for the completed year 1395.[82] Werkwoth's longitude (0;24° Gemini) was 1;24° ahead of that found by Odington ca. 1301, which would have implied an average precession of 1;24°/94 years = 1°/67 years. This was considerably faster than the result dictated by the *Alfonsine Tables* (where $\Delta\lambda$ = 0;59° for the same period), but still somewhat slower than the $0;1^{\circ/\text{year}}$ implied in the preceding text. The latter value harmonizes completely with the values for $\Delta\lambda$ obtained in Oxford in the years 1340, 1350, and 1357, which exhibit an increase of 0;10° during the 10 years between 1340 ($\Delta\lambda = 18^{\circ}$) and 1350 ($\Delta\lambda = 18;10^{\circ}$) and another 0;7° between 1350 and 1357 ($\Delta\lambda = 18;17^{\circ}$). This is again at odds with the *Alfonsine Tables*, which only predict a growth of 0;10° over the entire 17-year period.

Similar anti-Alfonsine conclusions could be drawn by taking into account the total period since Ptolemy, who according to the text in *O* and *U* made his observations around AD 138. In the three cases at hand, the data for $\Delta\lambda$ implied average precession rates of 1°/66.78 years (138 to 1340), 1°/66.72 years (138 to 1360), and 1°/66.67 years (138 to 1357), which all came close to al-Battānī's 1°/66 years and even closer to the rate of $0;0,54^{\circ/\text{year}}$ or 1°/66;40 years known from Ibn Ezra and the *Pisan Tables*. Further evidence for the influence of this particular parameter can be obtained from a star table with coordinates 'verified' in Paris in 1314, which survives in two different versions. For most of the longitudes, the difference $\Delta\lambda$ relative to Ptolemy is 17;40°, which is roughly equivalent to an increase of $0;0,54^{\circ/\text{year}}$ for the 1176 years from AD 138 to 1314 ($0;0,54^{\circ}\cdot 1176 = 17;38,24^{\circ}$).[83] Of the three manuscripts known to contain this table, one carries autograph notes by Jean des Murs who, like Jean de Lignères, belonged to the first generation of Alfonsine astronomers in Paris. In commenting on the star table of 1314, Jean pointed out that its longitude for α Leo ($20;10^{\circ}$) implied $\Delta\lambda = 2;27^{\circ}$ if compared to the value recorded by John of Seville (*Johannes Hispalensis*) in 1132 (17;43°). The difference of 2;27° over 182 years suggested to him a precession of 1° in slightly more than 74+2/7 years, but John did not explain how this related

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[81] *O*, fol. 143r: 'Nam octava spera tempore Ptholomei usque in annum Christi 1357 pertransivit 18 gradus, 17 minuta, quia modo in quolibet anno pertransivit unum minutum. Ista patent secundum magistrum Simonem de Bredon, qui circa annum Christi 1340 equavit motum octave spere cum maxima diligencia'. The passage derives from John Ashenden, *De significatione coniunctionis Saturno et Martis* etc. (1357), as found in MS Oxford, Bodleian Library, Digby 176, fol. 45r. The claim in the final sentence receives no immediate support from Simon Bredon's preserved writings, which merely show that in 1347 he observed a conjunction of Venus and α Leo and a lunar occultation of α Tau, finding discrepant values of $\Delta\lambda$ since Ptolemy (18;45° and 18;4°). See Snedegar (1999): 297–98.

[82] *O*, fol. 143r: 'Et ego, Thomas Werkwoth, in anno Christi 1395 perfecto, inveni eam in Geminis 0 gradibus, 24 minutis'.

[83] MSS El Escorial, Real Biblioteca de San Lorenzo, O.II.10, fol. 219r (short version, 13 stars; annotated by Jean des Murs); Cambridge, University Library, Gg.VI.3, fol. 128r (short version, 13 stars); Erfurt, Universitäts- und Forschungsbibliothek, CA 4° 369, fol. 203v (long version, 27 stars). See North (1967): 82, Kunitzsch (1966): 133.

to the Alfonsine model he normally employed in his writings.[84] According to the *Alfonsine Tables*, $\Delta\lambda$ for the interval from 1132 to 1314 should have been no greater than 2°.

An unambiguous case of $0;0,54^{\circ/\text{year}}$ or $1^{\circ}/60;40$ years being accepted as a precessional constant and used to draw up a new star table is encountered in the popular *Canones tabularum Alphonsinarum* written in 1365 by the Swabian astronomer Heinrich Selder, who incepted as a Master of Arts in Paris in 1378.[85] Of the work's 55 chapters, one came with a table of longitudes and latitudes for 60 bright stars, all of which exhibited $\Delta\lambda = 18^{\circ}$ relative to Ptolemy. In the accompanying text, Selder stated that the table showed longitudes valid for the complete year 1340, while a note appended to the star table instructed readers to add 0;54° in order to adjust the longitudes to 1400.[86] This choice of parameters is especially intriguing in light of John Ashenden's combined testimony that his colleague Simon Bredon 'around the year of Christ 1340 equated the motion of the eighth sphere with the greatest diligence' and that 'certain famous men' working in that year had found $\Delta\lambda = 18$ since Ptolemy. In endorsing this very same value, Selder evidently assumed that Ptolemy's catalogue was valid for AD 140, since $18^{\circ} = 0;0,54^{\circ} \cdot (1340\text{–}140)$. He later mentioned this year in connection with Ptolemy in a chronological (and anti-astrological) *Tractatus de tempore dominice annunciationis, nativitatis et passionis*, which featured a list of stellar longitudes updated to the year 1400 using $\Delta\lambda = 18;54^{\circ}$. This was in full agreement with the instructions in the *Canones*, showing once more that Selder worked with a precession parameter of $0;0,54^{\circ/\text{year}}$.[87]

That Selder intended this value to be constant is very clear from the closing remarks to his *Canones*, where he asked readers to take note that the motion of the eighth sphere was almost, but not exactly, one degree in every 66 years.[88] As he acknowledged himself, the resulting linear precession model differed significantly from that used in the *Alfonsine Tables*, which his *Canones* were meant to elucidate. 'According to Alfonso the apogees of the planets and the fixed stars are determined in a different manner, for this Alfonso founded his tables on the motion of access and recess. Yet this motion

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[84] MS El Escorial, Real Biblioteca de San Lorenzo, O.II.10, fol. 219r. See also the notes on precession found on fol. 199v, where Jean obtains rates of 1°/56 years, 1°/60 years and 1°/64 years. For the longitude of α Leo in John of Seville's *Epitome totius astrologie*, see MS Paris, Bibliothèque nationale de France, lat. 7306, fol. 65r. The work is normally dated 1142 rather than 1132. See the list of manuscripts in Burnett (2008): 223–24. According to Charles Burnett (2008: 237–38) there are good grounds to doubt the work's attribution to John of Seville, the well-known Toledan translator. See also Burnett (2002): 75–76.

[85] See Nothaft (2016a).

[86] Heinrich Selder, *Canones tabularum Alphonsinarum* (3.16), MS Freiburg, Universitätsbibliothek, 28, fols. 111v–13v. Other copies of this chapter can be found, for instance, in MSS Berlin, SBPK, lat. fol. 246, fols. 168v–69r; Freiburg, Universitätsbibliothek, 537, fols. 16r–v; Paris, Bibliothèque nationale de France, lat. 7292, fols. 12v–13v. Stand-alone copies of Selder's star catalogue appear in MSS Bernkastel-Kues, Bibliothek des Sankt-Nikolaus-Stiftes, 211, fol. 22r; Erfurt, Universitäts- und Forschungsbibliothek, CA 2° 377, fol. 63v; Melk, Stiftsbibliothek, 601, fols. 252v–53; Paris, Bibliothèque nationale de France, lat. 7272, fol. 2r–v (with an addition of 2;24°); Vienna, Österreichische Nationalbibliothek, 5412, fols. 156r–57r.

[87] See MS Munich, Bayerische Staatsbibliothek, Clm 18298, fols. 11ra (Ptolemy ca. 140), 21ra–22rb (stellar longitudes), and the remarks in Nothaft (2016a): 274–75.

[88] Heinrich Selder, *Canones tabularum Alphonsinarum* (3.35), MS Freiburg, Universitätsbibliothek, 28, fol. 136v: 'Amplius notandum quod secundum opinionem meam pro quibuslibet 66 annis fere debet addi unus gradus ad arcum subtrahendum, quia motus octave spere in tanto tempore ascendit unum gradum, sed non ex toto'.

must by no means be admitted'.[89] The linear precession model Selder put in its place offered considerably higher values for $\Delta\lambda$ than what the *Alfonsine Tables* allowed for over the same intervals of years. Had Selder computed his star table for 1340 with the Alfonsine model, $\Delta\lambda$ relative to Ptolemy would not have exceeded 16;10°. Another potential implication hidden in a swifter precession rate was that of a shorter length for the tropical year. If Selder had used his $0;0{,}54^{\circ/\mathrm{year}}$ in combination with the Toledan sidereal mean motion of 0;59,8,11,28,27°, he would have been able to obtain $(360^\circ - 0;0{,}54^\circ) \cdot 1/0;59{,}8{,}11{,}28{,}27^\circ = 365;14{,}28{,}42{,}2^{\mathrm{d}} \approx 365\frac{1}{4} - \frac{1}{115}$ days. Observational evidence aside, this may be part of the reason why Selder, in the aforementioned *Tractatus de tempore dominice annunciationis, nativitatis et passionis*, favoured not the Alfonsine tropical year of $365\frac{1}{4} - \frac{1}{134}$ days, but a value of $365\frac{1}{4} - \frac{1}{112}$ days.[90] And similarly, it may explain why in the *Canones* he alleged that al-Battānī's value for the tropical year, which he quoted as $365\frac{1}{4} - \frac{1}{100}$ days, came closer to reality than the year in the *Alfonsine Tables*.[91]

What these numerical parameters do not reveal are the reasons that motivated Selder's categorical rejection of Alfonsine trepidation. The only hint contained in his *Canones* is a passage suggesting that the Swabian astronomer shared the belief of his predecessors, Jean de Lignères and the Parisian astronomer of 1347, that the postulated access and recess entailed unobserved changes in stellar latitude.[92] Further insight into Selder's reasoning and choice of parameters could possibly be gained if someone were to discover his treatise 'on the motion and form of the eighth sphere', which he announced as a future project to readers of his *Canones*.

> And if God prolong my life, in this year I intend to compose a book concerning the eighth sphere in which I will thoroughly discuss the form of the 48 constellations and the locations of all fixed stars that enter the astronomer's consideration, of which there are 1022, and the lunar mansions, of which there are 28. And there I will assume the motion of the eighth sphere according to the same method that I uphold here, together with a refutation of Thābit and of all others who assume a motion of access and recess.[93]

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[89] Ibid. (3.15), fol. 111r: 'Porro secundum Alfoncium auges planetarum et stelle fixe aliter sunt verificande, quia ipse Alfoncius fundavit tabulas suas super motum accessus et recessus, qui tamen motus per nullum eventum debet admitti'.

[90] MS Munich, Bayerische Staatsbibliothek, Clm 18298, fol. 31rb: 'Sit autem secundum veriores observationes huiusmodi mutatio in 112 annis fere per unum diem, ita ut quartus annus bisextilem diem intercalet'. See Nothaft (2016a): 274.

[91] Heinrich Selder, *Canones tabularum Alphonsinarum* (3.31), MS Freiburg, Universitätsbibliothek, 28, fol. 134r–v.

[92] Ibid. (3.15), fol. 111v: 'Sed latitudo earundem est arcus eiusdem circuli interceptus inter eclipticam et corporis ipsius stelle, que latitudo est sempiterne invariabilitatis, quia motus accessus et recessus salva reverencia ponencium eum prorsus est negandus'.

[93] Ibid. (3.16), fol. 112r: 'Et si Deus prolongaverit michi vitam hoc anno intendo componere librum de octava spera, in quo pertractabo disposiciones 48 ymaginum et loco<s> omnium stellarum fixarum intrancium consideracionem astronomi, que sunt 1022, et de mansionibus lune, que sunt 28. Et ponam ibidem motum octave spere secundum modum quem hic teneo cum reprobacione Thebith et omnium aliorum ponencium motum accessus et recessus'. Ibid. (3.35), fol. 136v: '...sicut invenies Deo michi prolongante vitam in libro quem edam de motu et dispositione octave spere'.

Unfortunately, it remains uncertain whether this book was ever written. The same holds true for a work planned by Jean de Lignères, who in 1335 noted his intention of expanding upon his critique of Thābit in a separate treatise.[94] Another such example is the treatise *De motu octavae spherae contra Tebith suosque sectatores* the future publication of which Johannes Regiomontanus announced in a broadside prospectus printed in his Nuremberg shop in 1474.[95] Nothing is known about the content of this work, although some vague idea of Regiomontanus's stance on the issue can be gleaned from a letter he wrote in 1464 to his Italian colleague Giovanni Bianchini. In it, the famed astronomer dismissed not just the older trepidation model attributed to Thābit, but also that of the *Alfonsine Tables* for their respective failure to agree with observed and historical values for the obliquity of the ecliptic.[96] As with Heinrich Selder, we are thus faced with the loss of a potentially important source, which would have likely shed more light on the critical undercurrent that was part of the history of trepidation in medieval Europe.

3.1 In lieu of a conclusion: Agostino Ricci vs. Ptolemy

The main purpose of the foregoing survey was to present a greater range of evidence than previously available for the existence of medieval Latin astronomers who were uncertain about or opposed to the prevailing idea that precession was variable or even a two-way oscillation. As the discussion has hopefully shown, such evidence is not restricted to cases where the rejection of trepidation was expressly verbalized (e.g. in the work of William of Saint-Cloud, Walter Odington, or Heinrich Selder), but may also include the choice of certain solar parameters (e.g. the tropical year of $365\frac{1}{4} - \frac{1}{131}$ days attributed to al-Ṣūfī) and the precessional constants implicitly used in the creation of new star tables. From these various strands of evidence, it can be clearly seen that the dissatisfaction with the theory of access and recess was especially widespread between 1290 and 1320, at a time when the growing inadequacy of the *Toledan Tables* in predicting tropical solar longitudes had also begun to cast doubt on the underlying trepidation model. The situation changed profoundly after 1320 with the advent of the *Alfonsine Tables*, whose predictions of solar and stellar longitudes were reliable enough to make most users accept the way these tables combined a linear term for the motion of the fixed stars and apogees with a variable motion of the eighth sphere. Even so, the ambiguities inherent in this model continued to engender doubts, as seen from the criticisms voiced in Paris in 1347 and in Oxford in 1350, and especially from the work of Heinrich Selder, who preferred to view precession as strictly linear. Selder's stance on the issue was certainly an unusually radical one, especially in so far as it implied the complete dismissal of all past observations that indicated a variable rate

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[94] MS Paris, Bibliothèque nationale de France, lat. 7281, fol. 167r: 'Alia in tractatu separatim satis diffuse explanabo'.

[95] The advert was last edited by Malpangotto (2008): 147–54.

[96] See the English translation and commentary in Swerdlow (1990): 170–72, 175–83, and the German translation and commentary in Gerl (1989): 102–4, 269–91. See also Regiomontanus's complaints about Alfonsine stellar longitudes in *Scripta clarissimi mathematici M. Ioannis Regiomontani*, ed. Johannes Schöner (Nuremberg 1544), fols. 42v–43r, repr. in Schmeidler (1972): 658–59.

of precession. Such a step would have appeared overly problematic to at least some of his contemporaries, in particular because it deprived the modern investigator of the basis to come up with a better theory. Regiomontanus put the point as follows:

> For if our predecessors were deceived by instruments, we too shall necessarily be deceived, since our observations, if they are not compared with the observations of the ancients, will never teach anything. But if we shall think that that hidden motion is inherent in the stars, it will be expedient to keep a constant eye on the fixed stars and also to deliver our observations in writing to our successors.[97]

Instead of just casting out large contingents of ancient data, critics of trepidation needed to explain why certain sources, such as Hipparchus or al-Battānī, could be counted on as trustworthy observers of stellar longitudes, whereas others, such as Ptolemy, had gone astray. Contrary to received wisdom, the first astronomer writing in Latin to present such an explanation was not Tycho Brahe, but the Jewish-born Agostino Ricci, who was active around 1513 as a courtier of William IX Palaeologus, the Marquess of Montferrat. Although principally known as a scholar of the kabbalah and friend of Heinrich Cornelius Agrippa, Ricci also made a major contribution to sixteenth-century astronomy in the form of a treatise *De motu octavae spherae*, first printed in 1513.[98] At the heart to Ricci's treatise was a thorough philosophical defence of the view that the motion of the heaven of the fixed stars was simple and linear and that it was enough to postulate eight instead of nine or even ten spheres to account for all the phenomena. The consequence of this defence was a devastating attack on the theory of access and recess, which Ricci engaged in both its Toledan and Alfonsine forms. In each case, a crucial part of his critique was the demonstration that neither model agreed with the available observational data. In the case of the older Toledan model, which Ricci attributed to Azarquiel and Thābit, the most obvious problem was the rapid deterioration of its predictions for tropical solar and stellar longitudes after the thirteenth century. Alfonsine predictions failed as well, but here an additional problem was that the twofold precession model could not even accurately recreate the data recorded by al-Battānī and Ptolemy, the accommodation of which had been the original *raison d'être* for models of access and recess.[99]

Many of the more sophisticated elements in Ricci's discussion reflect his privileged access to a European Hebrew tradition of advanced astronomy, which had barely registered with Christian practitioners of the art. An important case in point is the

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[97] See lib. 7, prop. 6, in *Epytoma in Almagestum Ptolomei* (Venice 1496), sig. h8r, repr. in. Schmeidler (1972): 177: 'Nam si maiores nostri suis decepti sunt instrumentis et nos decipiemur necessario, cum nostre considerationes, nisi antiquorum conferantur observationibus, nihil unquam edocebunt. At si occultum illum motum inesse stellis estimabimus, expediet oculum ad stellas fixas habere assiduum posterosque [*lege*: posterisque] itidem scriptas considerationes liberare'. The passage was inspired by al-Battānī, *De motu stellarum*, ch. 52 = Al-Battānī 1537, fol. 81r.

[98] Ricci (1513). In what follows, I shall cite the second edition, Ricci (1521), which was edited in Paris by Oronce Finé. On Agostino Ricci, see Thorndike (1923–58): V, 264, 284, Nauert (1965): 41, 132–33. Although Ricci's *De motu* is sometimes quoted for specific information, the book as a whole has received remarkably little attention since the summary published in Delambre (1819): 577–81. Examples for more recent references are found in Duhem (1913–59): II, 263–64, Vescovini (1984): 287, 288 n. 38, Vescovini (1992): 342–43, Lerner et al. (2015): III, 245, Flórez Miguel (1999): 452–59 (often misleading).

[99] Ricci (1521), fols. 27r–30r.

fourteenth-century theoretical and observational work of Levi ben Gerson, whose monumental work *Wars of the Lord (Milḥamot Adonai)* contains one of the most original medieval analyses of precession. Observations of lunar occultations of certain stars, carried out in his native Provençe in 1335 and 1339, had put Levi in a position to assert that the total precession since al-Battānī's time had been almost identical to the 1°/66 years al-Battānī himself had found for the period since Ptolemy. Significantly, Levi fortified his argument by showing that even if one accepted that the rate of change during the centuries before Ptolemy had been 1°/100 years, there was no description of the eighth sphere available that accounted for the specific rates and dates reflected in the astronomical record. Instead, he discerned systematic errors in Ptolemy's methods of establishing changes in stellar longitudes, which were inherent both in his comparisons of declinations and his measurements of longitudes based on lunar occultations.[100]

Although the pertinent passages of Levi's work were rendered into Latin during his lifetime,[101] there are virtually no signs that this translation had any impact on subsequent Christian writers. Things look slightly different when it comes to Ricci's teacher, the Iberian astronomer Abraham Zacut, who endorsed the Battānīan precession rate of 1°/66 years in his *Great Composition (ha-Ḥibbur ha-gadol)*, completed in 1478.[102] The chapter on precession was not included in the Latin version of this work, known as *Almanach perpetuum* (printed in 1496), although it was rendered into Castilian by Juan de Salaya, professor of astronomy at the University of Salamanca. Among Zacut's contributions to the debate was his realization that the star catalogue included in the Alfonsine *Libro de las estrellas de la ochava espera* was based on a linear precession of 1°/66 years.[103] Agostino Ricci picked up on this cue and went on to argue that the star catalogue reflected a change in heart on the part of the illustrious King Alfonso X: whereas his original tables, dated 1252, had endorsed the twofold precession model, the star catalogue drawn up four years later, in 1256, showed that he had abandoned this absurd theory and returned to a simpler motion of the fixed stars. That Alfonso had been right in doing so could be seen from a comparison of stellar longitudes recorded in the Alfonsine catalogue with those observed in antiquity by Menelaus and in more recent times by Levi ben Gerson and Abraham Zacut, which supported the view that there had been no substantial change in the rate of precession over the past 1500 years. Ricci, for his part, estimated its true value to be no swifter than 1°/66 years and no slower than 1°/70 years.[104]

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[100] The pertinent chapter is translated and discussed in Goldstein (1975). See, in addition, Mancha (2002): 48–53; Mancha (2012), who argues that Gersonides was familiar only with trepidation models of an earlier type than that espoused by pseudo-Thābit.

[101] Mancha (1992); Mancha (1997).

[102] The pertinent chapter was translated into German by Cohn (1918): 17–21, and into Castilian by Cantera Burgos (1931): 242–50. See ibid.: 138–41, for the Castilian translation by Juan de Salaya (1481). On the appearance of Zacut's name in Ricci's work, see Chabás and Goldstein (2000): 7 (n. 2), 161. Other references to Hebrew texts and authors are documented in Steinschneider (1852–60): 2143–45. For a similar treatment of precession by a Jewish contemporary, see Goldstein (2001): 231–33.

[103] On this catalogue, which was based on al-Ṣūfī's tenth-century *Book of Fixed Stars*, see Samsó and Comes (1988): 67–76; Chabás and Goldstein (2003b): 234–37. On the relevant passages in Zacut and Ricci, see also Wegener (1905): 182–84.

[104] Ricci (1521), fols. 6v, 30v–33r, 51v.

The challenge that remained was to explain the recalcitrant data Ptolemy had adduced back in the second century to support a much slower rate of 1°/100 years. In tackling this problem, Agostino Ricci took a close and critical look at the two main methods the Alexandrian astrologer had used to establish differences in stellar longitude over time. One of these was based on the changing declination of fixed stars relative to the equinoctial circle, which was the only type of information Ptolemy's predecessors had left him in most cases. Ricci patiently described the practical limitations of this method, which was extremely sensitive to error, as even tiny differences in declination of only a few minutes could indicate shifts in ecliptic longitude of one degree. Doubts as to Ptolemy's ability to measure the declination of stars to the required degree of precision were raised by the fact that he himself admitted to have found slight differences in latitude between Hipparchus's time and his own. On the assumption of a linear precessional motion around the poles of the ecliptic, which Ptolemy and Ricci both shared, such changes were impossible and could only be put down to observational error, whether committed by Hipparchus or Ptolemy himself.[105] Further warning signs were hidden in Ptolemy's values for the maximum declination of the sun, which implied an obliquity of the ecliptic of 23;51,20°. This value had failed to receive confirmation from the astronomers after him, most of whom agreed upon an obliquity of 23;33° or thereabouts. Without Ptolemy's exaggerated value, there was also no good reason to believe that the obliquity had changed over time, thus eliminating another reason for assuming a trepidation model of the kind Thābit and Azarquiel had championed.[106] Expanding on arguments already made by Levi ben Gerson in the fourteenth century, Ricci further highlighted Ptolemy's oftentimes sketchy and selective approach to the historical data at his disposal. In particular, he showed that an independent analysis of the change in declination for individual stars over the 265 years between Hipparchus and Ptolemy yielded implied precession rates between 1°/73 years and 1°/90 years, as opposed to the 1°/100 years proposed in the *Almagest*.[107]

Ricci's criticism was no less perspicacious when it came to the other chief method Ptolemy had used to determine the rate of precession: that of directly comparing stellar longitudes. Here the key problem was Ptolemy's abnormally high value for the length of the tropical year of $365\frac{1}{4} - \frac{1}{300}$ days, which medieval astronomers had treated as important evidence in support of trepidation. On the assumption of a fixed sidereal year, a longer tropical year was indicative of slower precession and a shorter tropical year of a faster precession. Ricci's careful reading of the *Almagest* made him recognize the circular nature of such an argument. Far from being the corollary of a changing rate of stellar precession, the tropical year had been there first in the order of investigation and had been instrumental in justifying Ptolemy's slow rate, as seen from the arguments put down in Books III and VII of the *Almagest*. All of Ptolemy's stellar longitudes ultimately depended on his solar longitudes, which were in turn founded on an exaggerated length of the tropical year. Ricci calculated that Ptolemy's value for

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[105] Ricci (1521), fols. 33v–36r.

[106] Ricci (1521), fols. 36r–37r, 43v–44r.

[107] Ricci (1521), fols. 37r–38r. Cf. Goldstein (1975): 34–35, 39–40.

the tropical year would have made him systematically underestimate the total value of $\Delta\lambda$ for the 265 years that separate him from Hipparchus by 1;6°. If this gap was added to Ptolemy's observed stellar longitudes, the implied rate of precession increased from 1°/100 years to 1°/71 years and the error nearly vanished.[108]

Agostino Ricci's critique of observational error in Ptolemy's *Almagest* was highly incisive and touched on nearly all the major points that still inform our understanding of how and why Ptolemy had arrived at an incorrect value for precession. In light of this fact it will be important to sketch with greater precision the nature and extent of his reception in sixteenth-century astronomy. It seems clear that this reception received a boost with the 1521 re-edition of *De motu octavae spherae* by the Parisian mathematician Oronce Finé, who was attracted chiefly by Ricci's austere eight-sphere cosmology—a view of the spheres that has even left certain traces in Christopher Marlowe's *Doctor Faustus*.[109] Ricci's more technical findings on the nature and rate of precession were endorsed by the Bavarian humanist Jacob Ziegler in 1527,[110] while his detailed information on diverging astronomical schools and opinions later served to underpin the sceptical disquisitions of his friend Heinrich Cornelius Agrippa and the anti-astrological polemic of the Castilian theologian Miguel de Medina.[111] It is more difficult to find examples of the period's foremost mathematical astronomers engaging with the content of *De motu octavae spherae*. Erasmus Reinhold knew the book and cited it in his commentary on Peurbach's *Theoricae novae planetarum* (first edition 1542) for its information on the origins of the Alfonsine precession model and the precessional constant underlying King Alfonso's star catalogue.[112] When it came to the actual thesis Ricci sought to defend in his work, however, Reinhold remained terse and dismissive.

> But as for Ricci's supposition that the fixed stars always proceed forward in a uniform motion, this seems to be refuted plainly enough both by stellar observations and the varying length of the year.[113]

How far Agostino Ricci's influence on sixteenth-century debates of precession really extended, and why it did not suffice to stem the tide of trepidation, is clearly a question that would reward future study.

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[108] Ricci (1521), fols. 38v–40v.

[109] On Finé and Marlowe, see Johnson (1946), Ross (1971): 85–87, Pantin (1995): 341–42, 442.

[110] MS Munich, Bayerische Staatsbibliothek, Clm 28144, fols. 1r–3r. See Schottenloher (1910): 359–62, 388–89.

[111] See ch. 30 in Agrippa of Nettesheim (1531), sigs. f3r–6v, and Medina (1564), fol. 18r–v. Medina's remarks on astronomy, which were dependent on Ricci through Agrippa, are summarized in Westman (2011): 199–202, who does not comment on the source.

[112] Reinhold (1542): sigs. c5r, f6v–7r. See also ibid: sig. g6r, for a reference regarding Levi ben Gerson's discovery of a proper motion of the solar apogee. I am grateful to Noel Swerdlow for bringing Reinhold's commentary to my attention.

[113] Reinhold (1542): sig. f7r: 'Quod vero Ricius existimat stellas inerrantes semper aequabili motu procedere in consequentia, id satis aperte refutare videntur & observationes stellarum & anni varia quantitas'.

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