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THE SIGNIFICANCE OF ICONOGRAPHY IN THE PRINT CULTURE OF THE LATE-FIFTEENTH-CENTURY MUSIC THEORETICAL DISCOURSE: THE *THEORICUM OPUS MUSICE DISCIPLINE* (1480) AND *THEORICA MUSICE* (1492) OF FRANCHINO GAFFURIO IN THE CONTEXT OF HIS TRILOGY

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In memory of Howard Mayer Brown (1930–1993)

In scholarly examinations of the *disciplina musicae*, iconography has generally focussed on two principal subdisciplines: *musica practica*, with its concentration on the depiction of musical instruments and related issues of performance practices, and *musica theorica*, with its emphasis on the graphic representation of music-theoretical systems. With the launching of Gaffurio's *Theoricum opus music discipline* (Naples, 1480),[1] the first printed volume of a lengthy and venerable music theoretical discourse,[2] which focussed largely on the juxtaposition of the Greek *systema teleion*[3] and the Guidonian system of solmization and hexachords,[4] the study of iconography encountered a considerable broadening and intensification with regard to graphic representation. This occurred in a drastically altered context with regard to the intent of the image. While iconographic depictions of both *musica theorica* and *musica pratica* continued to exist side by side, the expansion of music theoretical discourse after 1480, provoked by the invention of printing and the rapid dissemination of information, gave rise to a prominent reliance of music theorists on the visual capturing of details pertaining to aspects of music theoretical systems in iconographic representations — a topic that has hitherto been ignored in secondary literature.[5]

The music-theoretical corpus of the eminent musical humanist Franchino Gaffurio (1451–1522), in particular his trilogy comprising the *Theorica musice* (Milan, 1492),[6] the *Practica musicae* (Milan, 1496),[7] and the *De harmonia musicorum instrumentorum opus* (Milan, 1518),[8] attest to this enhancement of the already exceptionally broad music theoretical discourse through depictions. With the publication of his *Theoricum opus musice discipline*, the significance accorded to the study of manuscripts in pre-sixteenth-century Europe shifted to the examination of printed materials. The print culture was so powerful that in music-theoretical writings, unlike in composition, this new means of transmission accounted for the rapid waning of the manuscript tradition which had served scholarship as a secure means of documenting the dissemination of the large and important body of music-theoretical writing since antiquity. Unlike the dissemination of music in manuscript form, which in comparison to the preservation of the musical legacy in print, continued on through subsequent centuries, especially in autographs and performing parts, the latter invariably prepared by scribes often closely associated with the particular composer (as in the case of Wolfgang Amadeus Mozart[9]), the preservation of music theoretical discourse rested almost exclusively in the form of the book, with the manuscript tradition becoming more or less obsolete. The transmission of Gaffurio's treatises substantiate this claim. Characterized by the sparse and incomplete transmission in manuscripts — respectively, from Lodi, Lyons, Naples, Paris, and Vienna in the case of the *De harmonia musicorum instrumentorum opus*, in a sole manuscript, namely in Bologna, Civico Museo Bibliografico Musicale, in the case of the *Theoricum opus musice discipline*, and merely in one manuscript, namely in Bergamo, Bibliotheca Civica, Σ IV.37, in the case of the

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1a."De inventione consonanciarum" [The discovery of the consonance]. Franchino Gaffurio, *Theoricum opus musice discipline* (Napoli, 1480), book 1, ch. 8. — 1b. "De exquisitione et inventione musicarum consonantiarum" [The investigation and discovery of musical consonances]. Franchino Gaffurio, *Theorica musice* (Venezia, 1492), book 1, ch. 8.

Practica musicae — the dissemination of Gaffurio's trilogy among both contemporaries and successors of the humanist tradition,[10] all of whom quote from him, relies almost exclusively on printed material, with the book in essence taking on the significance of the earlier manuscripts.[11]

Included in Gaffurio's trilogy are a number of woodcuts of a Milanese or Brecian design with the blocks presumably cut in Lombardy. The illustration at the end of chapter 8 of book 1 of the *Theoricum opus musice discipline*, the precursor of the *Theorica musice*, includes six figures with an anvil (Tubal Cain),[12] each figure holding a hammer of different weights, each capturing one aspect of the Pythagorean legend [fig. 1a].[13] The second woodcut, inserted in the middle of chapter 6 of book 5 of the same treatise, shows an organist, presumably Franchino Gaffurio himself, seated at the organ console. Included in this woodcut are the organ pipes, measured against the Greek *systema teleion* — also known as the Greater Perfect System or the double octave system — featuring the Greek gamut and the solmization syllables from the *Micrologus* of Guido of Arezzo (ca. 991/992–after 1033).[14] On the whole, the woodcut reveals the overall program of the *Theoricum opus musice discipline*, and of its revised version, the *Theorica musice*, specifically in the juxtaposition of two seemingly different systems, which had been reserved for a full exploration in the closing volumes of the 1480 and 1492 publications — an inquiry which for Gaffurio was presumably influenced by a comment in the *De architectura* 5.3 of Vitruvius (fl. in first century BCE), who stated that the discipline of Greek harmonics "is an obscure and difficult subject to read and write about, particularly for those who do not know Greek letters".[15]

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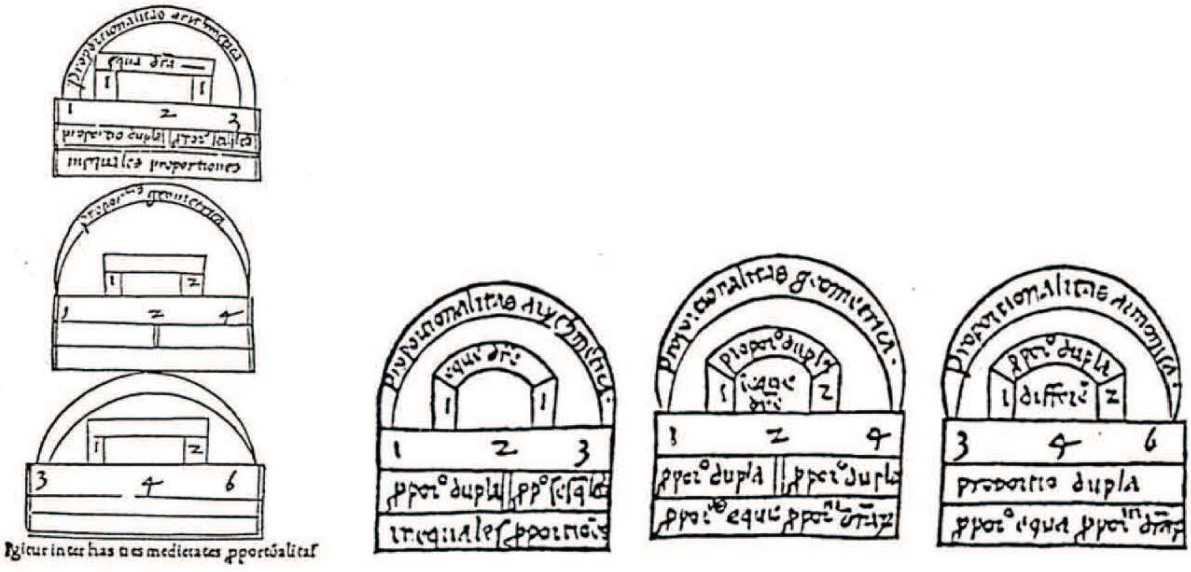
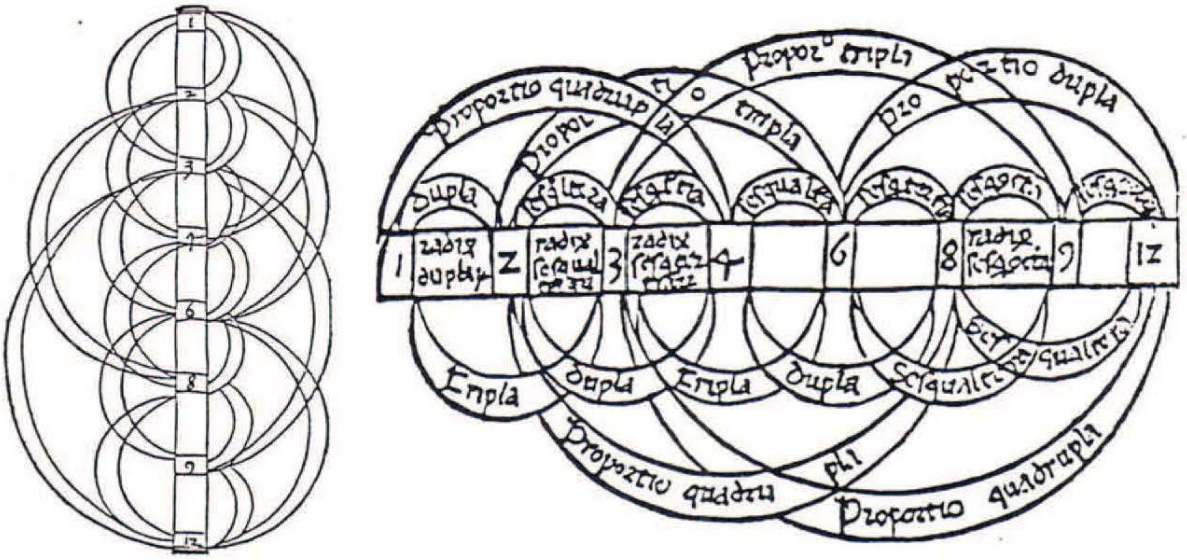


2a. "De proportionalitate" [The proportionality]. *Theoricum*, book 3, ch. 2. — 2b. "De triplici proportionalitate" [The threefold proportionality: Arithmetic, geometric, and harmonic proportionalities]. *Theorica musice*, book 3, ch. 7.



3a. "De formatione proportionum musicas consonantias producentium" [The formation of proportions that produce musical consonances]. *Theoricum*, book 3, ch. 8. — 3b. "De proportionibus conducentibus musicas consonantias" [The proportions that lead to musical consonances]. *Theorica musice*, book 3, ch. 8.

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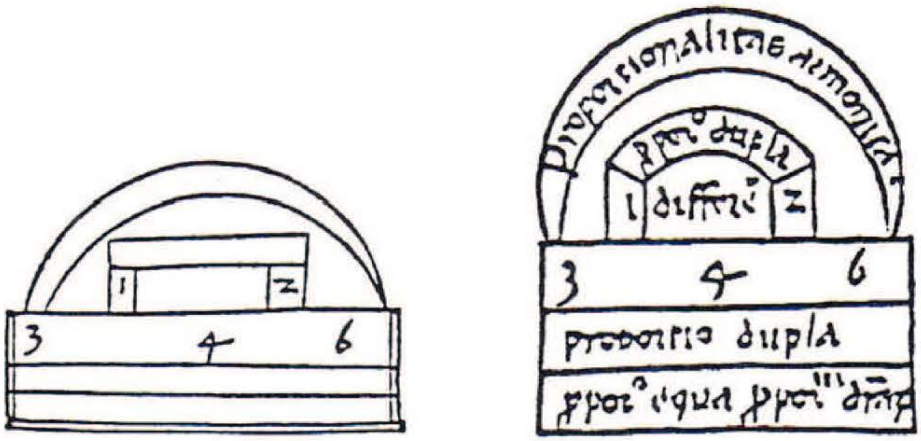
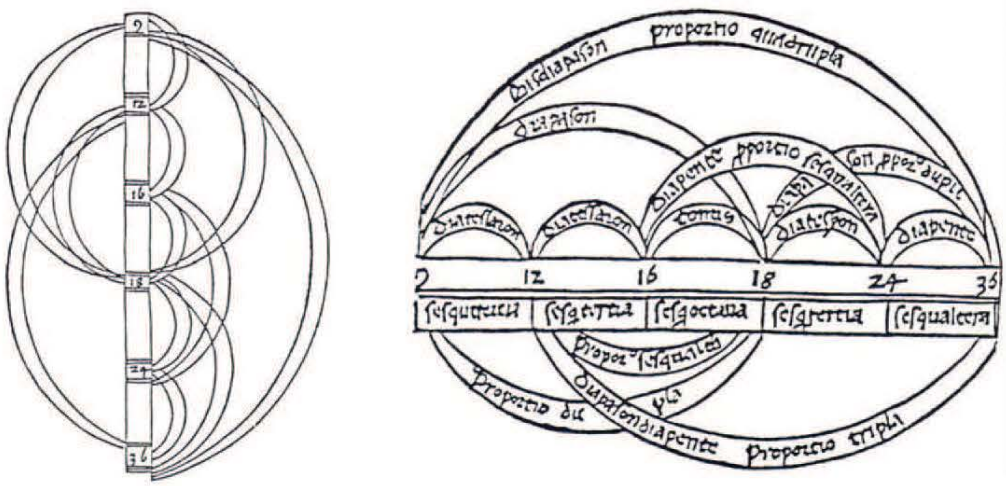


4a. "De natura et formatione consonantiarum ex proportionibus" [The natura and formation of the consonances from the proportions]. *Theoricum*, book 4, ch. 2. — 4b. "De natura et formatione consonantiarum ex proportionibus" [The nature and the formation of the consonances from the proportions]. *Theorica musice*, book 4, ch. 2.



5a. "De natura et formatione consonantiarum ex proportionibus" [The nature and the formation of the consonances from the proportions]. *Theoricum*, book 4, ch. 2. — 5b. "De natura et formatione consonantiarum ex proportionibus" [The nature and the formation of the consonances from the proportions]. *Theorica musice*, book 4, ch. 2.

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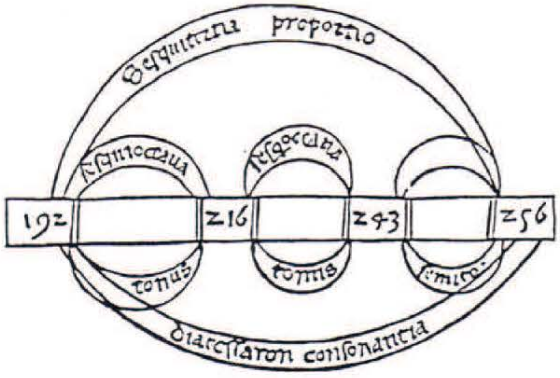
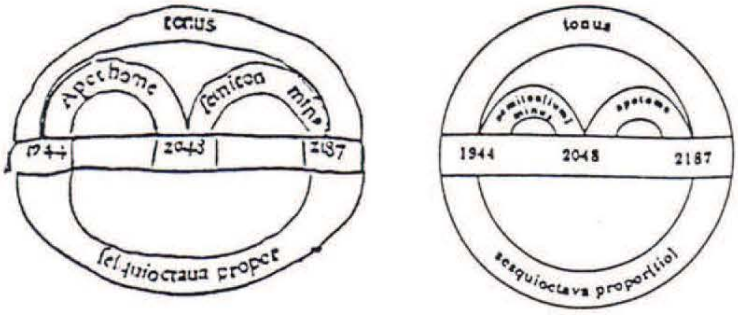
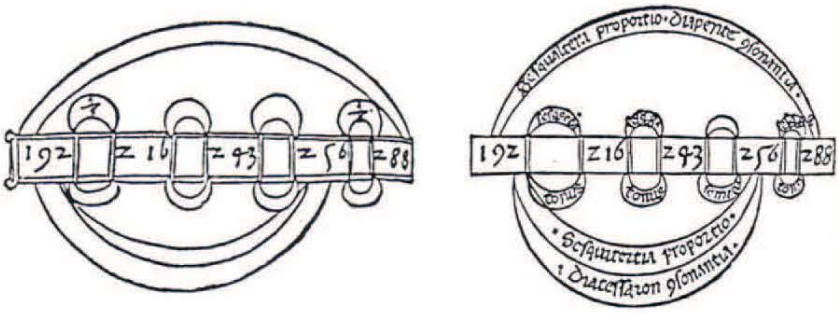


6. "De natura toni et semitoniorum" [The nature of the tone and the semitones]. *Theorica musice*, book 4, ch. 3.



7. "De natura toni et semitoniorum" [The nature of the tone and the semitones]. *Theorica musice*, book 4, ch. 3. On the left is a corrected version.



8a. "De natura diapentes et diapason" [The nature of the diapente and the diapason]. *Theoricum*, book 4, ch. 4. — 8b. "De natura diapentes et diapason" [The nature of the diapente and the diapason]. *Theorica musice*, book 4, ch. 4.

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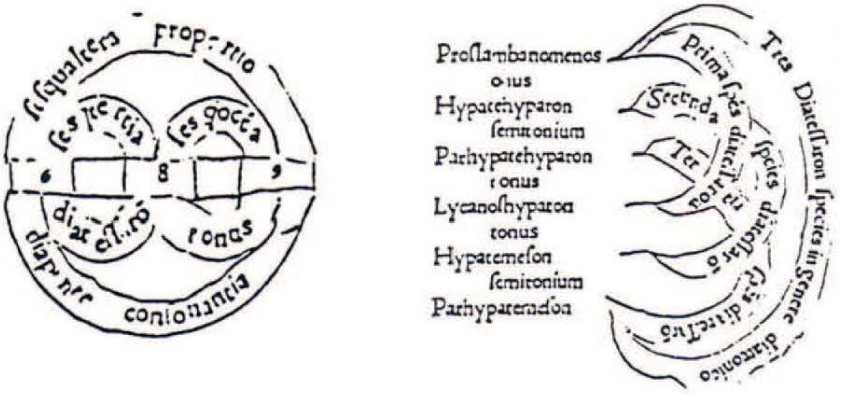
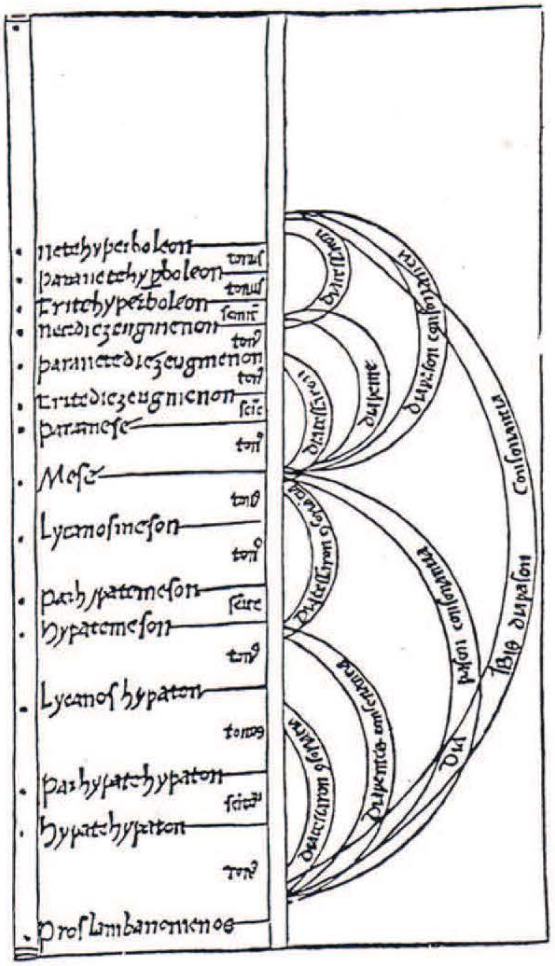


9. "Diapason consonantia ex alio quam multiplici genere evenire non potest" [The consonance of the diapason can only come from thge multiple genus]. *Theorica musice*, book 4, ch. 6. — 10. "De tribus tetracordorum generibus" [The three genera of tetrachords]. *Theorica musice*, book 5, ch. 2.



11. "De intervallis quindecim cordarum" [The intervals of the fifteen steps]. *Theorica musice*, book 5, ch. 3.

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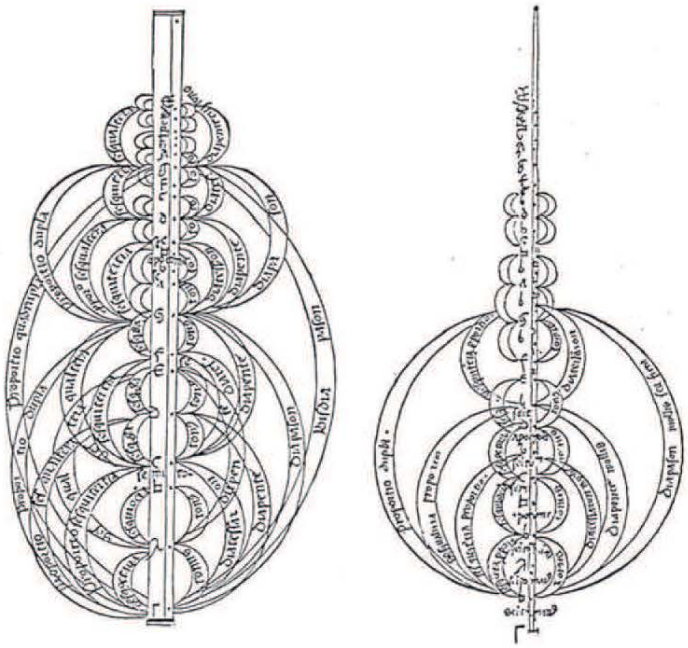
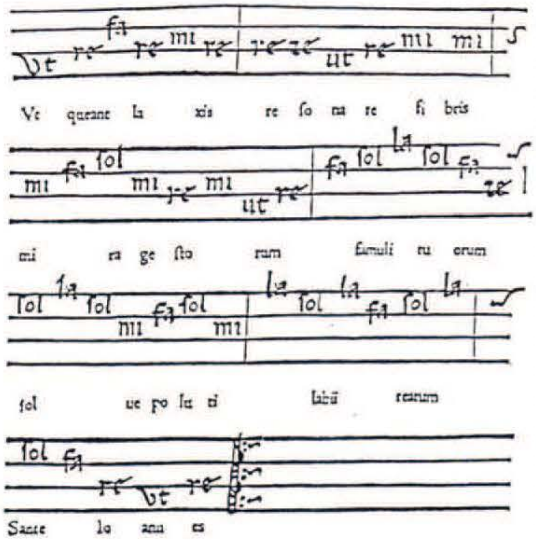


12. "De dimensione cordarum per tonos et consonantias ex proportionibus deductuas" [The measurement of the steps by the tones and the consonances derived from the proportions]. *Theorica musice*, book 5, ch. 4. — 13. "De divisione tonorum per propria semitonia" [The distribution of the tones by appropriate semitones]. *Theorica musice*, book 5, ch. 5.



14. "De applicatione litterarum et syllbarum ad cordas secundum Latinos" [The application of letters and syllables to steps according to the Latins: The solmization hymn *Ut queant laxis*]. *Theorica musice*, book 5, ch. 6.

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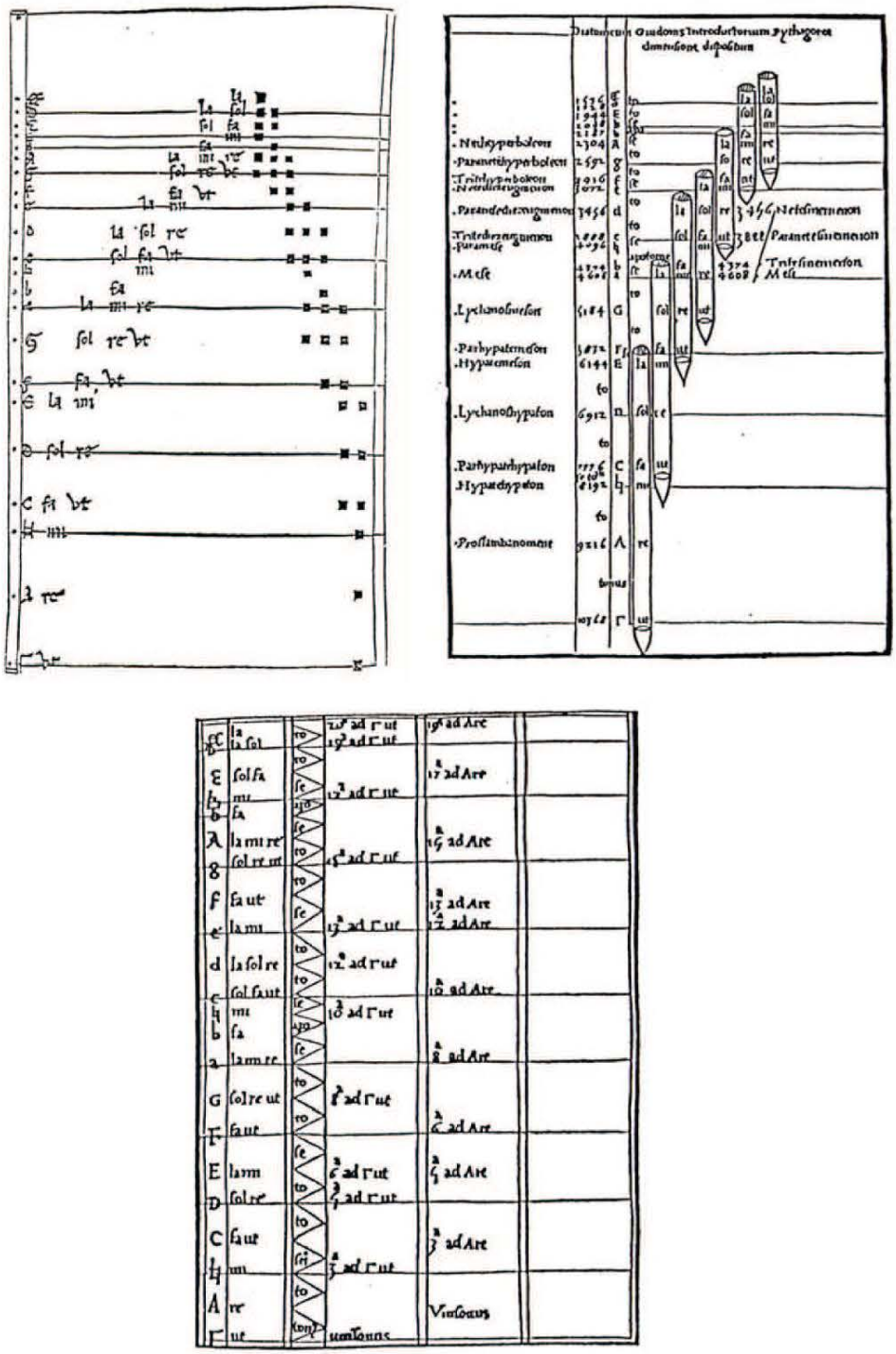

15a. "De applicatione litterarum et syllabarum ad cordas secundum Latinos" [The application of letters and syllables to steps according to the Latins: The Guidonian hexacords]. *Theorica musice*, book 5, ch. 6. — 15b. "De introductorio ad musicam exercitationem necessario" [An introduction necessary to the practice of music: The Guidonian hexachords]. Franchino Gaffurio, *Practica musicae*, book 1, ch. 1. — 15c. "De denominatione extremorum sonorum in concordantiis [Naming of widely separated consonant intervals in the Guidonian hexachords]. *Ibid.*, book 3, ch. 8.

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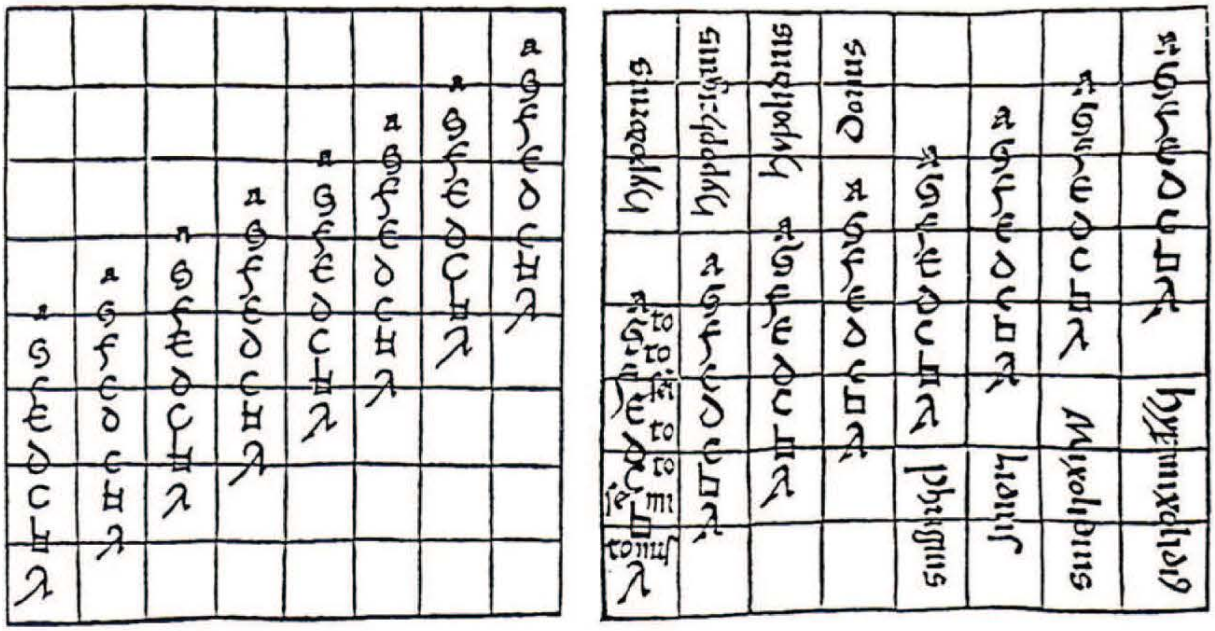

16a. "De speciebus diapason" [The species of the consonance of the diapason: The Latin modes]. *Theoricum*, book 5, ch. 8. – 16b. "De speciebus diapason consonantiae" [The species of the consonance of the diapason: The Latin modes]. *Theorica musice*, book 5, ch. 8.

While this particular woodcut was reissued in the *Theorica musice* as well as in the 1518 edition of the *De harmonia musicorum instrumentorum opus* – a fact which obviously underscores and substantiates the notion of the trilogy articulated in the secondary literature[16] – the woodcut recounting the Pythagorean legend was entirely recut in the *Theorica musice* [fig. 1b]. Here, four illustrations help to unfold the story, from the initial discovery of the proportions of the consonances by the smithy, to their application to other instruments and materials. Notwithstanding the reproduction of the six figures each holding an anvil, with the central message of the corresponding woodcut from the *Theoricum opus musice discipline* retained in the first illustration, Gaffurio expands on the 1480 publication by adding in the representation of Iubal. This inclusion attests to Gaffurio's awareness of a second tradition, namely the Scriptures,[17] and in essence provides a poignant reference to the age-old inquiry into the origin of the *disciplina musicae*,[18] by elegantly juxtaposing the two contenders, Iubal and Pythagoras,[19] in adjacent vignettes of one and the same woodcut.

In the second illustration, Pythagoras is testing the numerical ratios in bells and vessels filled with proportionate amount of fluids.[20] The extension of the *tetraktys*, presented by the numerical series 6, 8, 9, and 12, and by the numbers 4 and 16, is justified by Gaffurio's references to the *Commentarius in Ciceronis somnium Scipionis* (2.1.9–14), by Macrobius (fl. first half of 5th century CE),[21] and to the *Timaeus a Chalcidio translatus commentarioque instructus* (section 45) by Calcidius (fl. 4th or early 5th century CE).[22] Both authors attribute not only the discovery of the numerical proportions for the three basic consonances of the *diapason* (octave), *diapente* (fifth) and *diatessaron* (fourth), and also the *tonus* (whole tone), but also those consonances that are situated outside the interval of the diapason, namely, the *diapason-plus-diatessaron* (octave plus fourth), *diapason-plus-diapente* (octave plus fifth), and *bisdiapason* (double octave), to Pythagoras.

In the third illustration, Pythagoras is positioned in front of the six-string canon (*regula*), to which Gaffurio, curiously enough, alludes to with his handwritten comment in the margin of the *Theoricum opus musice discipline*.[23] In this illustration, each of the strings is suspended by a weight representing the numerical series of the extended *tetraktys*, that is, 4, 6, 8, 9, 12, and 16. Finally, in the fourth vignette, Pythagoras is joined by

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the Pythagorean Philolaus (fl. ca. 450–400 BCE) in the simultaneous sounding of the consonances on pipes whose lengths objectify the same series of numbers. Besides discovering the three consonances, Pythagoras, according to the account provided in the *Manual of Harmonics* by Nicomachus of Gerasa (fl. late 1st–early 2nd century CE),[24] also perceived a dissonance between the *diatessaron* and the *diapente* in the *disiunctio* between *tetrachordum* and *pentachordum*, while acknowledging their addition to form the consonance of the *diapason*.

Although there is, on the whole, agreement between the respective texts of Nicomachus, Boethius (ca. 480–ca. 524), and Gaffurio, one major discrepancy does exist between the three authors concerning Pythagoras's perception of the interval between the *diatessaron* and the *diapente*. In his *De institutione musica* 1.10,[25] Boethius, as a means of clarifying this passage, introduces a fifth hammer that Pythagoras "rejected as being dissonant". Gaffurio, in his faithful reliance on Boethius' text,[26] not only reproduces this interpolation, but expands upon it, introducing yet a sixth hammer.[27] Rather than relying on the comments of the two aforementioned sources, namely, *Commenatarius ex Cicerone in Somnium Scipionis* by Macrobius, and the *Timaeus a Chalcidio translatus commentarioque instructus* of Calcidius (both documents receiving mention in the *Theorica musice*), Gaffurio spins an uncorroborated tale, according to which Pythagoras exchanged the inconsonant fifth hammer for a consonant sixth hammer, to which he assigned the weight of sixteen pounds. In this way, Gaffurio justifies the expansion of the *tetraktys* to allow for intervals that lie outside the compass of the diapason, represented by the *proportio tripla* and by the *proportio quadrupla*.

While the myth illustrates the paramount dependence of the basic intervals on numerical proportions, as expressed in the *tetraktys* (i.e., the first four numbers), each of the four vignettes of the woodcut, despite the anonymous designer's aim to convey this central message, as well as the corresponding text borrowed largely from the *De institutione musica* of Boethius (who in turn was indebted to Nicomachus and his account of the myth in his *Manual of Harmonics*),[28] relate information that displays, not congruity from one vignette to the next, but, rather, disparate views. The weighing of the hammers, responsible for the production of the harmonic intervals, and the arrangement of the four strings of equal length and thickness, suspended with weights corresponding to the hammers, were to have provided further confirmation of the earlier discoveries. However, actual experiments, which were undoubtedly not carried out by either Pythagoras or Nicomachus, would have exposed the falsity of their observations. Weights of hammers or other solids do not yield ratios of consonances in the same way as string divisions. String divisions are governed by multiplication and division, tension by squares and square roots, and the weight of the hammers is influenced by several factors, such as their material, surface, the specific areas stuck together, and their density, and weight. Thus, the pitches sounded on strings of equal length and thickness suspended by equal weights do not agree with those sounds produced by the striking together of hammers in the smithy. While the ratios of the hammers to strings correspond to the string lengths, they do not correlate with the proportions of the suspending weights and density of the hammers, as the weight of hammers would need not to be doubled (as related in the account) but rather quadrupled—an operation which would then allow an inverse rise in the frequency of vibration proportionate to the square root of tension of the string. While authors of the Middle Ages and the Renaissance, including Isidore of Seville (ca. 559–636) in his *Etymologiae sive origines* 3.16,[29] Macrobius in his *Commentarius in Ciceronis somnium Scipionis* 2.1.9–14, and even Gaffurio, were not aware of the rather divergent information contained within the myth, it was not until the latter part of the sixteenth century that Vincenzo Galilei (late 1520s–1591) in his *Discorso* (1589) first called attention to this misconception faithfully transmitted through Boethius and subsequent theorists.[30]

While the message of the woodcut recounting the Pythagorean legend in the 1480 edition relates to the discovery of the consonances through hammers, it stands in obvious isolation to the remaining illustrations of the *Theoricum opus musice discipline*, all of which refer to the string divisions on the monochord [figs. 2a, 3a, 4a, 5a, 8a, and 16a],[31] the third vignette of the respective woodcut in the *Theorica musice*, referring to the derivation of the consonances from the string division as projected onto the monochord, serves as a convenient point of departure for a comprehensive exposition of the *systema teleion*. Thereby, in a pedagogically sound fashion, it dissects the entire system into its various components, with its respective illustrations serving as summaries inserted within often rather verbose prose prepared by Gaffurio. Many of the graphic representations in the *Theoricum opus musice discipline* are not fully annotated; that is, they often merely reveal the numerical proportions without indicating the appropriate term describing its respective proportion. In

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essence, the sparse annotations here seem to find their counterpart in the more abbreviated presentation of the material. Likewise, Gaffurio's considerably expanded presentation of Greek and Latin speculative music theory in the *Theorica musice* appears to have a direct bearing on the significant increase of illustrations, as well as the paying of greater attention to detail in the respective representations.

Following the discussion of the elements of music, the physical properties, and the interaction in the formation of sound (*Theorica musice* 2.1–4), the review of the Pythagorean theory of proportions in the presentation of the consonances (*Theorica musice* 2.5),[32] as well as detailing the significance, definition, etymological derivations, classification, principles and progressions of number (*Theorica musice* 2.6–8),[33] Gaffurio then introduces the term *proportionalitas* (proportionality), denoting the conjunction of two or more successive proportions arranged from two or more successive terms, in a graphic illustration.[34] Here, the properties of the individual proportionalities – the arithmetic proportionality of equal differences and unequal ratios, represented in the numerical series 1, 2, 3, the geometric proportionality of unequal differences and equal ratios, captured in the series 1, 2, 4; and the harmonic proportionality of extreme terms equal to the proportion of difference, disclosed in the series 3, 4, 6 [figs. 2a & 2b] – are adopted from Boethius's *De institutione arithmetica* 2.40[35] and *De institutione musica* 2.12.[36] In contrast to the geometric mean, which can yield irrational numbers,[37] the arithmetic division of the *diapason* results in the *diapente*, and the harmonic division of the *diapason* yields the *diatessaron*.[38]

In *Theorica musice* 3.8, Gaffurio focuses primarily on the examination of various numerical series in an endeavour to isolate arithmetic proportions suitable for the formation of consonances. The derivation here of positive integers through numerical series continues on from earlier procedures witnessed in connection with the more abstract presentations of proportions. Gaffurio arrives at these numerical series [figs. 3a & 3b] through random manipulations rather than by mathematical formulas, expressing their means merely with the intent of extrapolating proportions to which consonances can be assigned. In *Theorica musice* 4.2, Gaffurio repeats the earlier graphic representation of harmonic proportionality (compare fig. 4a with 2a, and likewise fig. 4b with 2b] as a basis for expanding this series comprising both forms of the *tetraktys* in its simplest form or arithmetic series (1, 2, 3, 4), and expanded version or geometric series (6, 8, 9, 12),[39] to include four additional integers, that is, 16, 18, 24 and 36 [figs. 5a & 5b], with the latter two numbers, computed through multiplication of constituent numbers of the harmonic proportionality, and exhibited in the numerical series 3, 4, 6; that is, 4 multiplied by 6 and 6 squared yields the respective terms, while the origin of 16 and 18, the compound of the sesquioctaval proportion 9:8, forms part of the arithmetic conjunct series 16, 17, 18.

In *Theorica musice* 4.3, Gaffurio returns to the arithmetic conjunct series 16, 17, 18 in his consideration of the unequal division of the whole tone (with a ratio of 9:8) as well as in the numerical representation of the *diesis*, also known as the *limma* or *semitonium minus*, defined as the difference between a perfect fourth (4:3) and two whole tones (twice 9:8), thus 9:8×9:8=81:64×3:4=243:256 [fig. 6], and the *apotome* or *semitonium minus*, defined as the difference between the whole tone (with a ratio of 9:8) and the *semitonium minus* or *diesis* (with a ratio of 256:243), thus 9:8×243:256=2187:2048. However, Gaffurio evidently confused *apotome* and *diesis* [fig. 7], thus necessitating an emendation in the text of the corresponding passages in the *Theoricum opus musice discipline* and in the *Theorica musice*, as well as of the graphic representation on folio g1 verso of the 1492 publication.[40] But already in the graphic representation of the numerical series 192, 216, 243, 256 [fig. 6], a derivative of the numerical series 64, 72, 81, 192, resulting from random mathematical computations (i.e., 8×8=64+8=72; 9×9 =81; 64×3=192), Gaffurio lacks precision in his identification of the proportion 243:256, merely as a *semitonium*, rather than a *semitonium minus* or *diesis*. In chapter 4 of book 4 of both the *Theoricum opus musice discipline* and *Theorica musice*, Gaffurio recalls the numerical series 192, 216, 243, 256 of the preceding chapter [fig. 6] in his examination of the *diapason* and *diapente*, extending it by one integer, 288 (that is, the eight part of 256 added to this number) [figs. 8a & 8b].

In view of the earlier consideration of the whole tone with its unequal division, accounting for the expansion of smaller intervals through numerical proportions, Gaffurio is now in a position to offer a more precise explanation concerning the size and configuration of consonances, with the *diatessaron* (embracing the sesquitertial proportion) comprising two tones and a *semitonium* (i.e., *semitonium minus*), and the *diapente* (embracing the sesquialtera proportion) comprising three tones and a *semitonium* (i.e., *semitonium minus*). In

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Theorica musice 4.6, Gaffurio revisits the classification of intervals according to the numerical genera of *aequalitates maiores* (or larger inequality) first discussed in *Theorica musice* 3.3–4—describing five genera of proportions that invariably comprise a ratio of a large quantity to a smaller quantity (such as 2:1; 4:2; etc.). In the related schematic diagram, based on the arithmetic disjunct numerical series of proportions 6, 8, 9 [fig. 9], first introduced in *Theorica musice* 3.8 [fig. 3b], Gaffurio illustrates the correlation between intervals and arithmetic ratios, that is, the interval of the *diapente* affiliated with the sesquialtera proportion and the interval of the *diatessaron* affiliated with the sesquitertial proportion, with both proportions belonging to the *genus superparticularis*, a class of proportions whereby the smaller number plus an aliquot part equals the larger number, defined in mathematical terms as (n+1):n.

Integral to the understanding of the *systema teleion* are various groupings of the fifteen steps, namely, in tetrachords, as well as in larger divisions (cf. *Theorica musice* 5.1.): that is, the heptachord,[41] comprised of two conjunct tetrachords with a *coniunctio*, the octachord, comprised of two disjunct tetrachords with a *disiunctio*, the endecachord, comprised of two conjunct tetrachords and one disjunct tetrachord,[42] and the projection of these formations onto the monochord.[43] This is again relying on Boethius's *De institutione musica* 1.21 ff., which was one of the most readily accessible sources, although admittedly the discussion and systematization of the tetrachordal genera surfaces in the writings of numerous other Greek authors. These include Archytas of Tarentum (fl. first half of 4th century BCE), (as recorded in Boethius's *De institutione musica* 5.17 and in Ptolemy's *Harmonics* 1.13),[44] Eratosthenos (ca. 276–196 BCE), Didymus (fl. second half of 1st century BCE),[45] Ptolemy (after 83–161 CE) (*Harmonics* 1.12),[46] Aristoxenus (ca. 375–360 BCE),[47] Cleonides (fl. 2nd century CE) (*Introduction to Harmonics* 18.9 ff.).[48] and Nicomachus (*Manual of Harmonics* 11.10 ff.), as well as in the treatises of Latin authors, among them Boethius (*De institutione musica* 1.21 ff.; 4.4 ff.) and Martianus Capella (fl. early 5th century) (*De nuptiis Philologiae et Mercurii*).[49]

Gaffurio includes a graphic representation of the diatonic genus comprised of three species of *diatessaron* projected onto a particular segment of the Greek scale, and delineated by the succession of *tonus, semitonus, tonus, tonus*, and *semitonus*, beginning with *proslambanomenos* and extending to *parhypate meson* [fig. 10]. In the *Theorica musice* 3.3, Gaffurio returns to the diatonic genus in his projection of the intervallic structure of the entire fifteen-step gamut of the *systema teleion* [fig. 11]. For the first time in the *Theorica musice*, Gaffurio does not capture his prose in the graphic representation, but merely provides a small exposition of an infinitely richer discussion in the text, which includes a few comments on etymological derivations of the fifteen steps of the monochord division (*Theorica musice* 5.3), a specification of intervallic relationships with recourse to arithmetic, and the correlation between the planetary orders and the Lesser Perfect System,[50] thereby setting the system of correlation—as reported by Boethius (*De institutione musica* 1.27)—with Saturn corresponding to *hypate meson*, likewise Jupiter to *parhypate meson*, Mars to *lichanos meson*, the Sun to *mese*, Venus to *paramese*, Mercury to *trite diezeugmenon*, the Moon to *paranete diezeugmenon*, and the Earth to *nete diezeugmenon*[51]—in bold relief to that reported by Marcus Tullius Cicero (106–43 BCE) (*De republica* 6.17–18). Cicero's system was thus set in reverse order, beginning with the sphere of the fixed star corresponding to *mese*, with Saturn corresponding to *lichanos meson*, Jupiter to *parhypate meson*, Mars to *hypate meson*, the Sun to *lichanos hypaton*, Venus to *parhypate hypaton*, Mercury to *hypate hypaton*, and the Moon to *proslambanomenos*.[52]

Incidentally, Gaffurio decides in favor of the planetary order proposed by Cicero for inclusion in the woodcut at the opening of the *Practica musicae* [fig. 17].[53] For the illustration of the *systema teleion* in *Theorica musice* 5.4 [fig. 12], Gaffurio, indebted to Johannes Gallicus (ca. 1415–1473) (*Ritus canendi* 1.6),[54] replaces the Greek letter names of the preceding graphic representation by the actual pitch names of the individual steps, including the identification of the *apotome* between the *b rotundum* and the *b quadratum*, and likewise the *semitonium* (i.e., *semitonium minor*) between the *b quadratum* and the pitch c, although, curiously enough, only for the upper octave. Gaffurio's preoccupation with the intervallic structure of the *systema teleion* continues in *Theorica musice* 5.5. In reconciling the diatonic genus with the chromatic genus, after these two genera have received separate coverage in *Theorica musice* 5.2, Gaffurio outlines here the intervallic structure of the *systema teleion* in the diatonic genus [fig. 13], although without supplying the correct terminology in the *Theorica musice*, but rather reserving it for a full coverage, that is, with recourse to the arithmetic proportions in the *De harmonia musicorum instrumentorum opus* 1.10.[55] While Gaffurio identifies every conceivable interval and corresponding arithmetic proportion in the illustration in *Theorica musice* 5.4 [fig. 12], he relinquishes this focus

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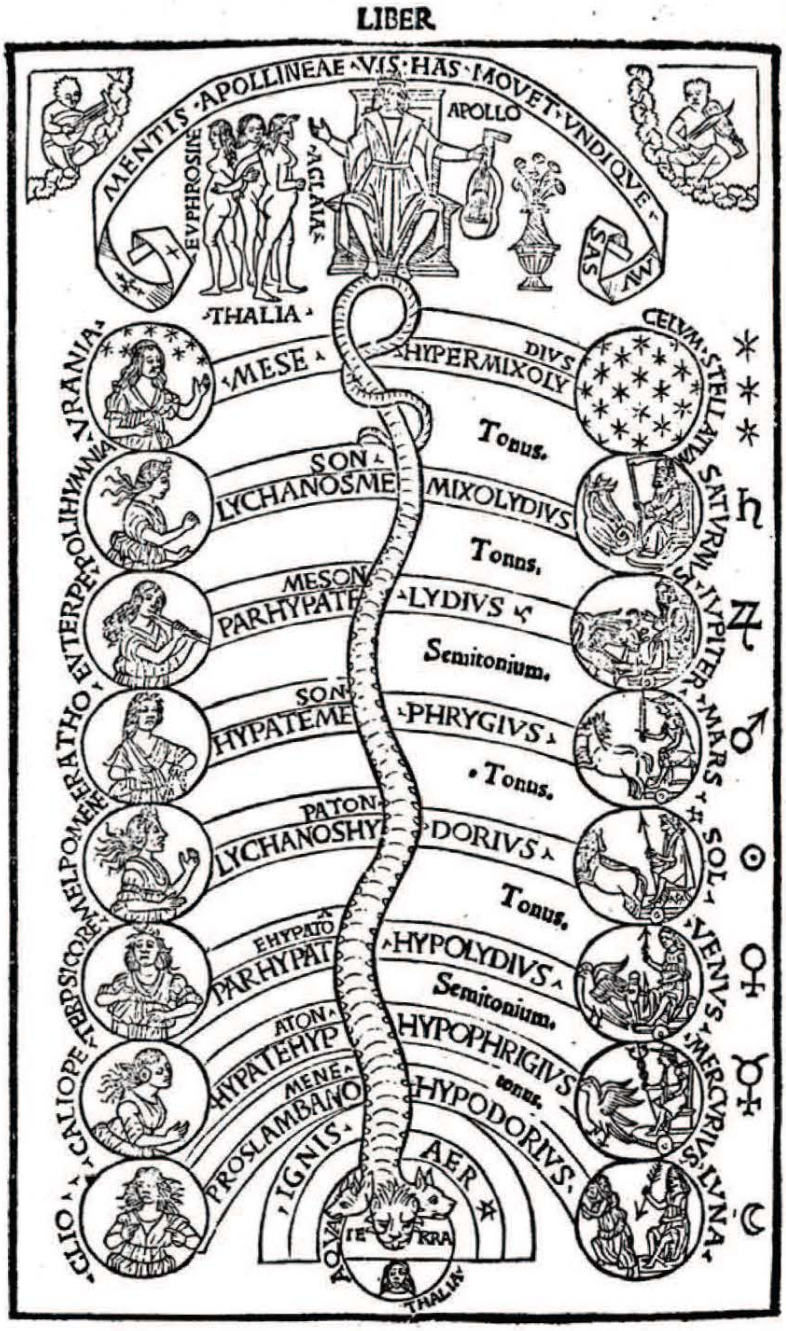

17. Projection of the *systema teleion* onto the Planetary Order reported in Cicero's *De republica*, book 6, ch. 17–18. Frontispiece of *Practica musica*, also reproduced in *De harmonia musicorum instrumentorum*, book 4, ch. 12.

in the graphic representation in *Theorica musice* 5.5 in favour a more sporadic labelling of intervals and proportions, and that as a means of furnishing the context of the new series of intervals, that is, the soft *diatessaron*, soft *diapente*, and soft or *ficta diapason*, resulting from the regularized identification of the *b rotundum*.

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With the inclusion of the solmization hymn *Ut queant laxis* in *Theorica musice* 5.6 [fig. 14],[56] Gaffurio signals a profound shift in focus away from an exclusive concentration on the *systema teleion* to a brief, highly compressed *excursus* into Latin music theory, not as a companion to Greek music theory, but rather as a further elucidation of the *systema teleion*. In this section on Latin music theory, Gaffurio completely changes his style of writing, in a direct reflection of his research method, thereby steering clear of his habit of amassing copious sources to shed light on any given topic. Rather, in this section of the *Theorica musice*, he concentrates predominantly on one music theorist, Guido of Arezzo, and three of his principal writings: the *Micrologus* (in *Theorica musice* 5.6.21 ff.), the most widely disseminated volume on music in the Middle Ages,[57] the *Regulae rhythmicae* (in *Theorica musice* 5.6.2 ff.),[58] and the *Epistola de ignotu cantu* (in *Theorica musice*, fig. 5.6.2).[59] These three documents were generally transmitted in the late fifteenth and sixteenth centuries under the title of the *Introductorium* of Guido. Gaffurio captures Guido's legendary fame as a pedagogue in the solmization hymn, with its unique characteristics—namely, that the beginning of each line of music is placed one step higher than the previous one, except for the last line—which served Guido as a mnemonic aid to teaching the reading of new melodies by means of the six syllables: ut, re, mi, fa, sol, and la (the initial syllable of each line of text), representing the central part of the *gamut*, that is, the notes D, E, F, G, and a.[60] while most fifteenth- and sixteenth-century theorists also resort to a graphic representation of the Guidonian hand,[61] generally accompanied with a detailed explanation of its pedagogical significance,[62] Gaffurio merely provides a cursory reference to the hand (*manus*), followed by the schematic representation of the two-octave scale in reference to the Guidonian hexachords [fig. 15a]. In Tinctoris's treatise, the seven *deductiones* (two of the *proprietas naturalis*, two of the *proprietas mollis* and three of the *proprietas dura*) are intimately linked with the mutation of the hexachords—a topic not addressed in the *Theorica musice*, but rather reserved for the *Practica musicae*, where Gaffurio's ultimate plan in the linking of Greek and Latin systems comes to full fruition [fig. 15b].[63]

At the same time, this illustration, in its inclusion of mathematical ratios for the individual steps, foreshadows the full exposition of Gaffurio's theoretical system in the final volume of his trilogy, the *De harmonia musicorum instrumentorum opus*, a work which in essence represents a revisiting and further alignment of *musica theorica* and *musica practica*—a tendency of sixteenth-century musical discourse.[64] However, with regard to the *practica musicae*, this highly unusual graphic representation with its poignant message places the *Practica musicae*—a volume so firmly rooted in the unfolding of the contemporary compositional practice with recourse to the Latin modality expressed and framed within the Guidonian theoretical practice [fig. 15c] —squarely at the center of his trilogy. However, it also serves as a reminder to contemporary readers of the debate, then still ongoing, about the Greek *systema teleion*. Gaffurio's final illustration, in what he himself allegedly acknowledges as the schematic representation of the Latin modes [figs. 16a & 16b], follows close on the heels of his confusion of the terms "octave species", "modes" and "*tonoi*".[65] The fact that Boethius, in his *De institutione musica* 4.15, does not explain the origin of the Greek modes (as the transposition of a whole system) from the octave species (as the transposition of a segment of a whole system), leads Gaffurio to the erroneous identification of the Boethian modes with a set of plainchant modes (utilizing the same designations) familiar to him from monophony and early polyphony.[66] After Gaffurio has erroneously turned the Greek octave species into modes, he summarizes the eight transpositions of the same A–a scale, identified by the Greek names *dorian*, *hypodorian*, and so forth.

In summary, Gaffurio's blending of his own discourse in both the *Theoricum opus musice discipline* and the *Theorica musice* with iconographic representations, in the form of woodcuts, is intimately linked to his trilogy, with these woodcuts serving as visual summaries of Gaffurio's diversified and for the most part dense prose. Rich in both detail and overall content, it is thus ideal for a study of theory's interaction with iconography as a means of providing a case study in the manifold approaches to scholarship, drawing from a large spectrum of written documentation, including compendia, commentaries, treatises, speeches, letters, and translations.[67] In his manifold recourse to iconography in his disclosing of the music theoretical discourse of antiquity, the Middle Ages,[68] and the early Renaissance,[69] Gaffurio here relies on a discipline that, by virtue of its elegant visual representation,[70] has attracted the attention of scholars and musicians alike in placing considerable focus on the *artes liberales*.[71]

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NOTES

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[1] Franchino Gaffurio, *Theoricum opus musice discipline* (Napoli: Francesco di Dino, 8 October 1480) [dedicated to Cardinal Giovanni Arcimboldo]; also as facsimile edition, vol. 100 of *Monuments of Music and Music Literature in Facsimile. Second series: Music Literature* (New York: Broude Brothers, 1967); facsimile edition prepared by Cesarino Ruini as vol. 15 of *Musurgiana: Collana di trattati di teoria musicale, storiografia e organologia in facsimile a cura dell'Istituto di Bibliografia Musicale di Roma*, issued under the auspices of the Istituto di Bibliografia Musicale di Roma (Lucca: Libreria Musicale Italiana, 1996).

[2] For an overview of the music-theoretical discourse on this topic, see Claude V. Palisca, *Humanism in Italian Musical Thought* (New Haven, Conn.; London: Yale University Press, 1985).

[3] For an overview of the *systema teleion*, see Annemarie Jeanette Neubecker, *Altgriechische Musik: Eine Einführung* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1977).

[4] For an overview of the Guidonian system, see Gaston G. Allaire, *The Theory of Hexachords, Solmization and the Modal System: A Practical Application*. Ed. by Armen Carapetyan ([Roma]: American Institute of Musicology, 1972).

[5] See, for example, Franzsepp Würtenberger, *Malerei und Musik: Die Geschichte des Verhaltens zweier Künste zueinander, dargestellt nach den Quellen im Zentrum von Leonardo da Vinci bis John Cage*. Ed. by Franzsepp Würtenberger (Frankfurt am Main: Peter Lang, 1979); Karoline Czerwenka-Papadopoulos, *Typologie des Musikerporträts in Malerei und Graphik: Das Bildnis des Musikers ab der Renaissance bis zum Klassizismus* (Wien: Verlag der Österreichischen Akademie der Wissenschaften, 2007).

[6] Franchino Gaffurio, *Theorica musice* (Milano: Filippo Mantegazza, impensa Io. Petri de Lomatio, 15 December 1492); also as facsimile edition by Gaetano Cesari (Roma: Reale Accademia d'Italia, 1934); facsimile edition as vol. 21 of *Monuments of Music and Music Literature in Facsimile. Second series: Music Literature* (New York: Broude Brothers, 1967); facsimile edition as vol. 2/5 of *Bibliotheca musica Bononiensis* (Bologna: Forni, 1969). For an English translation, see Walter Kurt Kreyszig, *Franchino Gaffurio: The Theory of Music*. Ed. by Claude V. Palisca (New Haven, Conn.; London: Yale University Press, 1993).

[7] Franchino Gaffurio, *Practica musicae* (Milano: Guilermus Signerre, 30 September 1496); also as facsimile edition (Farnborough: Gregg Press, 1967); and facsimile edition prepared by Giuseppe Vecchi as vol. 2/6 of *Bibliotheca musica Bononiensis* (Bologna: Forni, 1972). For an English translation, see Clement A. Miller, *Franchinus Gaffurius: Practica musicae*. Ed. by Armen Carapetyan ([Roma]: American Institute of Musicology, 1968); Irwin Young, *The Practica musicae of Franchinus Gaffurius* (Madison, Wisc.: University of Wisconsin Press, 1969).

[8] Franchino Gaffurio, *De harmonia musicorum instrumentorum opus* (Milano: Gotardus Pontanus Calographus, 27 November 1518); also as facsimile edition, vol. 97 of *Monuments of Music and Music Literature in Facsimile. Second series: Music Literature* (New York: Broude Brothers, 1967); facsimile edition prepared by Giuseppe Vecchi as vol. II/7 of *Bibliotheca musica Bononiensis* (Bologna: Forni, 1972). For an English translation, see Clement A. Miller, *Franchinus Gaffurius: De harmonia musicorum instrumentorum opus*. Ed. by Armen Carapetyan ([Roma]: American Institute of Musicology, 1977).

[9] Cliff Eisen, "The Mozarts' Salzburg Copyists: Aspects of Attribution, Chronology, Text, Style, and Performance Practice", *Mozart Studies*. Ed. by Cliff Eisen (Oxford: Clarendon Press, 1991), 253-307.

[10] Dietrich Tzwyvel (before 1490–after 1516), *Introductorium musicae practicae* (Münster, 1513); Heinrich Glarean (1488–1563), *Isagoge in musicen* (Basel, 1516); and *Dodecachordon* (Basel, 1547); Johannes Aventinus (Johannes Turmair; 1477–1534), *Musicae rudimenta* (Augsburg, 1516); Francisco de Salinas (1513–1590), *De musica* (Salamanca, 1577); Vincenzo Galilei (late 1520s–1591), *Dialogo dell musica antica e della moderna* (Firenze, 1581); Giovanni Battista Doni (1595–1647), *Compendio* (Roma, 1635); Athanasius Kircher (1601–1680), *Musurgia universalis* (Roma, 1650); and Leopold Mozart (1719–1787), *Gründliche Violinschule* (Augsburg, 1756, 1787, 1789).

[11] The reliance on print culture in humanist studies is vividly illustrated in Leopold Mozart's "Versuch einer kurzen Geschichte der Musik", part of section II of the introduction to his *Gründliche Violinschule* (Augsburg, 1789), 13-19. For an English translation of this part of Section II, see Editha Knocker, *A Treatise on the Fundamental Principles of Violin Playing by Leopold Mozart* (Oxford; New York: Oxford University Press, 1985), 19-24. A comprehensive examination of the aforementioned section of this treatise is provided in Walter Kurt Kreyszig, "'Leopold Mozart ... a man of much ... sagacity': The Revival of Humanist Scholarship in His *Gründliche Violinschule* (Augsburg, 1789)", *Music's Intellectual History*. Ed. by Zdravko Blažeković and Barbara Dobbs Mackenzie (New York: Répertoire International de Littérature Musicale, 2009), 43-156; see also Pierluigi Petrobelli, "La cultura di Leopold Mozart e la sua 'Violinschule'", *Mozart-Jahrbuch* (1989–90), 9-16.

[12] In the *Theorica musice*, Gaffurio replaces Tubal Cain by Iubal. On this dichotomy, see Paul E. Beichner, *The Medieval Representative of Music, Jubal or Tubalcain*. Ed. by Astrik L. Gabriel and Joseph N. Garvin (Notre Dame, Ind.: The Medieval Institute, University of Notre Dame, 1954); see also Björn R. Tammen, "Lebenswelten eines mittelalterlichen Bildmotivs: Jubal und Tubalkain in den Illustrationen zu Bibel, Weltchronik und Speculum humanae salvationis", *Musicologica Austriaca* 22 (2003), 103-134.

[13] On the Pythagorean legend, see Hans Oppermann, "Eine Pythagoraslegende", *Bonner Jahrbücher* CXXX (1925), 284-301; Marius Schneider, "Pythagoras in der Schmiede", *Festgabe zum 60. Geburtstag von Willi Kahl am 18. Juli 1953*. Ed. by Heinrich Hüschen (Köln: [no publisher], 1953), 126-129; see also Ruth Michels-Gebler, *Schmied und Musik: Über die traditionelle Verknüpfung von Schmiedehandwerk und Musik in Afrika, Asien und Europa*. Ed. by Martin Vogel (Bonn; Bad Godesberg: Verlag für Systematische Musikwissenschaft, 1984), 184 ff.

[14] Joseph Smits van Waesberghe, *Guidonis Aretini Micrologus*. Ed. by Armen Carapetyan ([Roma]: American Institute of Musicology, 1955); Warren Babb, *Hucbald, Guido and John on Music: Three Medieval Treatises*. Ed. by Claude V. Palisca (New Haven, Conn.; London: Yale University Press. 1978); see also Hubert Wolking, *Guidos 'Micrologus de disciplina artis musicae' und seine Quellen: Eine Studie zur Musikgeschichte des Frühmittelalters* (Emsdetten, Westfalen: Heinrich & J. Lechte, 1930); and Joseph Smits van Waesberghe, *De musico-paedagogico et theoretico Guidone Aretino eiusque vita et motibus* (Firenze: Leo S. Olschki, 1953).

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[15] As cited in Claude V. Palisca, "Introductory Notes on the Historiography of the Greek Modes", *The Journal of Musicology* III/3 (summer 1984), 221; Frank Granger, *Marius Vitruvius Pollio: De architectura* (Cambridge, Mass.: Harvard University Press; London: W. Heinemann, 1983–1985). Further on the significance of the Greek cyphers in the context of the *musica speculativa*, see Frieder Zaminer, "Griechische Musikaufzeichnungen", *Musikalische Edition im Wandel des historischen Bewußtseins*. Ed. by Thrasybulos G. Georgiades (Kassel: Bärenreiter, 1970), 9-27, esp. 12.

[16] On the surfacing of the trilogy in the secondary literature, see Luigi Salamina, *La Trilogia Gaffuria*. Ed. by Alessandro Caretta, Luigi Cremascoli and Luigi Salamina (Lodi: Edizioni dell' Archivio Storico Lodigiano, 1951), 137-153; Claudio Sartori, "Gaffurius (Gafori), Franchinus", *Die Musik in Geschichte und Gegenwart*. Ed. by Friedrich Blume and trans. by Anna Amalie Abert (Kassel: Bärenreiter, 1949–1986), vol. 4 (1955), col. 1241; Walter Kurt Kreyszig, "Research and Teaching During the Era of Musical Humanism: Defending the Scholar-Teacher in Response to the Principles of Creation and Dissemination of Knowledge in the Italian University Curriculum and Cultural Milieu of the Court of the Sforzas, with Special Reference to Franchino Gaffurio (1451–1522)", *What is a Teacher – Scholar? Symposium Proceedings, November 9 & 10, 2001*. Ed. by Ron Marken (Saskatoon: University of Saskatchewan, 2002), 97-132.

[17] Judith Cohen, "Jubal in the Middle Ages", *Yuval: Studies of the Jewish Music Research Centre* III (1974), 83-99.

[18] See Johannes Lohmann, "Der Ursprung der Musik", *Archiv für Musikwissenschaft* XVI (1959), 148-173, 261-291.

[19] See James McKinnon, "Jubal vel Pythagoras, qui sit inventor musicae?", *The Musical Quarterly* LXIV/1 (1978), 1-28; see also Philippe Vendrix, "Jubal, Orphée, Pythagore confrontés: Le mythe des sons originels à la Renaissance", *Art & Fact* 15 (1996), 8-15.

[20] On the pre-eminence of Pythagoras, see Frieder Zaminer, "Pythagoras und die Anfänge des musiktheoretischen Denkens bei den Griechen", *Jahrbuch des Staatlichen Instituts für Musikforschung Preußischer Kulturbesitz 1979/80* (Kassel: Merseburger, 1981), 203-211; see also Barbara Münxelhaus, *Pythagoras musicus: Zur Rezeption der pythagoreischen Musiktheorie als quadrivialer Wissenschaft im lateinischen Mittelalter*. Ed. by Martin Vogel (Bonn; Bad Godesberg: Verlag für Systematische Musikwissenschaft, 1976).

[21] Jacob Willis, *Saturnalia, apparatus critico instruxit, in Somnium Scipionis commentaries selecta varietate lectionis* (Leipzig: B.G. Teubner, 1970); see also Karl Mras, "Macrobius' Kommentar zu Ciceros Somnium: Ein Beitrag zur Geistesgeschichte des 5. Jahrhunderts n. Chr.", *Sitzungsberichte der Preußischen Akademie der Wissenschaften: Philosophisch-Historische Klasse 1933* (Berlin: de Gruyter, 1933), 231-286; see also P.M. Schwedler, *Die Philosophie des Macrobius und ihr Einfluß auf die Wissenschaft des christlichen Mittelalters*. Ed. by Clemens Bäumker (Münster in Westfalen: Aschendorff'sche Verlagsbuchhandlung, 1916).

[22] J.H. Waszink, *Timaeus a Calcidio translatus commentarioque instructus*. Ed. by Raymund Klibansky (London: Warburg Institute; Leiden: E.J. Brill, 1962).

[23] However, his 1480 publication does not include this particular aspect of the legend in the woodcut [fig. 1a], so that the particular handwritten annotation might have served as a reminder to include this vignette in the 1492 publication.

[24] Flora Rose Levin, *Nicomachus of Gerasa: Manual of Harmonics. Translation and Commentary* (Ph.D. diss., Columbia University, 1967); Andrew Barker, "Nicomachus", *Greek Musical Writings. II: Harmonic and Acoustic Theory*. Ed. by Andrew Barker (Cambridge: Cambridge University Press, 1989), 245-269.

[25] Gottfried Friedlein, *Anicii Manlii Torquati Severini Boetii: Institutione arithmetica libri duo e Institutione musica libri quinque accedit Geometria quae fertur Boetii* (Frankfurt am Main: Minerva, 1966); Calvin M. Bower, *Anicius Manlius Severinus Boethius; Fundamentals of Music*. Ed. by Claude V. Palisca (New Haven, Conn.; London: Yale University Press, 1989).

[26] Concerning the reliance on Boethius as *auctoritas*, see Claude V. Palisca, "Boethius in the Renaissance", *Music Theory and Its Sources: Antiquity and the Middle Ages*. Ed. by André Barbera (Notre Dame, Ind.: University of Notre Dame Press, 1990), 259-280; also in Claude V. Palisca, *Studies in the History of Italian Music and Music Theory* (Oxford: Clarendon Press, 1994), 168-188; see also Calvin M. Bower, "The Role of Boethius' *De institutione musica* in the Speculative Tradition of Western Musical Thought", *Boethius and the Liberal Arts*. Ed. by Michael Masi (Bern: Peter Lang, 1981), 157-174.

[27] On the legend of the hammers in the context of a broader tradition of myths perpetuated by the Greeks, and the subsequent transmission of these myths as *historiae*, see Peter G. Bietenholz, *Historia and Fabula: Myths and Legends in Historical Thought from Antiquity to the Modern Age*. Ed. by A.J. Vanderjagt (Leiden; New York: E.J. Brill, 1994), 21-61.

[28] Calvin Bower, "Boethius and Nicomachus: An Essay Concerning the Sources of the *De institutione musica*", *Vivarium: An International Journal for the Philosophy and Intellectual Life of the Middle Ages and Renaissance* XVI (1978), 1-45.

[29] Wallace Martin Lindsay, *Isidori Hispalensis episcope Etymologiarum sive originum libri XX* (Oxford: Clarendon Press, 1962); see also Heinrich Hüschen, "Der Einfluß Isidors von Sevilla auf die Musikanschauung des Mittelalters", *Miscelanea en Homenaje a Monsenor Higino Anglés* (Barcelona: Consejo Superior de Investigaciones Cientificas, 1958-1961), vol. 1, 397-406.

[30] Claude V. Palisca, "Scientific Empiricism in Musical Thought", *Seventeenth-Century Science and the Arts*. Ed. by Hedley Rhys (Princeton: Princeton University Press, 1961), 91-137, esp. 128-129; see also Daniel P. Walker, *Studies in Musical Science in the Late Renaissance*. Ed. by J.B. Trapp (Leiden: E.J. Brill, 1978), 23.

[31] Sigfrid Wantzloeben, *Das Monochord als Instrument und als System entwicklungsgeschichtlich dargestellt* (Halle an der Saale: Verlag von Max Niemeyer, 1911); Cecil Dale Adkins, *The Theory and Practice of the Monochord* (Ph.D. diss., State University of Iowa, 1963); idem, "The Technique of the Monochord", *Acta musicologica* XXXIX/1–2 (1967), 34-43; F. Joseph Smith, "The Medieval Monochord", *Journal of Musicological Research* V/1 (1984), 1-34.

[32] Charles André Barbera, *The Persistence of Pythagorean Mathematics in Ancient Musical Thought* (Ph.D. diss., University of North Carolina at Chapel Hill, 1980); Willi Apel, "Mathematics and Music in the Middle Ages", *Musica e arte figurativa nel secoli X–XII* (In Todi: Preso l'Accademia Tuchertina, 1973), 135-165.

[33] Leonard Eugene Dickson, *History of the Theory of Numbers* (Washington D.C.: Carnegie Institution of Washington, 1919).

[34] On the origin of the term *proportionalitas* (including a historic survey) and its significance for the derivation of the three means, see Werner Schulze, "Logos — Mesotes — Analogia: Zur Quaternität von Mathematik, Musik, Kosmologie und Staatslehre bei Platon", *Festschrift Rudolf Haase*. Ed. by Werner Schulze (Eisenstadt: Elfriede Rötzer Verlag, 1980), 107-180, esp. 114 ff.

[35] Michael Masi, *Boethian Number Theory: A Translation of the 'De institutione arithmetica'* (Amsterdam: Rodopi, 1983).

[36] For a comprehensive survey of the term *proportionalitas* in Boethius's *De institutione musica*, see Detlef Illmer, "Die Zahlenlehre des Boethius", *Rezeption des antiken Faches im Mittelalter*. Ed. by Frieder Zaminer (Darmstadt: Wissenschaftliche Buchgesellschaft, 1990), 242-249.

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[37] Heinrich Vogt, "Die Entdeckungsgeschichte des Irrationalen nach Plato und anderen Quellen des 4. Jahrhunderts", *Bibliotheca Mathematica* III/10 (1909–1910), 97-155; Gustav Junge, "Von Hippasos bis Philolaus; Das Irrationale und die geometrischen Grundbegriffe", *Classica et mediaevalia: Revue danoise de philologie et d'histoire* XIX (1958), 41-72. Further on the preoccupation with geometry in late sixteenth-century music theoretical discourse, see, for example, Claude V. Palisca, "Applications of Mathematics and Geometry in Galilei's *Dialogo* of 1581", *Music and Mathematics in Late Medieval and Early Modern Europe*. Ed. by Philippe Vendrix (Turnhout: Brepols, 2008), 235-252.

[38] Further on this topic, see C. André Barbera, "Arithmetic and Geometric Divisions of the Tetrachord", *Journal of Music Theory* XXI/2 (autumn 1977), 294-323; see also Theo Reiser, *Das Geheimnis der pythagoreischen Tetraktys* (Heidelberg: Verlag Lambert Schneider, 1967).

[39] The latter series is derived from the former series, with the respective correlation already intimated in Raffael's painting "The School of Athens"; see Julius Schwabe, "Hans Kaysers letzte Entdeckung: Die pythagoreische Tetraktys auf Raffaels 'Schule von Athen'", *Symbolon: Jahrbuch für Symbolforschung* V (1966), 92-102; see also Gerhard Jahoda, "Die Tonleiter des Timaios — Bild und Abbild", *Festschrift Rudolf Haase*. Ed. by Werner Schulze (Eisenstadt: Elfriede Rötzer Verlag, 1980), 43-80, esp. 52.

[40] Further on this emendation, see Walter Kurt Kreyszig, "Preparing Editions and Translations of Humanist Treatises on Music: Franchino Gaffurio's *Theorica musice* (1492)", *Music Discourse from Classical to Early Modern Times: Editing and Translating Texts*. Ed. by Maria Rika Maniates (Toronto; Buffalo: University of Toronto Press, 1997), 71-95, esp. 84.

[41] Gaffurio here wishes to assign special significance to the collection of seven pitches as "a metaphor of the seven planets producing the music of the spheres and the seven days of the week" (*Theorica musice* 5.1.16). Concerning the significance of the number 7 in Greek music theory, see Martin Vogel, *Die Zahl Sieben in der spekulativen Musiktheorie* (Ph.D. diss., Universität Bonn, 1953), 15-36.

[42] Further on this topic, see C. André Barbera, "The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism", *Journal of Music Theory* XXVIII/1 (spring 1984), 191-223; see also Giovanni Comotti, "L'endecacorde de Ione di Chio", *Quaderni urbinati di cultura classica* III (1983), 57-71.

[43] For an overview of these formations, see Martin Vogel, *Die Enharmonik der Griechen* (Düsseldorf: Verlag der Gesellschaft zur Förderung der Systematischen Musikwisssenschaft, 1963), vol. 1, 25 ff.

[44] Martin Vogel, "Über die drei Tongeschlechter des Archytas", *Bericht über den Internationalen Musikwissenschaftlichen Kongreß Hamburg 1956*. Ed. by Walter Gerstenberg (Kassel; Basel: Bärenreiter, 1957), 233-235.

[45] Vogel, *Die Enharmonik* 33ff.; 44ff.; Barker, *Greek Musical Writings. II*, 347.

[46] Claudius Ptolemaeus, *Harmonicorum Libri Tres*. Ed. by John Wallis (New York: Broude Brothers, 1977); Jon Solomon, *Ptolemy: Harmonics. Translation and Commentary* (Leiden; Boston, Mass.: Brill, 2000); see also Ingemar Düring, *Die Harmonielehre des Klaudios Ptolemaios* (Göteborg: Elanders Boktryckeri Aktiebolag, 1930).

[47] R.P. Winnington-Ingram, "Aristoxenus and the Intervals of Greek Music", *Classical Quarterly* XXVI/3-4 (July–October 1932), 195-208; Richard Crocker, "Aristoxenus and Greek Mathematics", *Aspects of Medieval and Renaissance Music: A Birthday Offering to Gustave Reese*. Ed. by Jan la Rue (London: Oxford University Press, 1967; also New York: Pendragon Press, 1978), 96-110.

[48] Giorgio Valla, *Cleonides: Harmonicum introductorium* (Venezia: Simon Papiens dictus Brevilaqua, 1497); Jon Solomon, *Cleonides ISAYOYI ARMONIKI: Critical Edition, Translation, and Commentary* (Ph.D. diss., University of North Carolina at Chapel Hill, 1980), 293 ff.

[49] Adolf Dick, *Martianus Capella* (Stuttgart: B.G. Teubner, 1969); also in English translation by William Harris Stahl and Richard Johnson, *Martianus Capella and the Seven Liberal Arts* (New York: Columbia University Press, 1991); see also Charles M. Atkinson, "Martianus Capella 935 and Its Carolingian Commentaries", *Journal of Musicology* XVII/4 (autumn 1999), 498-519.

[50] Marius Schneider, "Die musikalischen Grundlagen der Sphärenharmonie", *Acta musicologica* XXXII/2-3 (April–September 1960), 136-151; Hans Schavernoch, *Die Harmonie der Sphären: Die Geschichte der Idee des Welteinklangs und der Seelenstimmung* (Freiburg im Breisgau; München: Verlag Karl Alber, 1981); Friedrich Zipp, *Vom Urklang zur Weltharmonie: Werken und Wirken der Idee der Sphärenmusik* (Berlin; Kassel: Merseburger, 1985).

[51] Roger Bragard, "L'harmonie des sphères selon Boèce", *Speculum: A Journal of Medieval Studies* IV (1921), 206-213; see also Klaus-Jürgen Sachs, "Kosmisches Gesetz und musikalische Regel", *Musiktheorie*. Ed. by Helga de la Motte-Haber and Oliver Schwab-Felisch (Laaber: Laaber-Verlag, 2005), 31-72.

[52] Clinton Walker Keyes (ed.), *Cicero: De Re Publica* (Cambridge, Mass.: Harvard University Press; London: W. Heinemann, 1977); see also Walther Volkmann, *Die Harmonie der Sphären in Ciceros Traum des Scipio* (Breslau: G.P. Aderholz Buchhandlung, 1908); P.R. Coleman-Norton, "Cicero and the Music of the Spheres", *The Classical Journal* XLV (1949–1950), 237-241; idem, "Cicero Musicus", *Journal of the American Musicological Society* I/2 (summer 1948), 3-22.

[53] For a more detailed discussion of this woodcut, see James Haar, "The Frontispiece of Gafori's *Practica musicae* (1496)", *The Renaissance Quarterly* XXVII/1 (spring 1974), 7-22; reprinted in James Haar, *The Science and Art of Renaissance Music*. Ed. by Paul Corneilson (Princeton: Princeton University Press, 1998), 79-92; see also Clement A. Miller, "Gaffurius's *Practica musicae*: Origin and Contents", *Musica Disciplina* XXII (1968), 105-128.

[54] Albert Seay (ed.), *Johannes Gallicus Legrense: Ritus canendi vetustissimus et novus* (Colorado Springs: Colorado College Music Press, 1981).

[55] Further on the schematic representation of the diatonic genus in the context of the double octave system and the related numerical computations, see Herbert and Walter Kreyszig, "The Transmission of Pythagorean Arithmetic in the Context of the Ancient Musical Tradition from the Greek to the Latin Orbits During the Renaissance: A Computational Approach of Identifying and Analyzing the Formation of Scales in the *De harmonia musicorum instrumentorum opus* (Milano, 1518) of Franchino Gaffurio (1451–1522)", *Mathematics and Computation in Music: First International Conference MCM 2007. Berlin, Germany: Revised Selected Papers*. Ed. by Timour Klouche and Thomas Noll (Berlin; Heidelberg: Springer, 2009), 392-405.

[56] Carl-Allan Moberg, "Die Musik in Guido von Arezzos Solmisationshymne", *Archiv für Musikwissenschaft* XVI (1956), 187-206; Jacques Chailley, "*Ut queant laxis* et les origines de la gamme", *Musica disciplina* LVI (1984), 48-69.

[57] On the dissemination of the *Micrologus*, see Viatcheslav Kartsovnik, "Institutiones grammaticae and Mensura monochordi: A New Source of Guido of Arezzo's 'Micrologus'", *Musica disciplina* XLII (1988), 7-22.

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[58] Joseph Smits van Waesberghe, *Guidonis Aretini tres tractatuli editi cum apparatus critico: Regulae dictae Rhythmicae* (Buren: Frits Knuf, 1985); Dolores Pesce, *Guido d'Arezzo's Regule Rithmice, Prologus in Antiphonarium, and Epistola ad Michahelem: A Critical Text and Translation* (Ottawa: The Institute of Mediaeval Music, 1999), esp. 327 ff.

[59] Joseph Smits van Waesberghe, *Guidonis Aretini tres tractatuli editi cum apparatu critico: Epistola ad Michaelem* (Buren: Frits Knuf, 1976); Dolores Pesce, *Guido d'Arezzo's Regule Rithmice, Prologus in Antiphonarium, and Epistola ad Michahelem*, esp. 437 ff.

[60] Georg Lange, "Zur Geschichte der Solmisation", *Sammelbände der Internationalen Musikgesellschaft* 1 (1899–1900), 535-601; Eberhard Preussner, "Solmisationsmethoden im Schulunterricht des 16. und 17. Jahrhunderts", *Festschrift Fritz Stein zum 60. Geburtstag, überreicht von Fachgenossen, Freunden und Schülern*. Ed. by Hans Hoffmann and Franz Ruhlmann (Braunschweig: Henry Litoff's Verlag, 1939), 112-128.

[61] Susan Forscher Weiss, "'Disce manum tuam si vis bene discere cantum': Symbols of Learning Music in Early Modern Europe", *Music in Art* XXX/1–2 (2005), 35-74; see also Karol Berger, "The Hand and the Art of Memory", *Musica disciplina* XXXV (1981), 87-120; idem, "The Guidonian Hand", *The Medieval Craft of Memory: An Anthology of Text and Pictures*. Ed. by Mary Carruthers and Jan M. Ziolkowski (Philadelphia: University of Pennsylvania Press, 2002), 71-82.

[62] Albert Seay, "The *Expositio manus* of Johannes Tinctoris", *Journal of Music Theory* IX (1965), 194-232. Further on the importance of Tinctoris as a pedagogue, see Bonnie J. Blackburn, "A Lost Guide to Tinctoris's Teachings Recovered", *Studies in Medieval and Early Modern Music*. Ed. by Iain Fenlon (Cambridge: Cambridge University Press, 1981), 29-116.

[63] As substantiated, for example, in the *Expositio manus* of Johannes Tinctoris (ca. 1435–1511). The identification of Greek terminology relating to the fifteen steps of the *systema teleion* with the Guidonian letters is also found in later sources, for example, in Hadrian Junius's *Nomenclator* disseminated widely in editions published between 1567 and 1721; see Gottfried Scholz, "Musikalische Terminologie im Bildungsgut des 16. Jahrhunderts: Musikalische Begriffe im *Nomenclator* des Hadrian Junius. Übertragung und Kommentar", *Anzeiger der Philosophisch-Historischen Klasse der Österreichischen Akademie der Wissenschaften* CXXIX (1992), 145-194.

[64] Claude V. Palisca, "Theory, Theorists", *The New Grove Dictionary of Music and Musicians*. Ed. by Stanley Sadie (London: Macmillan, 2001), vol. 25, 373.

[65] Otto Gombosi, "Key, Mode, Species", *Journal of the American Musicological Society* IV/1 (spring 1951), 20-26; André Barbera, "Octave Species", *The Journal of Musicology* III/3 (summer 1984), 229-241; Calvin M. Bower, "The Modes of Boethius", *ibid.*, 252-263; Jon M. Solomon, "Towards a History of *Tonoi*", *ibid.*, 242-251; see also Charles M. Atkinson, *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music*. Ed. by Lawrence F. Bernstein (Oxford: Oxford University Press, 2009).

[66] On Gaffurio's familiarity with current compositional practices, see Knud Jeppesen, "Die 3 Gafurius-Kodizes der Fabbrica del Duomo, Milano", *Acta musicologica* III/1 (1931), 14-29; Claudio Sartori, "Il quarto codice di Gaffurio non è del tutto scomparso", *Collectanea historiae musicae* (Firenze: Leo S. Olschki, 1953), 25-44; Irwin Young, *Franchinus Gaffurius: Renaissance Theorist and Composer (1451–1522)* (Ph.D. diss., University of Southern California, 1954); see also Ernst Praetorius, *Die Mensuraltheorie des Franchinus Gafurius und der folgenden Zeit bis zur Mitte des 16. Jahrhunderts* (Leipzig: Breitkopf & Härtel, 1905).

[67] For an overview of these forms of literary discourse, see Paul Oskar Kristeller, "Der Gelehrte und sein Publikum im späten Mittelalter und in der Renaissance", *Medium aevum vivum: Festschrift für Walter Bulst*. Ed. by Hans Robert Jauss and Dieter Schaller (Heidelberg: Carl Winter, 1960), 212-230; also in English translation as "The Scholar and His Public in the Late Middle Ages and the Renaissance", *Medieval Aspects of Renaissance Learning: Three Essays by Paul Oskar Kristeller*. Ed. and trans. by Edward P. Mahoney (Durham: Duke University Press, 1974), 3-25.

[68] For a synopsis, see Walter Kurt Kreyszig, "Franchino Gaffurio als Vermittler der Musiklehre des Altertums und des Mittelalters: Zur Identifizierung griechischer und lateinischer Quellen in der *Theorica musice* (1492)", *Acta Musicologica* LXV/2 (1993), 134-150.

[69] See Walter Kurt Kreyszig, "Marchetto von Padua in musiktheoretischen Drucken des späten 15. Jahrhunderts: Terminologie und Etymologie aus rezeptionsgeschichtlicher Perspektive in Franchino Gaffurios *Theorica musice* (1492) und *Practica musicae* (1496)", *Festschrift für Floridus Röhrig*. Ed. by Karl Holubar (Wien: Mayer, 1997), 93-111.

[70] Further on the significance of iconography in underscoring the connection between music and arithmetic, see Ann E. Moyer, "Music, Mathematics and Aesthetics: The Case of the Visual Arts in the Renaissance", *Music and Mathematics in Late Medieval and Early Modern Europe*. Ed. by Philippe Vendrix (Turnhout: Brepols, 2008), 111-146.

[71] Werner Bachmann, "Bilddarstellungen der Musik im Rahmen der 'artes liberales'", *Bericht über den Internationalen Musikwissenschaftlichen Kongreß Hamburg 1956*. Ed. by Walter Gerstenberg (Kassel; Basel: Bärenreiter, 1957), 46-55; see also Friedmar Kühnert, "Die Reihenfolge der artes liberales in der Antike", *Wissenschaftliche Zeitschrift der Universität Rostock* XII (1953), 249-257; Edward A. Lippman, "The Place of Music in the System of the Liberal Arts", *Aspects of Medieval and Renaissance Music: A Birthday Offering to Gustave Reese*. Ed. by Jan LaRue (London: Oxford University Press, 1967; also New York: Pendragon Press, 1978), 545-559; Karl Gustav Fellerer, "Die Musica in den Artes liberales", *Artes liberales: Studien und Texte zur Geistesgeschichte des Mittelalters* V (1976), 33-49; Joseph Dyer, "The Place of *Musica* in the Medieval Classifications of Knowledge", *The Journal of Musicology* XXIV/1 (winter 2007), 3-71; see also Hans Martin Klinkenberg, "Artes liberales / artes mechanicae", *Historisches Wörterbuch der Philosophie*. Ed. by Joachim Ritter and Karlfried Gründer (Basel; Stuttgart: Verlag Schwabe, 1971-), vol. 1, 531-535, esp. 534.

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