THE ARCHICEMBALO OF

NICOLA VICENTINO

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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The Ohio State University
1966

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Adviser
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CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Cont.)

LIST OF EXAMPLES

LIST OF EXAMPLES (Cont.)

LIST OF PLATES

CHAPTER I

INTRODUCTION

In 1555, Don Nicola Vicentino[1] published L'antica musica ridotta alla moderna prattica. The work is divided into six books, the first called the "Book of Theory" and the remaining five the "Five Books of the Practice of Music."

In the book of theory, Vicentino discusses the three Greek genera of music, the diatonic, chromatic, and enharmonic, and the various species of the fourth, fifth, and octave. This information is attributed to Pythagoras. It appears that Vicentino's source was Boethius. In the first four books of the practice of music, Vicentino describes a method in which the three genera can be used in "modern" music. He gives examples of a notation for the three genera and examples of compositions employing the three genera.

In the sixth and final book of the text, the fifth book of the Prattica, Vicentino describes an instrument which he calls the Archicembalo. The Archicembalo is capable of playing the genera from any pitch level. This instrument, its construction, its tuning, and the notation necessary for its use, will be the concern of this paper.

In this fifth book of the Prattica, Vicentino assumes that

[1]For biographical information, see Henry W. Kaufmann, "The Life and Works of Nicola Vicentino (1511-1576)" (unpublished Ph. D. dissertation, Harvard University, 1960).

the reader understands the notation which has been described in the four previous books. Because much of what Vicentino says in the fifth book depends on a basic understanding of the notation, we shall deal with the notation first. This will be accomplished in Chapter II. Also included in the second chapter will be a description of the keyboard of the Archicembalo and the relation of the keyboard to the notation.

Chapter III describes the structure of the Archicembalo in detail. The purpose in this chapter is to solve all the necessary problems so that a working instrument could be constructed following Vicentino's design.

Chapter IV is concerned with the tuning of the instrument. The solution which is sought must fulfill two requirements. First it must meet Vicentino's requirements, i.e., it must produce the types of intervals which he describes. It must also be capable of being tuned, not with twentieth century electronic devices, but with methods employed in the sixteenth century.

Along with the description of the Archicembalo, the fifth book of the Prattica contains the instructions for tuning and exercises designed, not only to show how the system works, but also as practical studies for the performer. And in this book, Vicentino reviews much of the basic theory, because, as he states in Chapter LX (Vic. p. 143v),[2] a person might want to use this book separately.

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[2]All direct references to Vicentino's L'antica musica will be made in this manner.

The following is a brief outline of Book Five.

Chapter I: Preface.

Chapter II: Description of the Archicembalo.

Chapter III and IV: Names of the keys in the six orders of the instrument.

Chapters V through VII: Tuning of the Archicembalo.

Chapters VIII through XXXVIII: Finding all the perfect and imperfect consonant intervals above and below each key.

Chapters XXXIX through LII: Dividing the tone into semitones and dieses, beginning in any order.

Chapters LIII and LVIII: Transposition of diatonic scales to all possible levels.

Chapter LIX: Examples of all clefs.

Chapters LX through LXIV: A review of various intervals with their proportions.

Chapter LXV: Moving from one order to another.

Chapter LXVI: Discussion of fretted instruments.

The source used was the facsimile edition by Lowinsky.[3] However the lengths of the lines used in Chapter II to describe the dimensions of the Archicembalo and the photographs of the templates of the keyboards located at the end of the book were taken from an original copy located at the Sibley Library, Rochester, New York.

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[3]Nicola Vicentino, L'antica Musica . . ., ed. Edward E. Lowinsky (Kassel: Bärenreiter, 1959), pp. 99-164v.

CHAPTER II

RELATIONSHIP OF THE NOTATION TO THE KEYBOARD

Vicentino's templates, which are reproduced in Plates I, II, and III (infra pp. 5-7), are found at the end of the original text. Plate IV (infra p. 8) shows the left front corner of the first frame. These are helpful in understanding the design of the keyboard.

There are two manuals. Each manual has three rows or orders of keys. In the lower manual or first frame (Plate III), there is a total of 19 keys within the octave. In the upper manual or second frame (Plate II) there are 17 keys per octave. The combined total is 36.[1] The octave is not included in the count. In the first frame, there are seven white keys in the lowest row or first order, five black keys in the second order, and seven black keys in the third order. In the second frame the keys are the same except that the sixth order, i.e., the third row of keys on the second frame, has only five keys. The arrangement of black and white keys can be seen in the templates. For additional clarity, a three dimensional projection of one octave is presented in Figure 1 (infra p. 9) with the two frames of keys placed one above the other.

[1]Both Reese (Gustave Reese, Music in the Renaissance (New York: W. W. Norton & Co., Inc., 1954), p. 530) and Apel (Willi Apel, Harvard Dictionary of Music (Cambridge, Massachusetts: Harvard University Press, 1960), p. 47) say that the instrument had six manuals with 31 keys per octave.

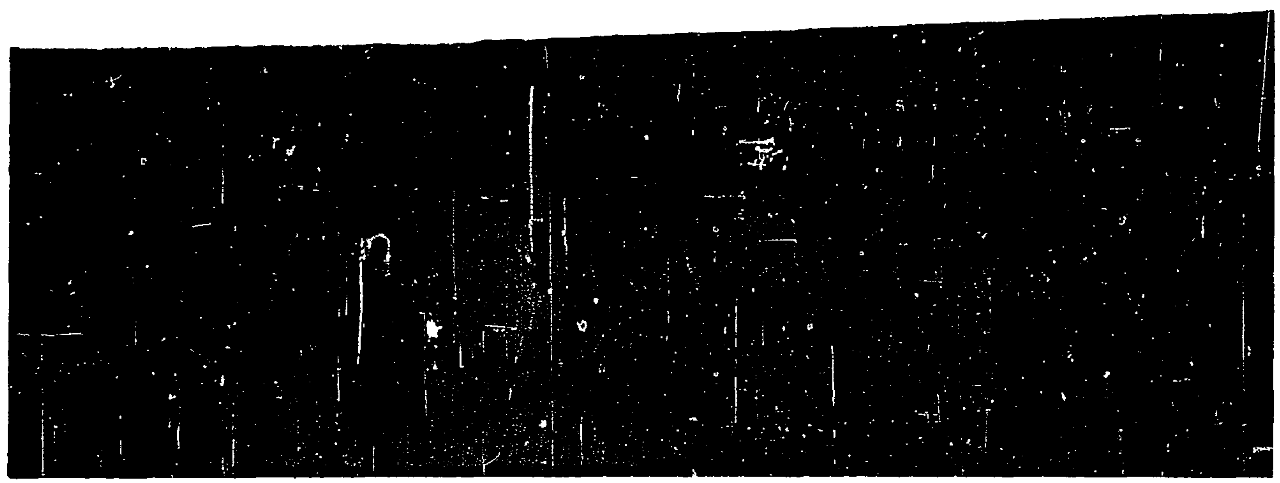


PLATE I

REGISTER TEMPLATE (Reduction 1: 4.25)

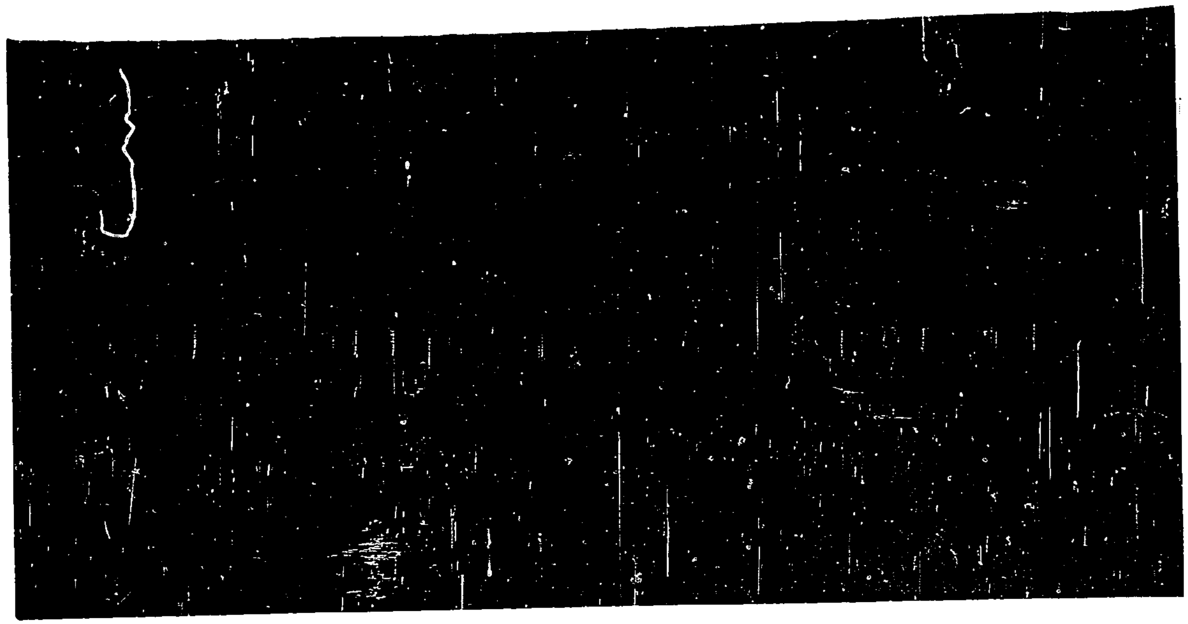


PLATE II

SECOND FRAME (Reduction 1: 4.25)

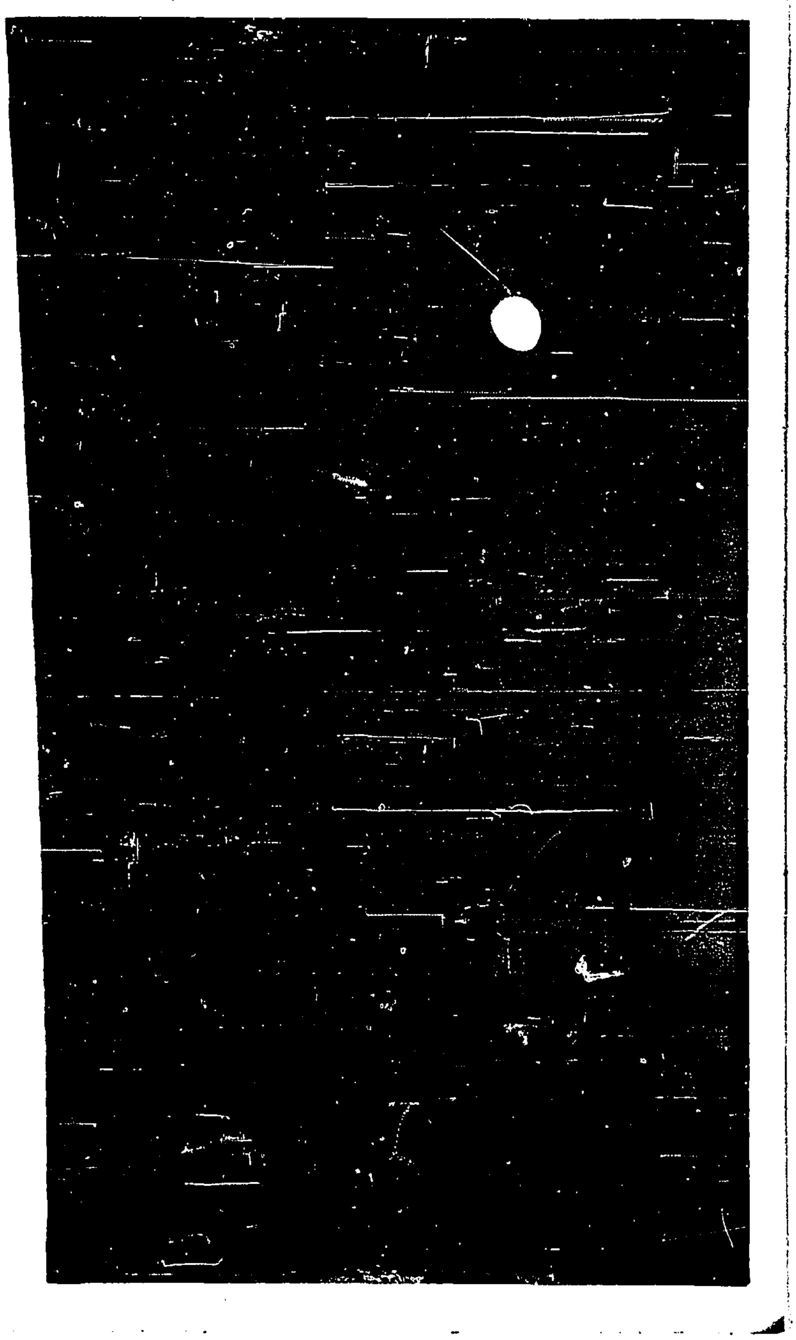


PLATE III

FIRST FRAME (Reduction 1: 4.25)

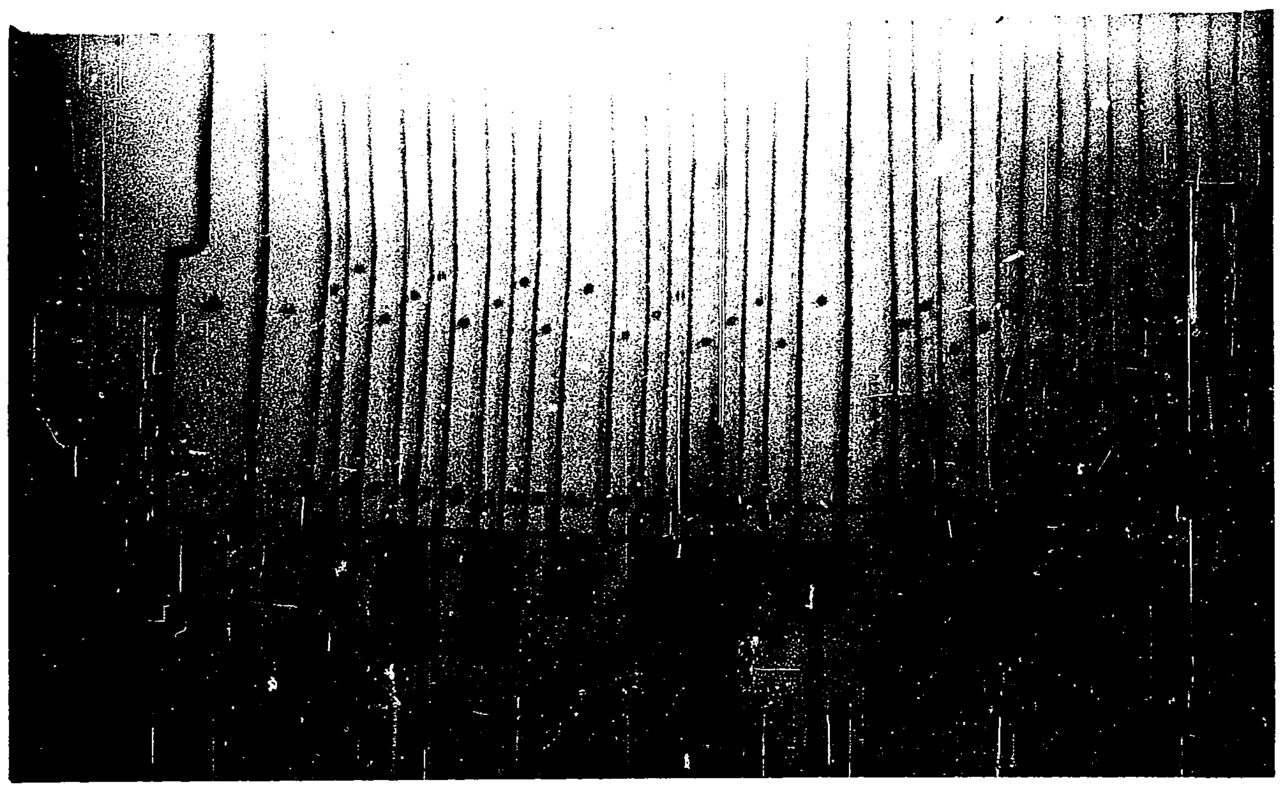


PLATE IV

DETAIL OF FIRST FRAME (Reduction 1: 1.91)

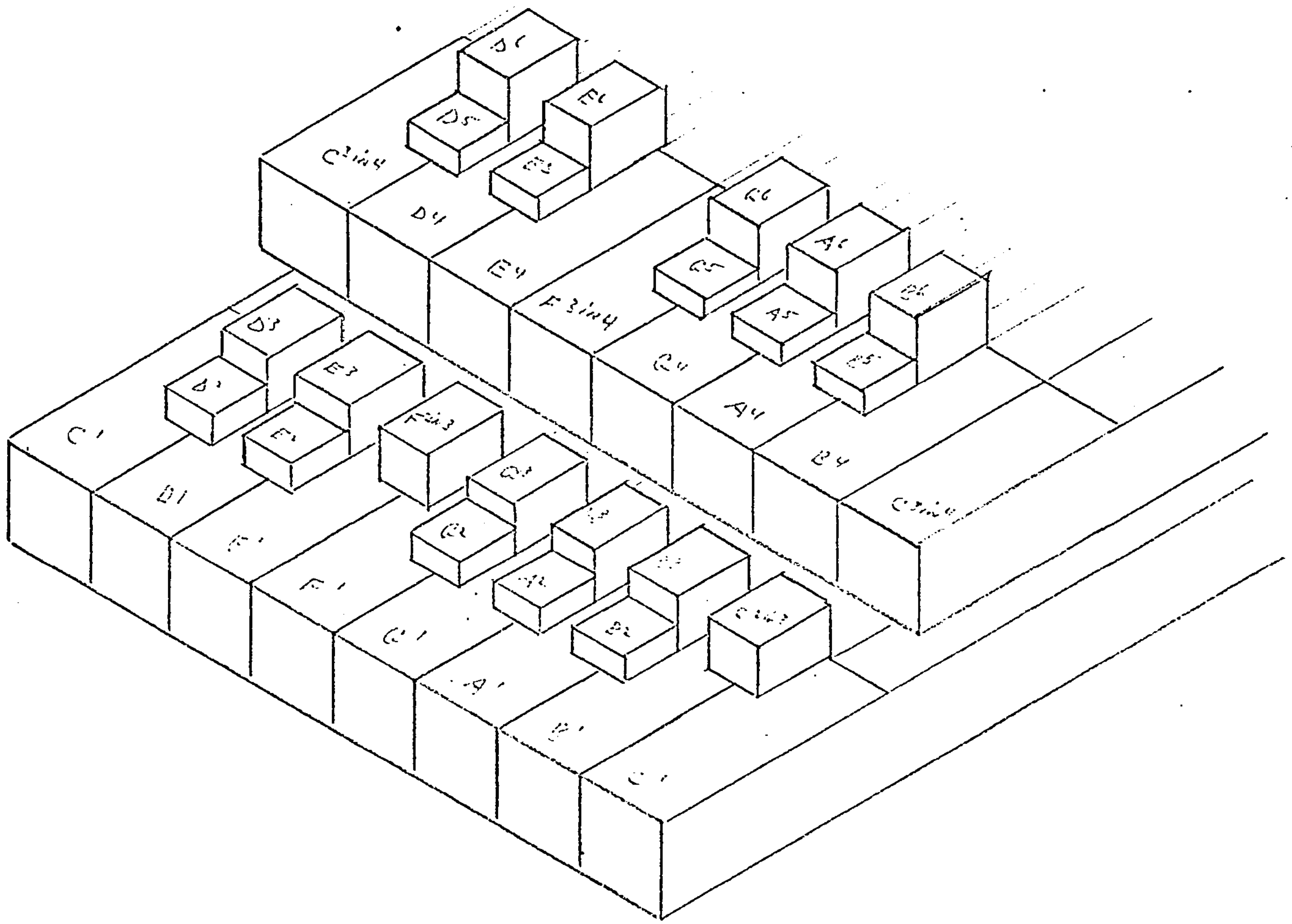


FIGURE 1. TWO FRAMES OF KEYS (SCALE APP. 2:1)

In Chapter IV (Vic. pp. 102-103v), Vicentino gives the names of the keys on the instrument. First he says that the first order of keys, i.e., the first row of white keys on the lower manual, are like those of the common keyboard. These are given the common letter names A through G. Whenever Vicentino refers to one of these keys he gives the letter name plus "first," meaning in the first order. For example, "A first" means "A in the first order of keys."

The black keys placed between the white keys are given the name of the white key to the right as one is facing the instrument. For example, the first black key between G and A, i.e., the black key in the second row or second order, is given the name A, and called "A second," meaning the A in the second order. "A third" is then the black key directly above "A second." "A fourth" is the white key in the second frame of keys, directly above "A first." "A fifth" and "A sixth" are the black keys between G and A in the fifth and sixth orders respectively. Thus in one octave on the instrument we find six A's, six B's, six D's, six E's, and six G's.

In the case of C and F, there are no keys in the second, fifth, or sixth orders. Thus Vicentino speaks of C second in the third order, meaning the second C on the instrument but in the third level of keys. Or he speaks of C third in the fourth order, meaning the third C on the instrument in the fourth level of keys. This would all be clear enough except for the fact that these names are sometimes shortened in the text to C second, C third, and C fourth. The C second and C fourth present no problem meaning C second in the third order and C third in the fourth order respectively. However the meaning of C third must be determined from context, but it is usually clear. The same is true for the F keys as is true for the C keys.

To add another problem to C and F, Vicentino, on several occasions, refers to C fourth in the sixth order and F fourth in the sixth order. These two keys were apparently in the original conception of the instrument, but in Chapter XVII (Vic. p. 109v) and again in Chapter LVIII (Vic. p. 133v), he states that there was no room for the jacks.

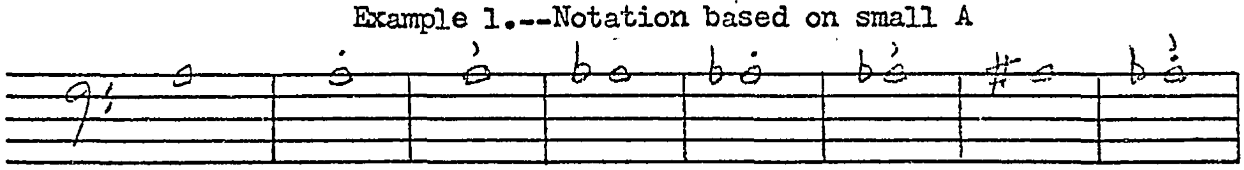
When it is necessary to indicate the specific octave of a key, Vicentino uses the hexachord syllables. For example, A re first is Large or Great A first, A la mi re first is Small A first, and high A la mi re first is One line A first.

The names given to the keys of the instrument can be confirmed by looking at the two templates of the lower and upper frames of keys. But the key names are, in a sense, a tablature, i.e., the names indicate only which keys are to be played on the instrument and do not give any direct indication of the pitch or relationship between any two pitches. Nor do they directly imply a one to one relation with the staff notation as we shall see.

As mentioned before, the keys are named with the seven letters, A through G. As we go through the text and find a first order key discussed, and this discussion is followed by a musical example, we find that the staff notation for this first order key is always identical to modern notation. Thus the familiar seven letter names of the natural notes, their location on the white notes of the lower keyboard, and their staff notation can serve as the point of departure into the understanding of the instrument.

In the course of the Fifth Book the following staff notation can be found in the examples for the note A. All are written in Example 1 on small A.

Example 1.--Notation based on small A



So that it will be possible to speak of these symbols in the text without always giving an example on the staff, the following names and abbreviations will be assigned to the symbols; A natural (A), A dot (A.), A flat (Ab), A flat dot (Ab.), A flat comma (Ab,), A sharp (A#), and A flat dot comma (Ab.,), respectively.

It quickly becomes apparent that in Vicentino's system of notation all of the accidental markings, sharp, flat, dot, and comma, apply only to the note that they are placed either before or above and do not have any effect on any other notes, even a note immediately following.

There are eight symbols based on A and only six keys named A. Therefore it must be the case that either the symbols used in the staff notation do not refer directly to the keys on the instrument, or that some symbols mean the same thing. But it is important that the distinction is kept between the staff notation and the names of the keys until this problem is solved. We do know at this time that the

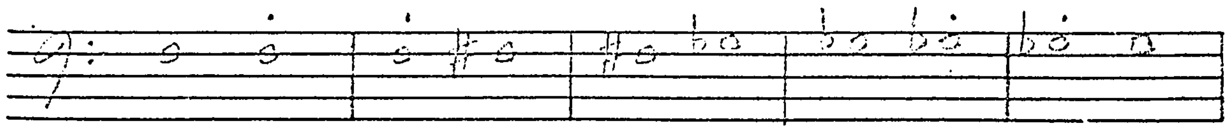
A natural does refer to A first, but we have no relationships between the other symbols and keys of the instrument.

At the end of Vicentino's Chapter LIX (Vic. p. 143), we find that a tone can be divided into five parts, called minor dieses or minor enharmonic dieses. From Vicentino's example, shown in Example 2 (infra p. 14), we can deduce that a dot above a note symbol raises that note by one minor diesis, a sharp raises it by two minor dieses or a minor semitone, a flat lowers it by two minor dieses, and a flat plus a dot lowers it by one minor diesis. With this information, plus that given in Chapter LX (Vic. p. 143v), that G natural to A flat is called a major semitone, i.e., three minor dieses, and B natural to C natural is also a major semitone as is E natural to F natural, we can write an ascending scale of dieses for one octave using Vicentino's notation (Example 3, infra p. 14).

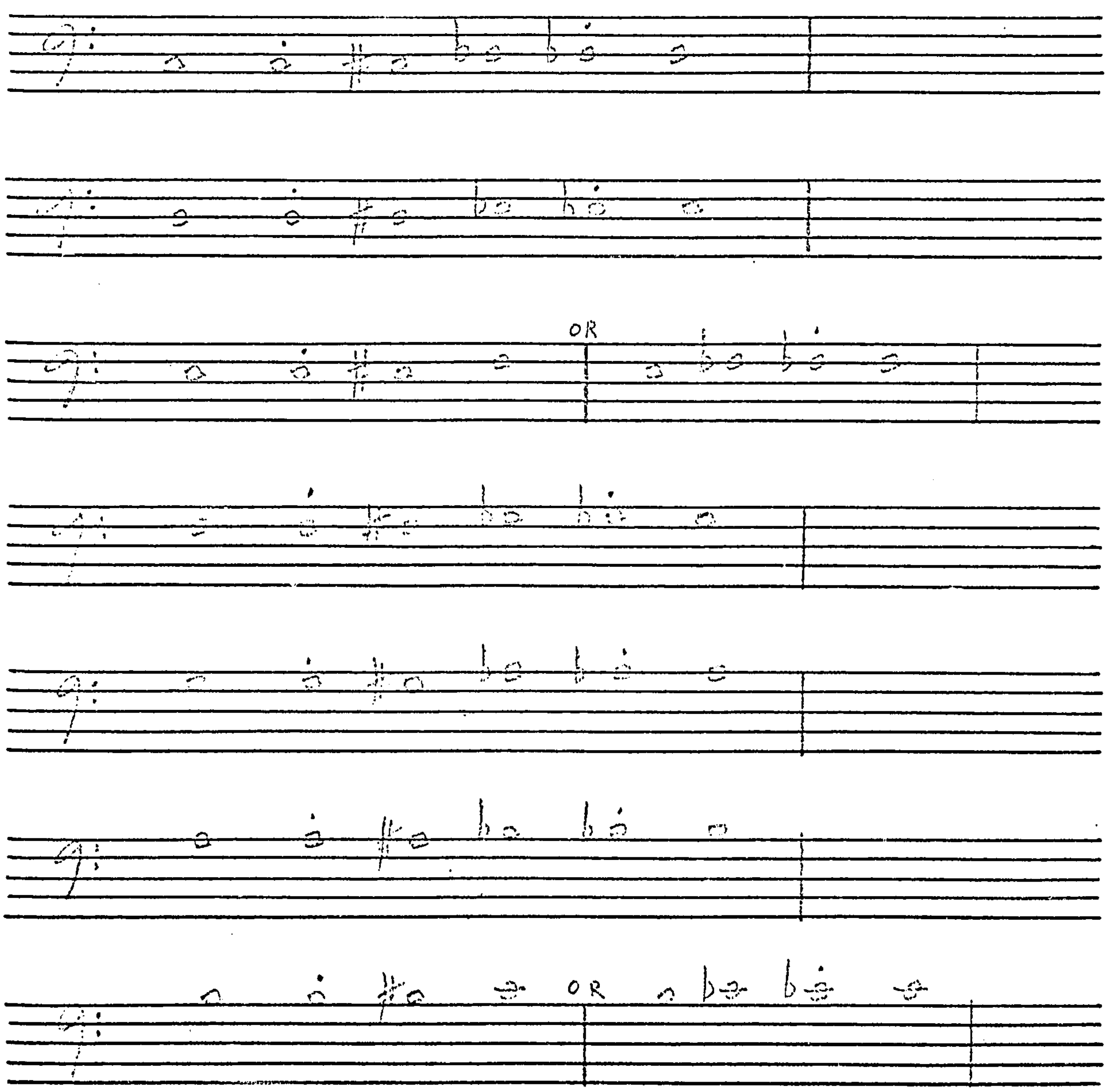
With the information available at this point, both notations given in Example 3 for B to C and E to F are logical possibilities. Vicentino usually employs the first shown in the example, but in the octave and clef studies, Chapters LIII through LIX, the second notation is occasionally used in such a way that it must be considered equivalent. For example in the clef studies, Chapter LIX, the low F flat is used as the octave below E dot (Vic. p. 140v).

We are also told at the end of Chapter LIX that the minor diesis can be divided into two commas, or a tone into ten commas. In Chapter XIII (Vic. p. 108) it can be determined that a comma (,) placed above a note symbol raises that symbol by the interval of one comma. Thus in the staff notation, the interval from A natural to

Example 2.--Division of a tone

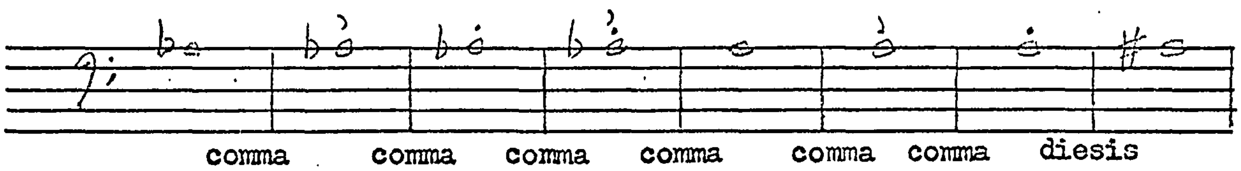


Example 3.--Scale of dieses



A comma is an ascending interval of one comma and then from A comma to A dot is also an ascending interval of one comma. It is now possible to go back to Example 1 (supra p.12) and arrange these eight symbols in an ascending order as shown in Example 4.

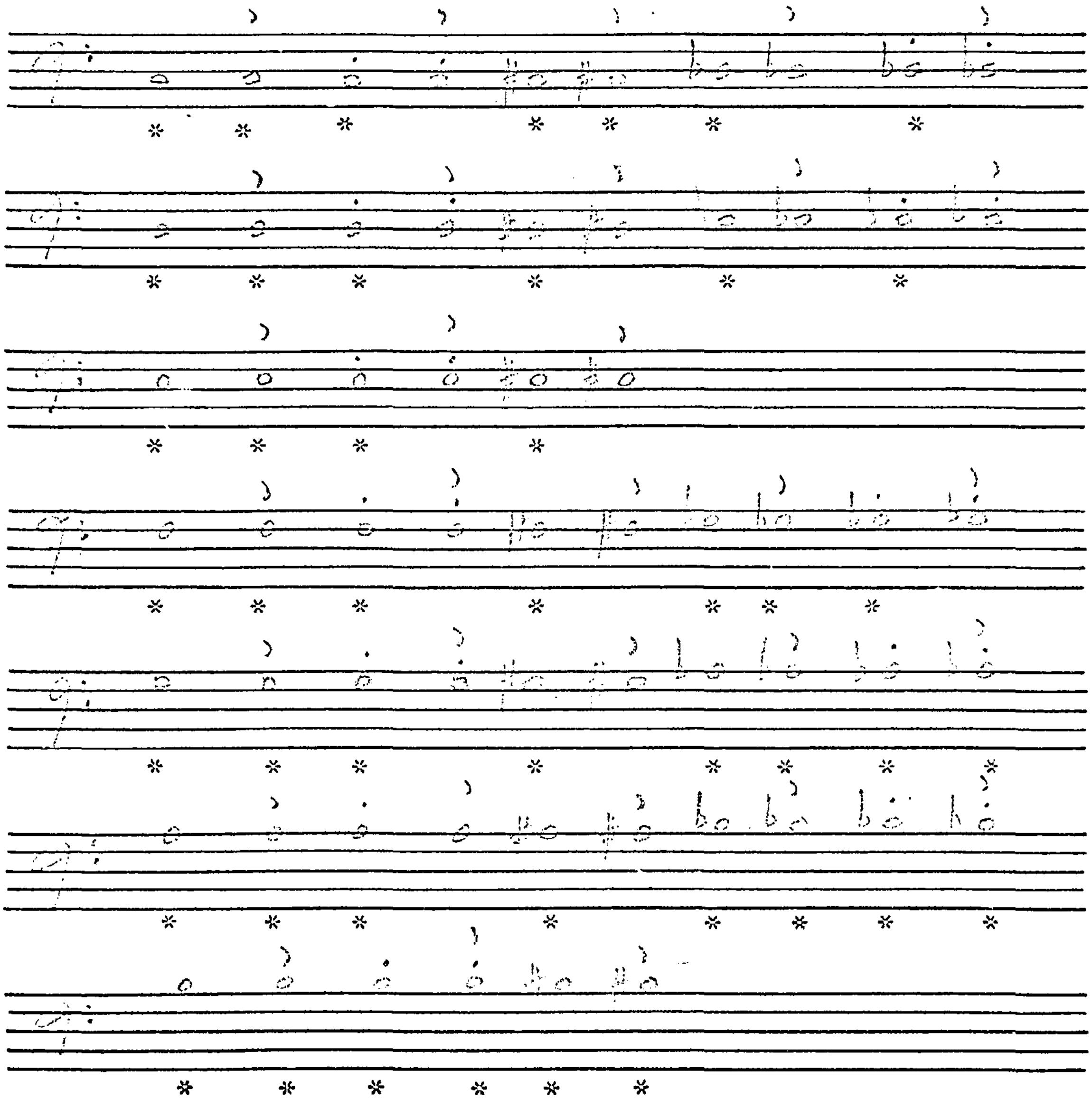
Example 4.--Notation in ascending order



Logically this system of notation produces an octave of 62 commas, 10 for each of five tones and 6 for each of two major semitones. The complete octave is shown in Example 5 (infra p.16). The 46 notes marked with the asterisk (*) are those which are employed by Vicentino in the Fifth Book.

In addition to the comma and the minor enharmonic diesis, Vicentino speaks of several other intervals smaller than a tone. These include the major enharmonic diesis, which is equal to two minor enharmonic dieses; the major semitone, which is equal to three minor enharmonic dieses; and the minor tone, which is equal to two minor semitones or four minor enharmonic dieses.

Example 5.--Octave of 64 commas



Vicentino also discusses several new consonant intervals. First there is the distinction between the common or altered fifth and the perfect fifth. These intervals are equal to 18 minor dieses and 18 minor dieses plus one comma respectively. Then each of the imperfect consonant intervals has a "near related" and a "nearest related" size.[2] The near related size is the imperfect consonance, either ascending or descending, plus one minor enharmonic diesis. The nearest related size is the imperfect consonance plus or minus one comma. Both kinds of the nearest related intervals may be found in the text, but Vicentino strongly prefers the imperfect consonance plus the comma. Table 1 (infra p. 18) gives a complete listing of all the intervals that are used in the text of the Fifth Book.

In order to establish the relationship of the staff notation and the keys of the instrument it will be necessary to examine the three chapters in which Vicentino discusses the tuning of the Archicembalo, Chapters V, VI, and VII (Vic. pp. 103v-105). At this time we shall not be concerned with the cents values of the pitches involved, but only there relative position in the scale. The problem of the cents values will be discussed in Chapter IV of this work.

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[2]Original "propinque" and "propinquissima," respectively. Kaufmann (op. cit., p. 182) translated these as "near" and "nearest." Barbour (James Murray Barbour, "Equal Temperament: Its history for Ramus (1482) to Rameau (1737)" (unpublished Ph. D. dissertation, Cornell University, 1932), p. 63) translated these as "neighbor" and "near neighbor."

TABLE 1

INTERVALS

Name	Description
1. Comma	Smallest interval in the system, 10 per tone, 62 per octave.
2. Minor enharmonic diesis	2 commas, 5 per tone, 31per octave.
3. Major enharmonic diesis	2 minor enharmonic dieses.
4. Minor semitone	2 minor enharmonic dieses (MED), equal to major enharmonic diesis.
5. Major semitone	3 MED.
6. Minor tone	4 MED, or 2 minor semitones.
7. Tone	5 MED, or one minor semitone plus one major semitone.
8. Major tone	6 MED, or one tone plus one MED.
9. Minor third	8 MED, or one tone plus one major semitone.
10. Nearest related of minor third.	Minor third plus or minus one comma.
11. Near related of minor third . .	9 MED, or minor third plus one MED.
12. Major third	10 MED, or two tones.
13. Nearest related of major third.	Major third plus or minus one comma.
14. Near related of major third . .	11 MED, or major third plus one MED.
15. Fourth	13 MED, or two tones plus one major semitone.
16. Common fifth	18 MED, or three tones plus one major semitone.
17. Perfect fifth	Common fifth plus one comma.
18. Minor sixth	21 MED, or three tones plus two major semitones.
19. Nearest related of minor sixth.	Minor sixth plus or minus one comma.
20. Near related of minor sixth . .	22 MED, or minor sixth plus one MED.
21. Major sixth	23 MED, or four tones plus one major semitone.
22. Nearest related of major sixth.	Major sixth plus or minus one comma.
23. Near related of major sixth . .	24 MED, or major sixth plus one comma.

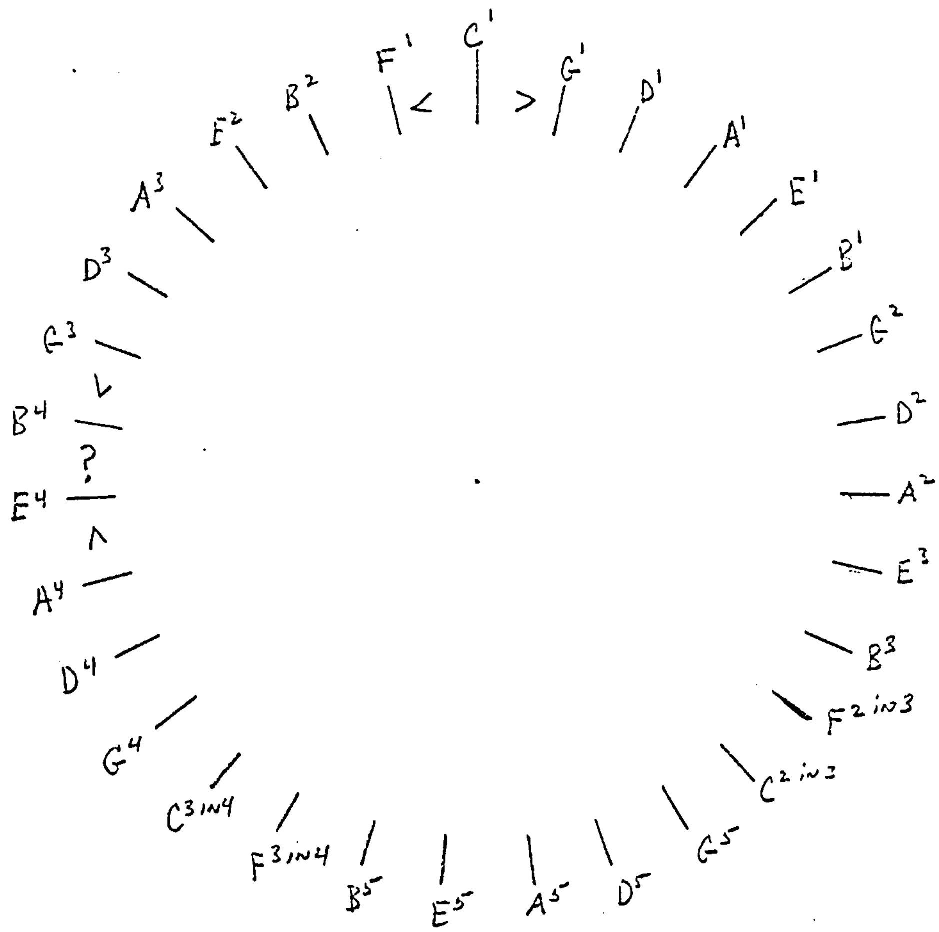
In the method of tuning Vicentino gives in his Chapter V (Vic. p. 103v), which shall be called the first tuning, the tuner is told to tune the first and second orders of the keys "in accordance with the usage of the other [common] instruments, with the fifths and fourths somewhat altered..." All that this tells us is that we have twelve keys, and we assume their octaves, the seven in the first order and the five in the second, tuned with fourths and fifths which apparently are not perfect. Then "...he will play the key of A la mi re second, that in practice one calls G sol re ut sharp, ... its fifth above, which will be high E la mi in the third order." Thus A second is G sharp. Then later, "He will play the key of high E la mi flat in the second order, and he will temper its fifth which will be A la mi re third." E second is E flat. Now there are exactly twelve pitches in a series of fifths from E flat to G sharp and these are the twelve pitches used in what are usually thought of as the common tunings of the period, i.e., meantone tuning. There is an exception, which should be mentioned, Arnold Schlick tuned from A flat to C sharp.[3]

If E second is E flat, then B second must be B flat. Then if A second is G sharp, then it must be that D second is C sharp and G second is F sharp. This accounts for all twelve keys within each octave in the first and second orders and all twelve pitches in the series of fifths from E flat to G sharp. The series of fifths written in terms of keys on the instrument would appear as E2, (i.e., E second) B2, F1, C1, G1, D1, A1, E1, B1, G2, D2, and A2.

[3]Arnold Schlick, Speigel der Orgelmacher und Organisten, ed. Ernst Flade (Kassel und Basel: Bärenreiter, 1951), pp. 34 and 35.

Vicentino then continues the series of fifths in both directions, but he only gives this series in terms of the keys on the instrument without examples of staff notation, and he only mentions the first five orders of keys. There is nothing said in this chapter, Chapter V, about the sixth order. The sequence of fifths which he gives is as follows, the places where octave shifts occur have been omitted. Descending by fifths: E2 to A3, to D3, to G3, to B4. Then ascending by fifths: A2 to E3, to B3, to F2 in 3 (i.e., F second in the third order), to C2 in 3, to G5, to D5, to A5 to E5, to B5, to F3 in 4, to C3 in 4. Then he says continue the fourth order as the first, i.e., C3 in 4 to G4, to D4, to A4, and to E4. The question arises immediately as to whether this sequence should continue to B4. B4 was tuned in the descending series. Is this fifth E4 to B4 supposed to be the same size as the rest of the fifths? We will return to this question in a moment, all that can be said at this time is that the first tuning appears to be based on a series of thirty fifths ascending from B4 to E4, involving all the keys in the first five orders. This series is shown in Example 6 (infra p. 21) in terms of the keys on the instrument.

The second tuning of the instrument, i.e., that described by Vicentino in Chapter VI, is the same as the first tuning in the first three orders of keys. Then the keys in the fourth, fifth, and sixth orders are tuned so that they will produce perfect fifths above the first, second, and third orders respectively. For example, from D first to A fourth is a perfect fifth, D second to A fifth is a perfect fifth, and D third to A sixth is a perfect fifth. Then



Example 6.--Series of 30 fifths

Vicentino states that one can play the upper keyboard as one does the lower. This would imply that the upper frame of keys has the same internal relationships as the lower frame of keys, but the two frames are separated in pitch by the difference between a perfect fifth and a common fifth, i.e., by a comma.

Now the question must be asked, are these two methods of tuning supposed to yield the same result? The answer can be found in the final statements of Chapter VI, the chapter on the second tuning.

> One will be able to make an organ that will be divinely tuned with the first tuning without the perfect fifths. And then one will add a register with the perfect fifths tuned in the above method, according to the arrangement of the perfect fifths. And in the organ it will not be necessary to move the tuning of this instrument as one will do in the Archicembalo. (Vic. p. 104v).

Thus these two methods of tuning, while they produce similar results in the lower frame of keys. i.e., the first three orders, produce distinctly different results in the upper frame.

Therefore in the discussion it will be necessary to keep the two methods of tuning separate, and since the second tuning is based in part on the first, we shall discuss the first tuning at this time and later return to the second. Vicentino's third chapter on tuning, Chapter VII, is only a condensation of the first tuning and yields no new information. Vicentino included it for the benefit of the tuner and in it only indicates where the fifths are located that have one member in one order of keys and the other member in some other.

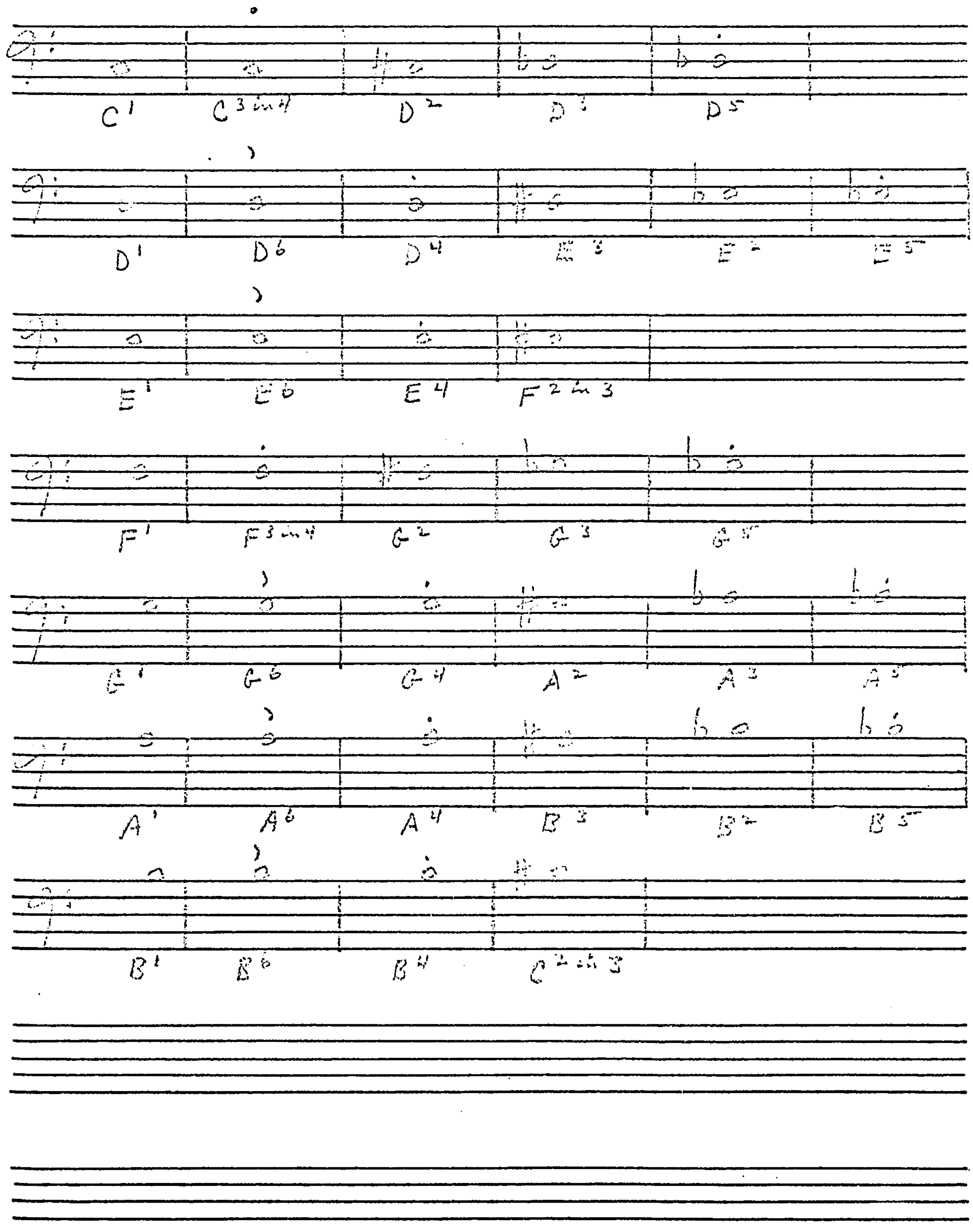
In Chapters VIII through XXXVIII (Vic. pp. 105-118v), Vicentino discusses the consonant intervals above and below each key and gives musical examples of the intervals. In total he presents 31 keys, five are missing, F third in the fourth order, G sixth, E sixth, D sixth, and B sixth. In the case of F third in the fourth order, this appears to be an unintentional omission. First it is the only key in the first five orders which is not discussed individually. Secondly, if it were included, it should follow the discussion of F second in the third order, which can be determined by the sequence used for the other keys. It is noted that the last three intervals given for F second in the third order, while they are incorrect for F second in the third order, are correct for F third in the fourth order. In the case of the other four keys, Vicentino omits them because, as he states in Chapter XIII (Vic. p. 108), he does not think there is enough to talk about in regard to their consonant intervals. But since he does speak of most of the keys separately and gives examples, it is then possible to determine the relationship between the keys of the instrument and the staff notation even though Vicentino fails to designate which of the two tunings he is using.

This can easily be determined by careful reading of the text concerning the key of G third in Chapter XVI. If Vicentino is discussing the first tuning, then the common fifth below this G3 should be given as B4. If this is the second tuning, then there would be no fifth below G3 since this is the final key in the descending series of common fifths before going into the fourth order. B4 in the second tuning is a perfect fifth above E1. Chapter XVI (Vic. p. 109) gives the common fifth below G3 as B4. This would indicate the first tuning and the first tuning can be confirmed by going through all the keys in Chapters VIII through XXXVIII and finding the common fifths above and below each key. This process produces exactly the sequence of fifths given for the first tuning.

It is now possible to relate the keys with the staff notation for the first five orders of the instrument in the first tuning. In the case of the sixth order keys, which were omitted in the chapter on the first tuning as will be recalled, only one of these is discussed specifically in Chapter XIII, the key of A sixth. But from this chapter we can determine the notation for A sixth in the first tuning, which is A comma, and then by implication, the other four keys in the sixth order, which would also be notes raised by a comma. The staff notation of one ascending octave for the first tuning is given in Example 7 (infra p. 25) together with the key on which each note can be found.

The notation of the second tuning is not as clearly described as that of the first. However there is enough information to produce an accurate notation for this tuning. First we know that the tuning of the first three orders is the same in the second tuning as in the first. The notation for these three orders can be assumed to be the same. In terms of the sequence of fifths given in Example 6 (supra p. 21), this would include the keys from G3 (G flat), to C2 in 3 (B sharp).

Example 7.--Notation for first tuning



In Chapter XIII, concerning A6 in the first tuning, we are told that A6 down to D1 is a perfect fifth. In the second tuning A4 down to D1 should be a perfect fifth. Since A6 is notated A comma in the first tuning, then it must be the case that A4 in the second tuning is notated A comma since it is supposed to have the same relationship to D1. If this is true then it would follow that in the second tuning G4, a perfect fifth above C1, is notated G comma; B4, a perfect fifth above E1, is B comma; C4, above F1, is C comma; D4, above G1, is D comma; E4, above A1, is E comma; and F4, above B1, is F comma. But in this latter case we find that a conflict of information develops.

If F4 is a perfect fifth above B1 and E4 is a perfect fifth above A1, then E4 to F4 equals A1 to B1 or one whole step. But Vicentino says (Vic. p. 104) that one should play the upper keyboard as one does the lower. This would imply that E4 to F4 equals E1 to F1 or one major semitone. Vicentino also states (Vic. p. 104) that the fifths are symmetrical and are found in the same orders of the upper and lower frame. These latter two statements would imply that since the common fifth above B1 is G2, _i.e._, F sharp, then the common fifth above B4 and the perfect fifth above B1 is found on G5. The balance of information seems to be in favor of G5 as the perfect fifth above B1. The notation for G5 is then F sharp comma. The F4, F comma, is the perfect fifth above B2, _i.e._, B flat.

The fifth order would then consist of F sharp comma, G5; G sharp comma, A5; B flat comma, B5; C sharp comma, D5; and D sharp comma or E flat comma, E5. In this latter case D sharp comma would be the perfect fifth above G sharp, second order to fifth as are the others. But if the two frames of keys are kept symmetrical then E5 should be E flat comma since E flat is in the second order not the third. On the basis of the previous decision, the more probable choice is E flat comma for the key E5.

The sixth order would then be G flat comma, G6; A flat comma, A6; A sharp comma, B6; D flat comma, D6; and D sharp comma, E6. The notation for all 36 keys in the second tuning is shown in Example 8 (infra p. 28) arranged in ascending order.

Now let us examine a statement which occurs near the end of Chapter VIII (Vic. p. 105v), concerning the consonant intervals below and above A first. Taken out of context it is at best confusing, but taken in light of what has just been presented concerning the intervals, the two systems of tuning, and the notation of each, it acts like the final piece to a puzzle.

> Now the discussion is finished which includes all the consonances of A la mi re first both below and above, and thus of perfect as well as imperfect, and in addition their near related. . . . The nearest related consonances occur when the instrument will be tuned with the perfect fifths as I have already said in describing the tuning of it.

The reader will recall that in this chapter Vicentino is finding intervals in the first tuning. In this the related of the minor third and minor sixth above A1 are C4 and F4 respectively, or C dot and F dot. Now in the second tuning. i.e., the tuning with the perfect fifths, C4 and F4 are C comma and F comma respectively, i.e., the more related of the minor third and minor sixth, and neither pitch was avaible in the first tuning. It would also be true that the more

Example 8.--Notation for second tuning.

related of the major third and major sixth above A would be avaible in the second tuning and not in the first, C sharp comma, found on D5, and F sharp comma, found on G5.

CHAPTER III

THE ARCHICEMBALO

As previously stated in the first chapter, the dimensions presented in the translation that will be used in this chapter were taken from an original copy of "L'antica Musica" located at the Sibley Music Library, Eastman School of Music, Rochester, New York. Although all the dimensions given and used in the subsequent calculations are in terms of one hundredths of an inch, there are two factors which make such implied accuracy impossible. First, the quality of the printing is such that the ends of the given lines are not always square and clear. Second, the pages of the book have become somewhat wrinkled. Therefore all dimensions should be read assuming a possible error of plus or minus 0.03".

It is impossible to take accurate measurements from the three templates. The printed lines of the templates are relatively thick and are not straight. Also the templates are printed on large sheets of paper which have been folded to the size of the other pages of the book. Because of the age of the paper, these folded pages will not lie flat.

The facisimile edition of "L'antica Musica" was found to be quite accurate in the reproductions of the lines within the body of the text. Variations in the lengths of these lines, compared to the Sibley Library copy, were less than 0.05". However the templates have not been reproduced at the original size. Comparing the facsimile templates to the original at the Sibley Library, the template of the first frame of keys is about 0.70 times the size of the original, and the template of the second frame of keys is about 0.95 times the original. The template of the register is reproduced at the original size.

The three templates in the original are bound in such a way that the three can be unfolded and placed one on top of the other, the register template on top, the second frame of keys second, and the first frame of keys third. When this is accomplished, it is possible to see the keyboards of the first and second frame in there proper relationship, i.e., as they would appear on the instrument (See Plate V infra p. 32). Because of the different sizes of the reproductions, this is impossible in the facsimile edition.

To begin the discussion of the Archicembalo, we shall first make a list of the important points of Vicentino's instructions as they are found in Chapter II (Vic. pp. 100-101).

1. Along with the instructions, there are to be found at the end of the book, three templates or patterns, drawn to actual size.
    a. The first template discussed, and the last printed in the book shows the first or lower frame of keys. This frame of keys is to be constructed such that it is possible to remove it and the keys as a unit. There are 69 keys shown on the first frame (Plate III, supra p. 7).
    b. The second template discussed and the second printed in the book shows the second or upper frame of keys. This

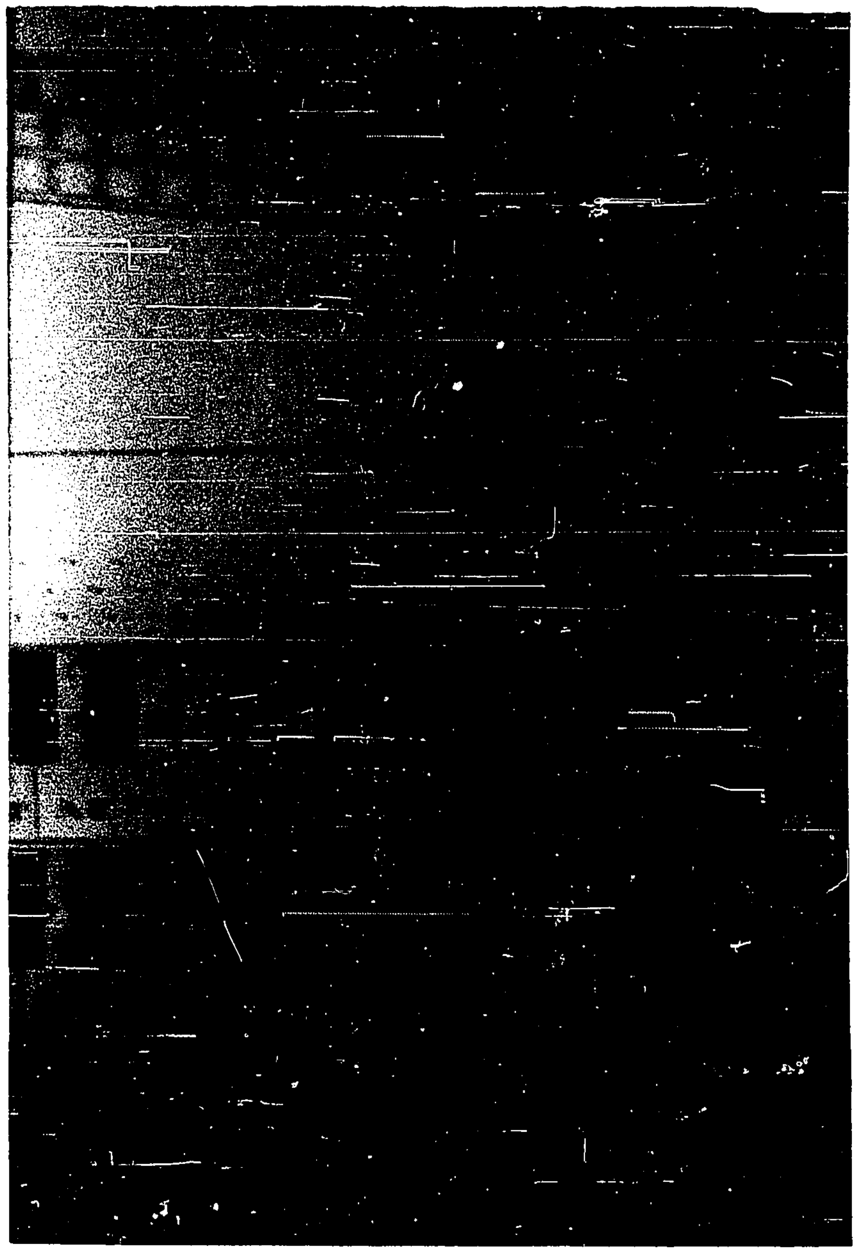


PLATE V

RIGHT SIDE OF THREE TEMPLATES PLACED

ONE ABOVE THE OTHER (Reduction 1: 2.50)

frame is also constructed so that it can be removed as a unit. There are 63 keys shown on the second frame. Also this keyboard will be pierced so that some of the long jacks, i.e., jacks from the lower frame, are able to pass through this frame of keys. These holes are not shown on the template. (Plate II, supra p. 6).

c. The third template discussed and the first printed shows the top view of the soundboard from the nameboard to the jack guide or register, including the tuning pins, nut, some strings, and some jacks. (Plate I, supra p. 5). However there are only 130 places for the jacks shown in the jack guide for the 132 jacks.[1] The problem raised by these missing jacks will be discussed later (infra p. 57).

2. The length of the instrument is to be 20 times a line 3.86" or 77.20", and the width is 8 times the same line or 30.88".

3. The depth of the instrument is 2 times a line 4.26" or 8.52".

4. The line which indicates the height of the top of the instrument from the soundboard is missing. This is also the height from the lower level of the instrument to the first keyboard.

[1]The facsimile edition shows only 128 places for the jacks.

5. The height of the jack slide[2] and that part by the sides of the keys, i.e., the keyblocks, is measured as 2.71".
6. The height of the two keyboards one above the other is 2.06".
7. Two times 5.36" is the distance from the rose to the jacks, i.e., 10.72", and 3 times the same line is almost the length of the right hand side of the instrument, i.e., 16.08".
8. The width of the rose is 3.51" and this is also the distance from the nut to the tuning pins.
9. The distance "where the strings rest" from the bridge to the hitch pins is 4.16" at the right hand side of the instrument and becomes wider beyond the middle of the instrument.
10. The lengths of the six orders of keys, first to sixth order resp., are 2.68", 1.54", 0.83", 2.42", 1.49", and 0.85". These lengths do not agree completely with those shown on the templates. The mean lengths on the templates are 2.48", 1.46", 0.82", 2.42", 1.34", and 0.74", resp. The former set of numbers will be used subsequently for reasons already explained.
11. The height of the second order above the first frame is 1.34", and the height of the third order is 1.61".
12. Lead is placed in the distal end of the long keys.

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[2] Orig., <u>morto</u>, see following discussion (infra p. 48).

13. Shammy should be placed under the jacks.
14. Four irons are used to support the second frame and pass through four holes bored near the middle of the frame. These holes are not shown on the templates.
15. The length of the highest string is two times 6.19", or 12.38".
16. The distance from the nut to the first jack is 2.48".
17. The length of the long jacks is 4.32", the short jacks, 2.90".
18. The tuning pins are placed about 0.76" from the name-board.
19. The first row of jacks will contain both long and short jacks, the second row will contain only one length.
20. The frame and keys will be bored for the pins, which will be covered with shammy, except those at the distal end of the second frame which are covered with cloth.
21. The quills are soft and short.
22. "And then all these previous measurements may be slightly altered according to the judgement of the good instrument maker."

Vicentino does not mention such items as the bracing of the harp or sound box of the instrument, the placement of the ribs under the soundboard, the mechanical action of the jacks, and the design of the frame upon which the keys are placed. It is obvious that Vicentino is not telling his readers how to build the Archicembalo beginning with the raw lumber, but rather telling the reader how to build the instrument providing the reader already knows how to build a mid-16th century Italian harpsichord.

For such items mentioned above we are forced to rely on other sources. Fortunately there is very little variation in the building of Italian harpsichords in the 16th and 17th centuries so that it is possible to gain a clear idea of the kind of instrument that Vicentino must have had in mind as the basis for his Archicembalo.

> Whatever laudable traits the Italian builders may have had, they cannot be considered to have been progressive. Their instruments of the mid-16th century hardly can be distinguished from those made around 1700.[3]

There is an instrument called an Archicembalo made by Vito Trasuntino of Venice, 1606. Reynolds gives two pictures of the instrument in his book.[4] However this instrument contains only 31 keys per octave arranged in five levels and it appears that all of the keys are set onto one frame. Therefore this Archicembalo is of no significant help as a model for Vicentino's Archicembalo.

In the following discussion of the Archicembalo, as there are items which Vicentino does not describe and which would be known by the

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[3]John D. Shortridge, "Italian Harpsichord Building in the 16th and 17th Centuries," Contributions from the Museum of History and Technology: Paper 15, (Washington, D.C.: Smithsonian Institution, 1960), p. 95.)

[4]Raymond Russell, The Harpsichord and Clavichord, (London: Faber and Faber, 1959), Plates 13a and 13b.

16th century builder, we shall turn to the harpsichord made in 1665 by Giacomo Ridolfi as the primary guide. This harpsichord is now in the collection of the U.S. National Museum and is clearly described by Shortridge.[5]

This discussion of the Archicembalo will be divided into two major divisions, first the side view and followed by the top view. Because Vicentino gives more information about the keys of the instrument, the discussion will begin with these, first the playing surfaces of the keys, the action, and then the frames upon which the keys are placed. At this point it is possible to determine information concerning the dimension that is missing in the Vicentino text (Vic., p. 100). Although it is not possible to state an exact dimension at this time, maxima and minima can be determined.

Following this, the length given for the jack guide will be discussed and shown to be incorrect. Another length must be found for the jack guide. The length originally given for the jack guide then is found to meet the requirements for the missing dimension. The remaining vertical dimensions and horizontal dimensions needed to complete the side view are then discussed.

The first problem in constructing the top view of the instrument is the superposition of the three templates provided by Vicentino. This will also include the assigning of the keys to the jacks and finding places for the jacks which are not included on the register template. The final portion of the discussion concerns itself with the location of the bridge and the nut.

[5]Shortridge, op. cit., pp. 100-107.

The Side View

Because there is more information given by Vicentino about the keys of the instrument than any other part, we shall begin with these, trying first to develop the side view of the action of the instrument. There is the statement that the height of the two keyboards, one above the other is 2.06". If this is interpreted to mean that from any given point on the lower frame of keys to a similar point on the upper frame is 2.06", *e.g.*, from the upper surface of the first frame white keys to the upper surface of the second frame white keys, then a conflict develops. Since all of the strings are on the same level, as shown in the register template (supra p. 5), and since there are two sizes of jacks in the first row of jacks, then it must be the case that the tops of both sizes of jacks are level. Then the height between the distal ends of the keys of the upper and lower frames must be equal to the difference between the two sizes of jacks, *i.e.*, 4.32 - 2.90 = 1.42". It is reasonable to assume that the two frames are level and parallel from back to front, so that the distance from any point on the lower frame to a similar point on the upper frame must be 1.42", not 2.06". Then the 2.06" must be the total distance from the surface of the first order keys to the surface of the sixth order keys. Since the length of the six orders of keys is also known, *i.e.*, the part of the key that extends outside of the instrument, the sketch shown in Figure 2 can be constructed showing the upper surfaces of the keys.

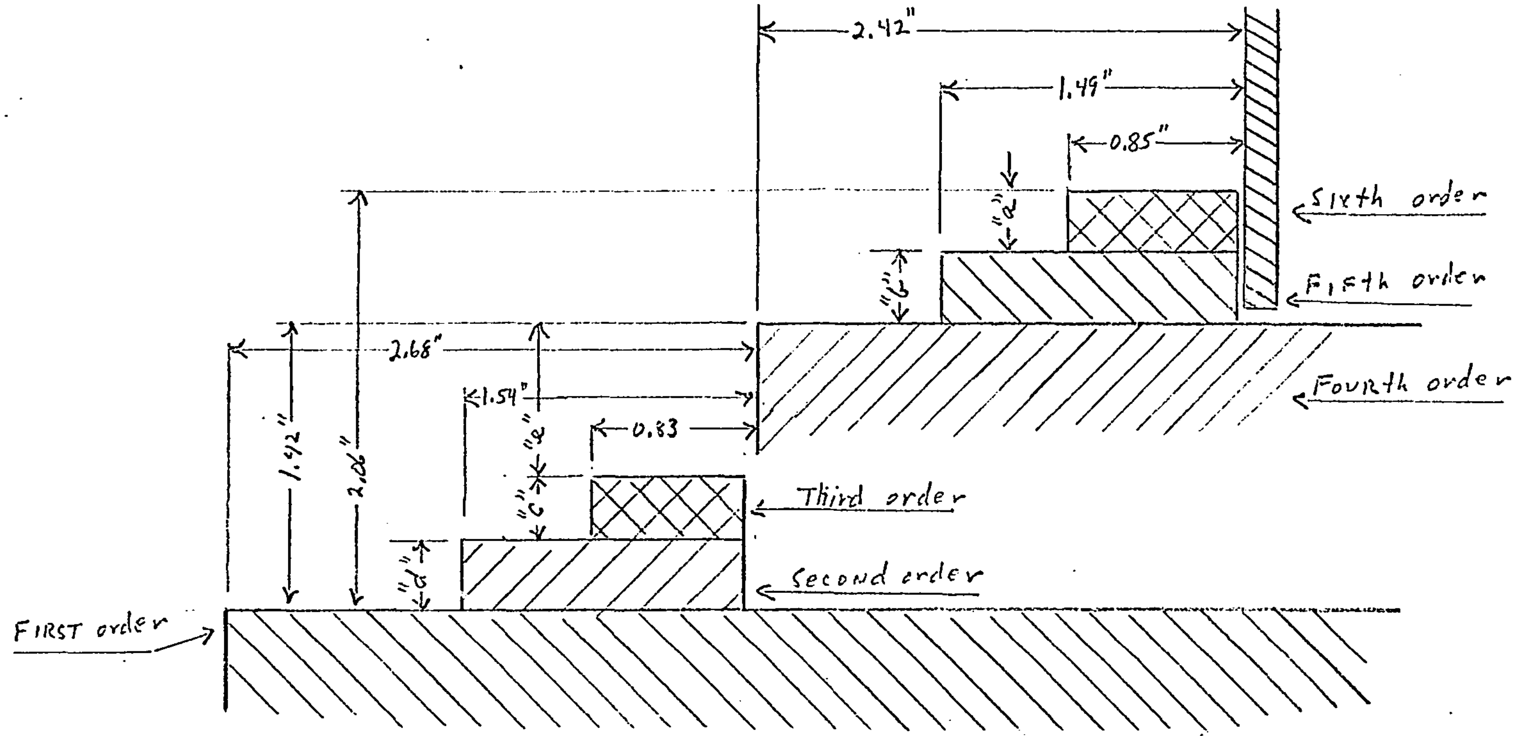

FIGURE 2. SIDE VIEW OF KEYS (SCALE 1:1)

The letters "a" through "e" in Figure 2 represent missing dimensions. In solving for these dimensions, the additional information that the height from the second frame to the top of the second order is 1.34" and the height from the frame to the top of the third order is 1.61" is all that is needed. Wherever the lower point of these dimensions may be, *i.e.*, the upper or lower surface of the frame, the difference between them should represent the height of the third order above the second (letter "c" in Figure 2). Since Vicentino only gives this information about the first frame, then it is quite probable that it would also be the height from the fifth order to the sixth (letter "a"). Thus "a" and "c" are equal to 1.61 - 1.34 = 0.27". The distance from the surface of the fourth order to the surface of the fifth (letter "b") is equal to the distance from the first order to the sixth, minus the distance from the first to the fourth, and minus the distance from the fifth to the sixth, *i.e.*, "b" = 2.06 - (1.42 + 0.27) = 0.37". This would also be the height from the first order to the second (letter "d"). The height from the third order to the fourth (letter "e") is then 1.42 - (0.37 + 0.27) = 0.78".

With the above information we can develop the action of the keys. Using the Ridolfi harpsichord as a guide, the frame consists of five pieces, one at each end running front to back, and three running left to right. The two end boards and the one at the front are of the same thickness, as shown in the Ridolfi Harpsichord.[6] The center board running from left to right is the balance rail and the back board

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[6]Shortridge, *loc. cit.*

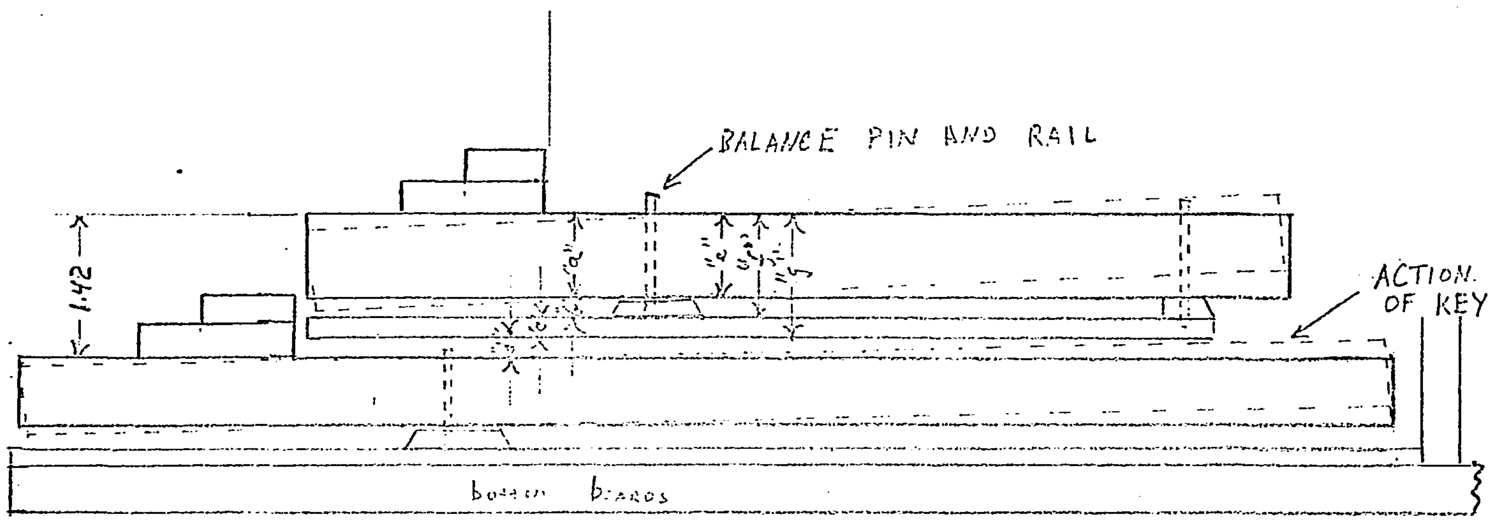

FIGURE 3, SIDE VIEW OF ACTION (SCALE≈1:2) supports the keys at rest. These latter two boards are of the same thickness, greater than the others.

Within the distance from the upper surface of the keys on the first frame to the similar points on the keys of the second frame, 1.42", four items must be taken into account, namely; (1) the thickness of the key (See Figure 3, letter "a"); (2) the height that the balance rail extends above the smaller boards of the frame (letter "b"); (3) the thickness of the smaller boards of the frame (letter "c"); and (4) clearance for the distal end of the lower frame keys so that they do not touch the upper frame when depressed (letter "d").

The depth of the action becomes critical at this point, and Vicentino is of no direct help. A limit can be set however. Since the height of the sixth order of keys above those of the fifth is 0.27", the action must be less than 0.27" at the proximal end of the key. This is a little less than the present depth recommended for harpsichord action of 5/16".[7] The Ridolfi harpsichord shows an action of 0.19" at the distal end being even less at the proximal end. If we let 0.19" be adopted as the amount of action at the distal end of the keys, "d" would have to be greater than 0.19 so that the lower keys would not touch the upper frame. Also "b", the height of the balance rail, would be about equal to 0.19", since the action would be less at the proximal end, but some room must be allowed for clearance so that the key would not touch the frame at that point.

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[7]Frank Hubbard, Harpsichord Regulating and Repairing, (Boston, Massachusetts: Turner's Supply, Inc., 1963), p. 24.

Vicentino states that the height of the second order, and therefore the fifth order, is 1.34" above the frame. Since we know that it is 0.37" from the second order to the first, then the first is 0.97" above the frame. But it is not clear whether this means to the top of the balance rail (letter "e", Figure 3), the top of the frame (letter "f"), or the bottom of the frame (letter "g"). The process of trial and error (See Appendix I) shows that "f" equals 0.97 is the correct interpretation.

If 1/16" is allowed for the clearance between the bottom of the upper frame and the top of the lower key when the key is depressed, then "d" can be set at 0.19 + 0.06 = 0.25, making "c" = 0.20, with "b" = 0.19" and "a" = 0.78". These are all workable dimensions even though 0.20" seems rather thin for the thickness of the frame. But two facts should be noted. (1) "a" is based on a straight line surface of the key from the front to the back, but if the key is veneered with a thin strip of ivory or some kind of wood, then the actual dimension of "a" would decrease by the thickness of the veneer. At the same time "c" could increase by the same amount. Since Vicentino gives exact dimensions for the amount of the key which extends outside the instrument, but also says that the width and length of the key can be varied, then perhaps the variable dimension refers to some material covering the key. The Ridolfi Harpsichord shows such a covering on the white keys of the instrument.[8] (2) Vicentino also states that four iron posts

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[8]Shortridge, loc. cit. See also Plate 5 in Russell (op. cit.) showing an Italian harpsichord by Jerome of Bologna, 1521, which appears to have a covering or veneer on the white keys.

are to be placed near the middle to support the second frame, which implies that the second frame needs support.

Combining the information which we now have with the side view of the Ridolfi Harpsichord[9] and Figures 10 and 11 of Hubbard's _Harpsichord Regulating and Repairing_,[10] we can develop a picture of the two frames with their keys. Figure 4 (infra p. 45) shows a portion of the two frames with several keys on each frame.

With the present information concerning the vertical dimensions of the two frames, it is now possible to determine some facts about the missing line. This missing line, which shall be called "x" is to be, "the height from the top to the soundboard" and also "the height of the first keyboard or the first frame to the lower level of the instrument."

Working first with the height from the top to the soundboard, it is possible to establish a minimum for the missing dimension. This height is equal to the total of the height of the nut, the part of the jack which extends above the strings and contains the damper, the upward action of the jack, the thickness of the jack rail, and the distance from the top of the jack rail to the top of the instrument.

The height of the nut of the Ridolfi instrument[11] varies from 3/8" to 7/16". The mean, 0.40", will be used here. The height of the part of the jack above the string is not known but a minium of 0.25" is reasonable. The Ridolfi instrument[12] shows about 0.25" for this

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[9]Shortridge, _op_. _cit_., p. 103.

[10]Hubbard, _op_. _cit_., p. 10.

[11]Shortridge, _op_. _cit_., p. 102.

[12]_Ibid_.

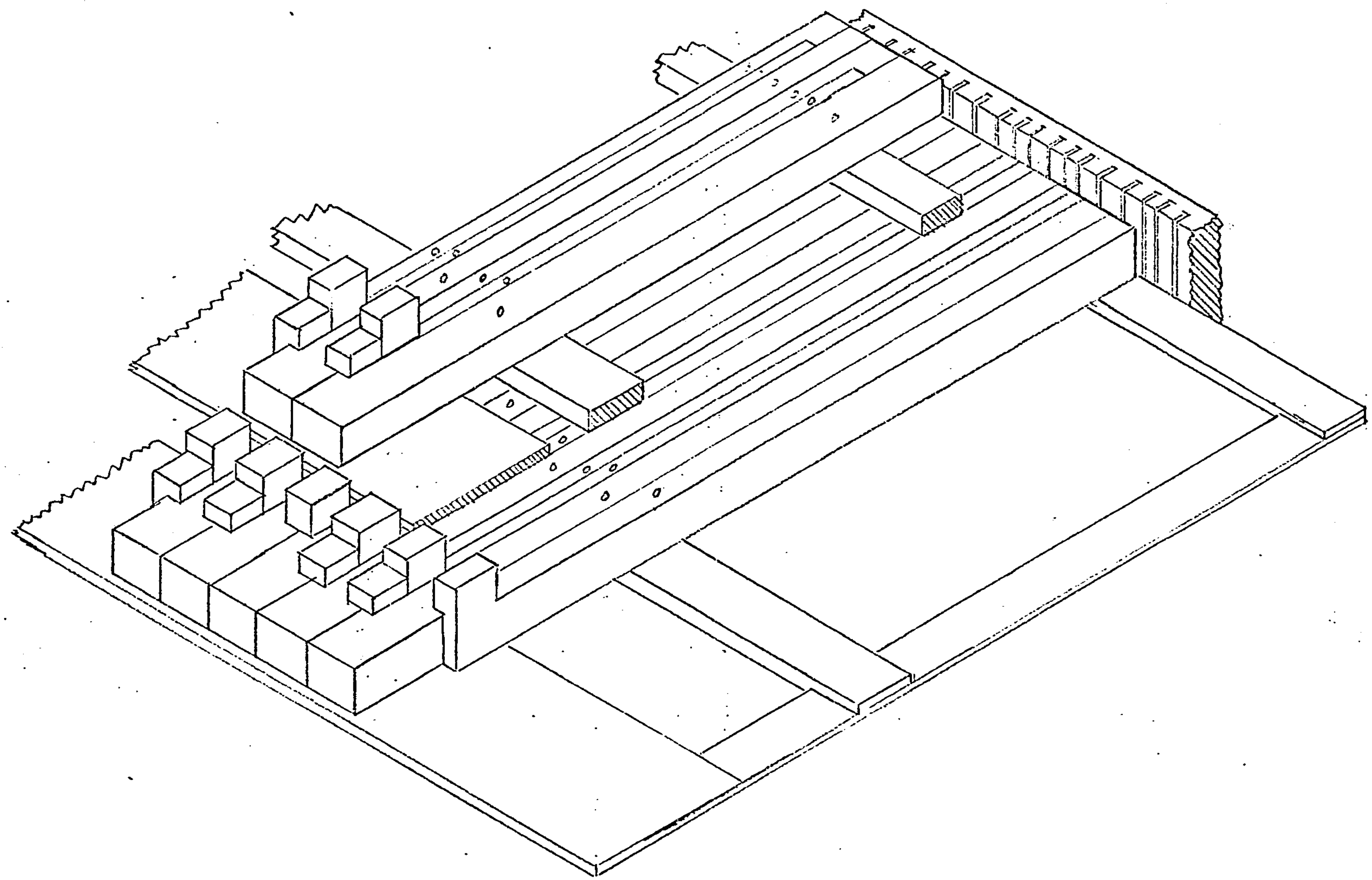

FIGURE 4. TWO FRAMES (NOT TO SCALE)

dimension. A jack from an instrument of Kirckman of 1775[13] shows about 0.40". For the action, 0.19 will again be used. The thickness of the jack rail is again unknown. The Ridolfi Harpsichord shows a thickness of 3/4" at the center and almost an inch at the edges where moulding has been added.[14] Since a minimum is being established, let 0.50" be used, for a smaller dimension would not be rigid enough over the distance from side to ~~side~~ of the instrument of some 31". The harpsichords of the period also show that the top of the jack rail is not level with the top of the instrument but is placed below it. A minimum 0.25" will be used.[15] Thus the missing dimension is greater than 0.40 + 0.19 + 0.25 + 0.50 + 0.25, or greater than 1.59".

Turning our attention to the second use of the missing dimension, i.e., the height from the lower level of the instrument to the first keyboard, it is not clear where the limits of the dimension are located. On one hand, the "piano basso", i.e., the lower level, could mean the upper surface of the bottom boards or it could mean the lower surface of the same. On the other hand "the height of the first keyboard, or the first frame" could mean that the upper limit of the dimension is the surface of the frame, or the upper surface of the first white keys, or even the upper most surface of the keys, i.e., the third order black keys.

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[13]Reynolds, op. cit., Plate 1.

[14]Shortridge, op. cit., p. 103.

[15]Ibid., p. 103. Also see Reynolds (op. cit.), Plates 3, 5, and 13a.

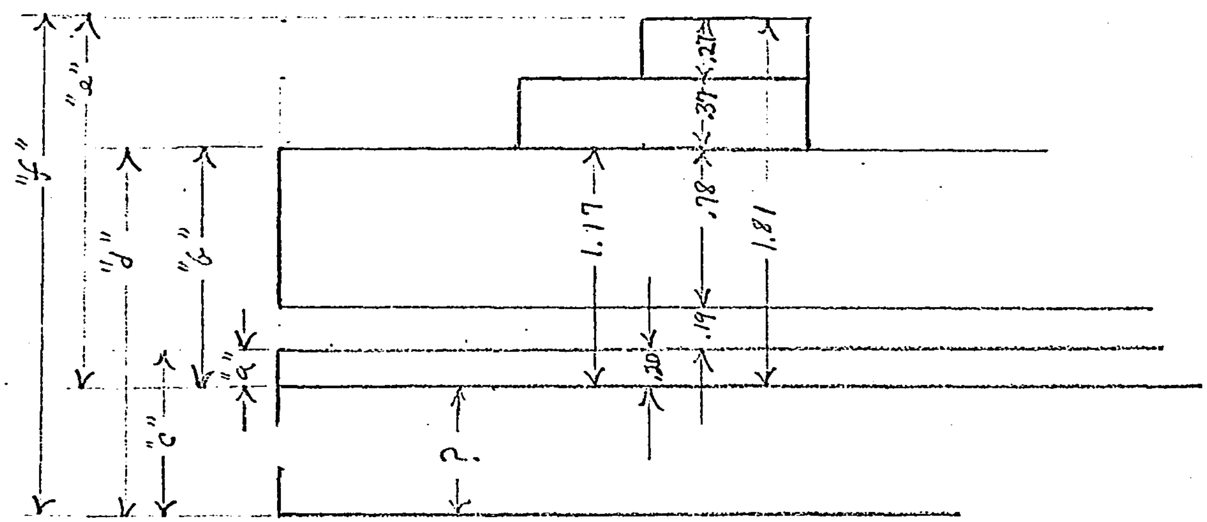

FIGURE 5. FIRST FRAME AND BOTTOM BOARDS (SCALE 1:1)

Using the two possible lower limits for the missing dimension and the three possible upper limits, six possibilities are established. First from the top of the bottom boards to the surface of the first frame (See letter "a", Figure 5). Second, from the top of the bottom boards to the upper surface of the first order keys (letter "b"). A third possibility is from the lower edge of the bottom boards to the surface of the first frame (letter "c") and a fourth is from the lower edge of the bottom boards to the upper surface of the first order white keys (letter "d"). A fifth is from the upper surface of the bottom boards to the surface of the third order keys (letter "e"). And a sixth would be from the lower edge of the bottom boards to the upper surface of the third order keys (letter "f").

It can be shown (See Appendix II) that "a" and "b" can not be correct interpretations of the missing dimension. However the four remaining possibilities can be used to establish a maximum limit for the missing dimension. If "c" is the missing dimension, then the maximum value is 2.44"; if "d", 2.92"; if "e", 1.81"; and if "f", 3.25". The largest possible value for "x" is 3.25" when the missing dimension is measured from the top surface of the third order keys to the bottom surface of the bottom boards, i.e., "f" in Figure 5. Thus the limits of the missing dimension are $1.59 \leqq x \leqq 3.25$. We shall return to the missing dimension after discussing another problem.

The term "morto" presents a problem for the translator. Literally it means "dead". Kaufmann translated it as "upper works".[16]

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[16]Kaufmann, op. cit., p. 272.

In Mersenne's Harmonie Universelle, The book on Instruments", we find the statement, "...they the jacks enter by forty-nine small holes, which are pierced in a wooden ruler called the mortise."[17] Thus morto must mean the jack guide and since it has height, 2.71", then it must be the box type of guide as described by Hubbard[18] as opposed to the more common arrangement of having two thin strips of wood, one just above the keys and the other at the level of the soundboard. Hubbard also states that the box type of guide is found in some old Italian instruments and they vary in size from 1 to 3 inches.[19] The Ridolfi harpsichord uses this box guide.[20]

The height Vicentino gives for the jack guide is 2.71". The length Vicentino gives for the short jack is 2.90", and this length must not only allow for the jack guide, but also the part of the jack containing the dampers, the height of the bridge, because the top of the jack guide would be level with the soundboard,[21] the action of the jack, and the clearance between the jack guide and the keys of the second frame. Even without this latter item, which is unknown at this time, the minimum total of the first four items is 2.71 + 0.25 + 0.40 + 0.19 = 3.55", which is much greater than the 2.90" given for the short jack.

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[17] Marin Mersenne, Harmonie Universell, The Book on Instruments, trans. Roger E. Chapman (The Hague: Martinus Nijhoff, 1957), p. 163.

[18] Hubbard, op. cit., p. 11.

[19] Hubbard, op. cit., pp. 11 and 33.

[20] Shortridge, op. cit., p. 102.

[21] Reynolds, op. cit., Plates 5, 6, and 7.

The mistake must be either in the length given for the short jack, our translation of _morto_, or the length given for the _morto_. First the length given for the short jack has seemed sensible up to this point, at least in relation to the length given for the long jack and so is probably correct. Also, if there were an error in the length of the short jack, it would have to be made longer to fit the jack guide, but at the same time this would reduce the difference between the lengths of the two sizes of jacks and thus the space between the two frames of keys. It will be remembered that if the space between the two frames is reduced by more than a few hundredths, then two working frames of keys becomes impossible.

Second, it has been impossible to find another reasonable meaning for _morto_. But before deciding that the length given for the jack guide is incorrect, it should be remembered that this length also serves as the height "of that part which is by the sides of the keys." This would be the key blocks at the end of the keyboards. It is not known whether the key blocks rest on the bottom boards or on a part of the frame, but even if they rest on the bottom boards, it would mean that the key block would extend 2.71 - 1.17 = 1.53" above the surface of the white keys. In the Ridolfi instrument[22] and other instruments of the period[23] the key blocks do extend above the surface of the white keys, but the distance is more in the order of 0.25", rather than 1.53". It again appears that the length given for the _morto_ is incorrect. What should this dimension be, and what is the 2.71" for?

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[22] Shortridge, _op. cit._, p. 100.

[23] Reynolds, _op. cit._, Plates 5, 7, and 13a.

A maximum limit for the jack guide can be determined. We know the length of the short jack is 2.90" and this must be equal to the portion of the jack containing the dampers (minimum, 0.25"), plus the height of the nut (0.40"), plus the jack guide, plus the action (0.19"), plus clearance between the jack guide, and the distal end of the second frame keys (minimum, 0.06"). Thus the height of the morto is $<$ 2.00". Although it is impossible to directly set up a minimum limit for the height of the jack guide, it is certain that it could not be a small dimension, since its function is to keep the jacks in a vertical position with no side play when the key is depressed. As previously mentioned (supra p. 49) Hubbard states that the box type of jack guides vary in size from 1 to 3 inches. It seems reasonable that the height of Vicentino's jack guide falls between 1 and 2 inches.

Now let us turn our attention to the second use of this dimension, i.e., the height of the key blocks. There is a rectangular shape shown at the left and right side of the keys in Vicentino's template of the first frame of keys, and one shown on the left side of the keys of the second frame (supra pp. 6 , 7). These are no doubt the key blocks in question. The one on the right of the second frame was omitted. It is noticed that the blocks on the first frame extend beyond the visible portion of the keys to a position underneath the blocks of the second frame. Now if it is assumed that the first frame blocks rest on the bottom boards and the second frame blocks rest on the first, and that the blocks are the same height above the keys of their frames, then the height of the second frame block would be exactly

1.42", *i.e.*, the height between any two similar points on the first and second frames. Then there is no reason to assume that the upper and lower blocks would not be the same dimension. The 1.42" dimension would place the upper surface of the lower frame block slightly above the surface of the first order keys, 1.42 - 1 17 = 0.25", which is similar to other instruments of the period. Also the 1.42" falls within the limits set up for the height of the jack guide.

Then what is the 2.71" dimension, which was originally given for the height of the key blocks and the jack guide, to be used for? First it is noted that the missing line, *i.e.*, from the top of the soundboard and from the lower level of the instrument to the first keyboard, is supposed to be under the third description in the Vicentino text, and the 2.71" line is found under the fourth description. Second, 2.71" falls within the range of values which have been determined for the missing dimension. Could it be the case that this 2.71" line, which was given as the height of the jack guide is the value for the missing dimension, and the height of the jack guide is the real missing value?

If the 2.71" dimension is the missing value, then it should be possible to simply substitute 2.71 in place of the "x" in the previous equations and have a working instrument. But it will be remembered that there were four possible interpretations of the "x" at the lower level of the instrument. (See Figure 5, p. 47 , letters c, d, e, and f). Thus it is necessary to again take the four possible interpretations, one by one, to see if any or all of them work with the 2.71".

First, the minimum and maximum values for "c" are 1.59 and 2.44, respectively (supra p. 48). Thus if "c" is the correct interpretation for the missing dimension, then the 2.71" line is too large. But it can be shown (See Appendix III) that if the values for "c" are used for the missing dimension, then the thickness of the bottom boards is between 1.74" and 2.02". The Ridolfi harpsichord[24] shows the bottom boards with a thickness of 9/16 inches on an instrument with 24 strings per octave. Since the Archicembalo has 36 strings per octave, and increase in thickness and therefore strength could be expected, but not an increase to between 1.74" and 2.02". Thus "c" must not be the correct interpretation for the missing dimension.

The limits for "d" are 1.59 to 2.92, which of course includes 2.71. If 2.71 is substituted for "d" (See Appendix IV), the bottom boards of the instrument are still quite thick, 1.53" and the distance from the top of the distal end of the second frame keys to the top of the soundboard is 1.68". This 1.68 is almost too small to account for the jack guide (1.42), the action (0.19), and clearance between the jack guide and the keys (minimum 0.06), a total of 1.67".

The third possibility, "e", cannot equal 2.71, because it was previously established as 1.81". However, if this 1.81 is used as the missing dimension, the bottom boards are extremely thick, being between 1.87 and 2.43 (See Appendix V). Thus "e" must not be the correct interpretation for the missing dimension.

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[24]Shortridge, op. cit., p. 100.

The range of values for "f" are 1.59 to 3.25. If 2.71 is substituted for "f", then the thickness of the bottom boards is 0.90" (See Appendix VI). The distance from the top of the soundboard to the top of the second frame keys is 2.32, which allows sufficient space for the jack guide, action of the keys, and clearance. The height of the jack which extends above the string is 0.18".

We now have two possible interpretations for the missing dimension, namely, "d" and "f". The choice between the two seems fairly clear. First with "d", the bottom boards were still quite thick, 1.54, with "f", 0.90. The latter compares much more favorably with the Ridolfi instrument with bottom board of 9/16". Also in the first case the action of the instrument was somewhat crowded which is not true in the second case. However in the second case, there is the problem of the amount of jack which extends above the soundboard, 0.58". Up to now a minimum of 0.65, 0.40 for the height of the nut and 0.25 for the portion of the jack containing the damper, has been allowed for this dimension. It will be remembered that this dimension was never stated by Vicentino, but was taken from the Ridolfi instrument. Also Vicentino states that under each jack there will be shammy which would raise the height of the jack above the soundboard by the thickness of the shammy. Thus it must be the case that "f" is correct.

There are a few remaining vertical dimensions which must be considered. For the thickness of the jackrail, the height that the nameboard extends above the level of the soundboard, and the thickness of the soundboard, we are forced to use sources other than Vicentino.

These dimensions taken from the Ridolfi instrument[25] are 0.75" at the center for the jackrail, about 2.50" for the top of the nameboard to the level of the soundboard or the top of the nameboard is about 0.22" below the sides of the instrument, and 0.13" for the thickness of the soundboard. The thickness of the wrestplank or tuning pin block, can be determined. The lower edge of the wrestplank will be level with the lower level of the jack slide. The upper edge of the jack slide is level with the top of the soundboard. It was Italian practice to cover the wrestplank with soundboard wood so that it appeared that the soundboard extended to the nameboard.[26] Thus the thickness of the wrestplank would be the jack slide minus the soundboard, or 1.42 - 0.13 = 1.29".

The height of the rack, or that slotted piece at the back of the first frame that keeps the keys from moving sideways, would be the order of 2.00", *i.e.*, the height of the first frame, the keys, and the action.

Therefore, concerning the vertical dimensions of the instrument, we shall conclude that; (1) the jack guide is of the order of 1.42"; (2) the 2.71" line given for the height of the jack guide is really the missing dimension; and (3) that the correct interpretation of the dimension "the height of the first keyboard or first frame to the lower level of the instrument", means from the upper surface of the third order keys to the lower surface of the bottom boards.

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[25] Shortridge, *op. cit.*, p. 102-103.

[26] Russell, *op. cit.*, p. 28.

Now we must direct our attention to the horizontal dimensions of the side view. Since more information is given about the first string at the right side of the instrument, the side view will be developed at that point. First the length of the visible portion of the keys is known, 2.68 and 2.42, as is the distance from the nameboard to the tuning pins, 0.76. The distance from the tuning pins to the nut is given as 3.51, and the nut to the first jack is 2.48. The distance from the bridge to the hitch pin is 4.16 at the righthand side of the instrument. The length of the first string is 12.38 and it is known that the righthand side of the instrument is to be greater than 16.08.

Working from the front of the instrument to the back, the distance to the nameboard would be the total of the first and fourth order keys, i.e., 2.68 + 2.42 = 5.10. The thickness of the nameboard is unknown, but would no doubt be the same as the sides of the instrument. Though the thickness of the side of Italian instruments apparently varies somewhat, the size is relatively small compared with instruments of other countries.[27] We shall use the 0.16" found in the Ridolfi instrument for the thickness of the sides. The distance from the nameboard to the tuning pins is given as 0.76, but the register template shows the tuning pins placed in three rows. Measurement of the template shows that this 0.76 must refer to the distance to the middle row of pins. The distance between the rows of pins is about 0.37. The length of the first string,

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[27]Russell, op. cit., p. 28. Russell states, "the case work is generally about a quarter of an inch thick..." Shortridge gives the thickness of the Ridolfi instrument as 5/32 (Shortridge, op. cit., p. 100.

which the register template shows originating from the nearest row of tuning pins, is given as 12.38. This is obviously too long to mean the sounding length (infra p. 80) and must refer to the total length of the string. The distance from the hitch pin at the back of the instrument is unknown, but from the Ridolfi instrument, it must be the order of 0.25. For the thickness of the back of the instrument, 0.16 will again be used. The total length of the Archicembalo at the first string is 5.10 + 0.16 + 0.76 - 0.37 + 12.38 + 0.25 + 0.16 = 18.44. The length at the right side of the instrument would be less than 18.44 because of the curve, but still greater than 16.08 (See Figure 6, infra p. 58).

The depth of the jacks and the space between them must be determined from the template. They are 0.37 and 0.12, respectively. The total width of the jack guide is 3(0.37) + 4(0.12) = 1.62. The placement of the jack guide relative to the nameboard can now be determined. We know that from the nut to the end of the first jack is 2.48" and the template shows this first jack to be in the third row of jacks, i.e., the row most distant from the keyboard. We are also told that the distance from the nut to the tuning pins is 3.51". But there are three rows of tuning pins. Also the template shows that the nut is farther from the nameboard at the right side of the instrument than at the left and that the jack and jack guide are nearer to the nameboard at the right side than at the left. However when Vicentino gives the 3.51 dimension, it is in a paragraph which concerns the distance of the bridge to the hitch pin at the right side of the instrument. Therefore

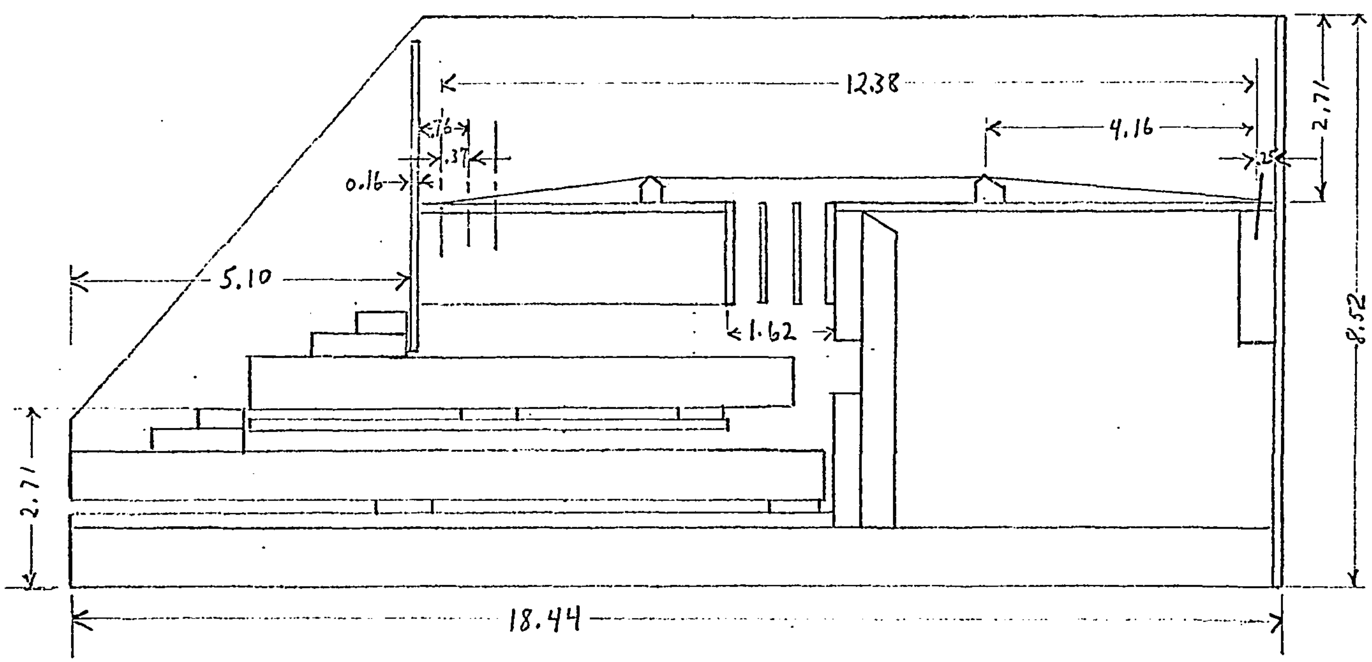

FIGURE 6, SIDE VIEW AT FIRST STRING (SCALE ≈ 1:3)

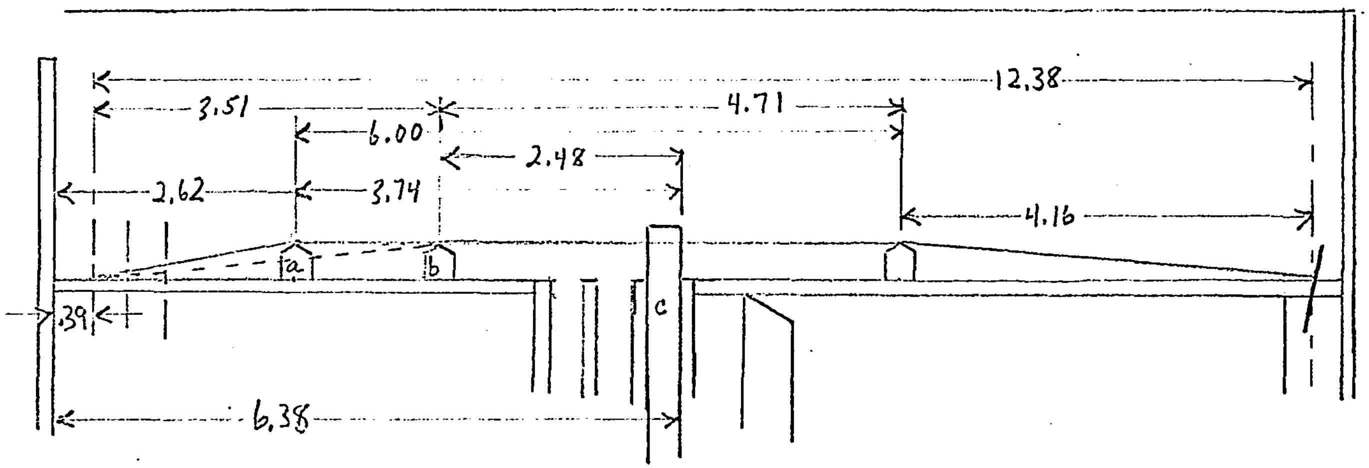

FIGURE 7. POSITION OF NUT a. Nut from template. b. Nut from lines. c. First jack. (SCALE 1:2)

we shall assume that the 2.48 also refers to the right side of the instrument, and specifically the upper most string. This string, it will be recalled, originates from the row of tuning pins nearest the nameboard. Thus the distance from the nameboard to the end of the first jack is 0.39 + 3.51 + 2.48 = 6.38 (See Figure 7, supra p. 59). The register template shows this distance to be 6.36, which checks very well.

The positioning of the nut relative to the nameboard does present a problem. If the distance is measured on the template at the point at which the first string crosses the nut, the nut is 2.62" from the nameboard. This distance according to the dimensions given in the lines should be 0.39 + 3.51 = 3.90" (See Figure 7). The solution can be found in the length of the string that is to vibrate. In the case of the dimensions from the template, this string length is 6.00". The length from the lines is 4.71". The discussion of this problem will be reserved for the section dealing with the top view of the instrument.

Two other items will also be reserved for the top view. First in Figure 6, it will be noticed that the second frame of keys is just long enough to extend under the first two rows of jacks. This will be shown to be the case. And the exact position of the balance rail and balance pins will also be discussed in connection with the top view of the instrument. These items are reserved for later because the angle of the jack guide makes the dimensions variable relative to the side view.

<u>The Top View</u>.

In the construction of the top view, the first step will be to superimpose the three templates. However it should be remembered that the condition of the templates makes it impossible to obtain accurate measurements, especially these concerning the width of the instrument. Some variance must be expected.

The width of most of the white keys of both frames is 0.94, the black keys 0.54. The templates appear to have been printed from two or three separate plates rather than one large plate for each template. Thus some of the horizontal lines which should be straight are broken and a few of the keys are larger or smaller than the rest due to the putting together of the several plates.[28]

Since there are 27 white keys on each frame, the total width of the keys is about 25.38. The measured width of the first frame is 25.69, the second frame, 25.25. The width of the keyblocks at the ends of the first frame is 1.72 at each end. The block is not shown on the right side of the second frame template and the block on the left could not be measured because of a tear in the page. The total width of the first keyboard template is then 28.82". However we are told that the width of the instrument is to be 30.88, a difference of about 2". The template width would not include the sides of the instrument, but if the sides are 0.16 wide as is typical in Italian construction, this would not be enough to make up the difference. There are two possibilities;

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[28]See for example small F and C'' on the first frame which are somewhat wider than the rest (supra p. 7).

either the templates were not drawn very accurately or the 30.88 is to include the dimensions of the outer case of the instrument as well as the inner case. We shall reserve judgment until more information is available.

If a measurement is taken on the first frame from the front edge of the first key on the right, perpendicular to the front edge and along the righthand side of the key, to a point at the edge of the distal end of the keys, this distance is 11.54" (See Figure 8, infra p. 63). The similar dimension on the left is 14.44". The length of the back of the keys, measured along the slanted line, is 26.44. The extreme right hand edge of the distal key at the right is 0.86 from the perpendicular drawn from the proximal end of the key. From the position of the balance pins it can be determined that for the second and third order keys, the second order keys are connected to the distal key to the left of the split key, and the third order keys are connected to the distal key on the right. The distance of the balance pins, measured from front of the key, follow three lines. The balance pins for the first order keys are 4.53 on the right and 5.08 on the left, for the second order, 4.83 on the right, measured at the point of the first pin, and 5.27 on the left, for the third order, 5.10 on the right and 5.55 on the left.

Similar dimensions for the second frame are given in Figure 9 (infra p. 64), plus the added dimension of the distance from the front of the keys to the pins at the distal end, which are 6.65 at the right perpendicular and 8.80 at the left.

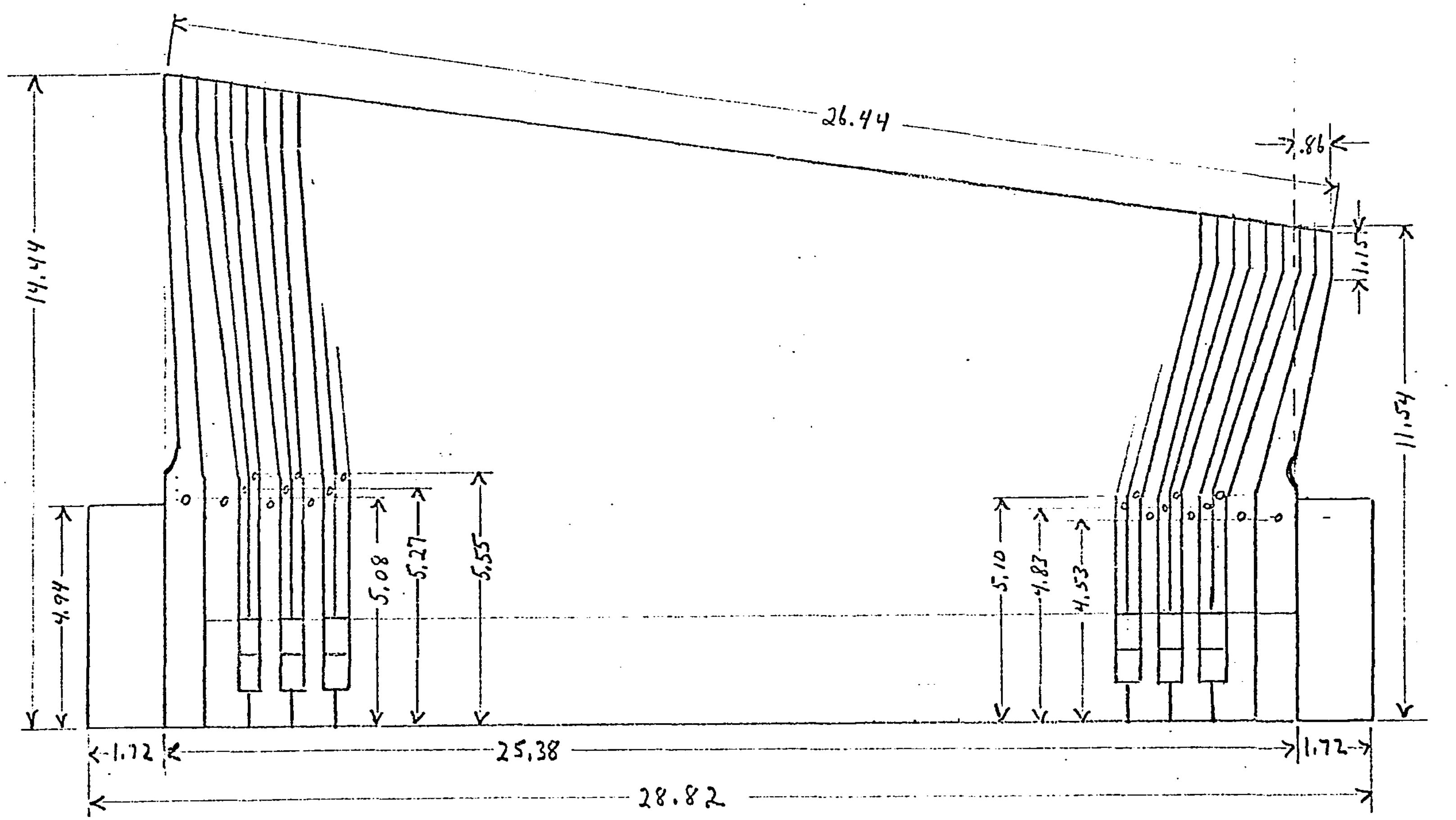


FIGURE 8. FIRST FRAME (SCALE 1:4)

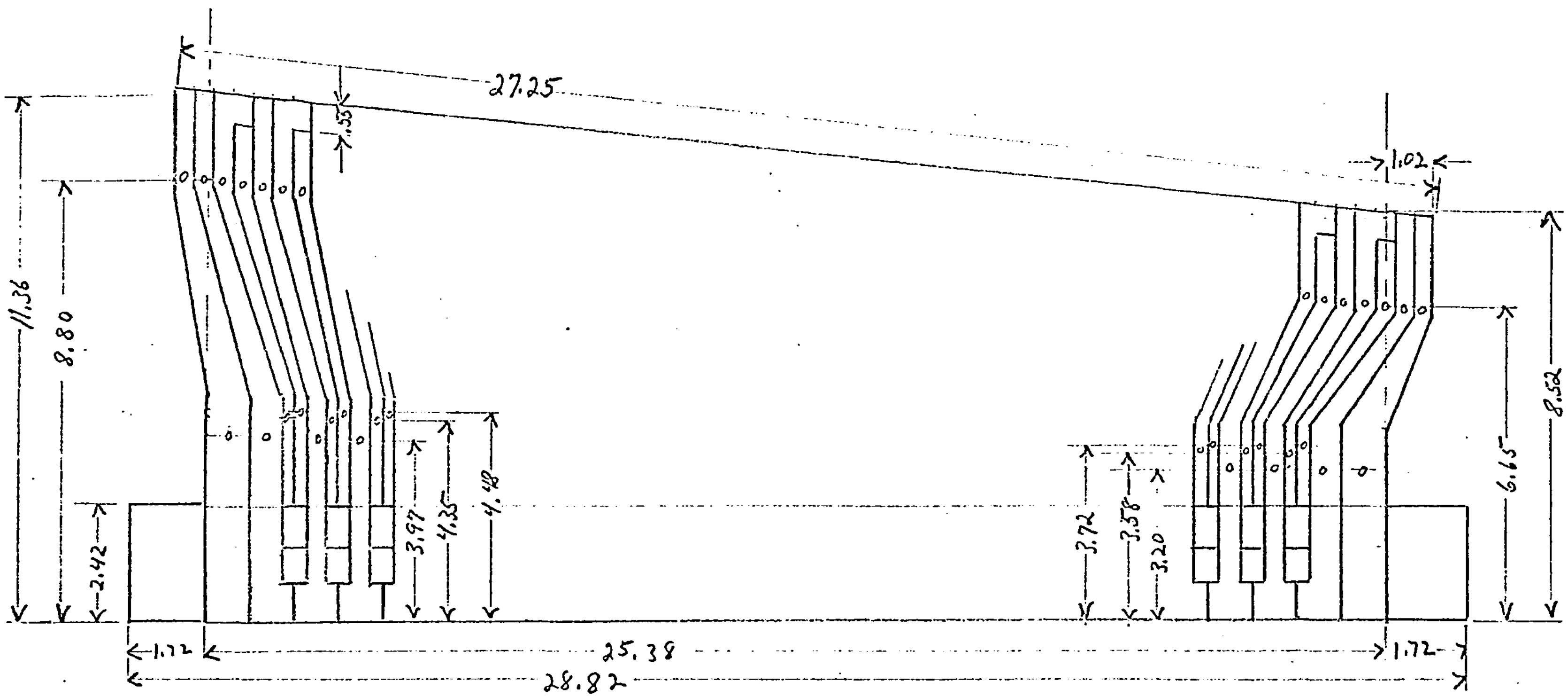


FIGURE 9. SECOND FRAME (SCALE 1:4)

Placing the two frames of keys one above the other such that the keyboards are aligned, we find that the distal end of the second frame keys are wider than those of the first and extend 0.75" beyond those of the first on the left hand side and 0.16" on the right. It is also the case that the first frame keys extend farther at the distal end than those of the second. If the lengths of the visible portion of the keys are taken from the lines, which give the length of the first order keys as 2.68", then the distal end of the first frame keys extends 0.40 on the right and 0.46 on the left. If the length of the first order key is taken from the template, which gives the length of the first order key as about 2.48, then the extension is 0.60 on the left and 0.54 on the right. The mean of these latter two dimensions was used in Figure 10 which shows the extreme left and right portion of the distal end of the frames. The width of the distal end of the keys on the individual frames appears to be equal on the template. Measured along the slant, this would mean that the first frame keys should be 26.44/69 = 0.382" and the second frame keys should be 27.25/63 = 0.434" wide.

Without recalling any additional information except that we are going to have three rows of jacks, it would seem, looking at Figure 10, that the first frame of keys would govern the third or most distant row of jacks, the longer keys of the second frame, the second row of jacks, and the shorter keys of the second frame, the first row of jacks.

When the third template is combined with the two of the two frames of keys, the result is an impossible puzzle where the pieces do not fit. There is however one set of dimensions that checks with those

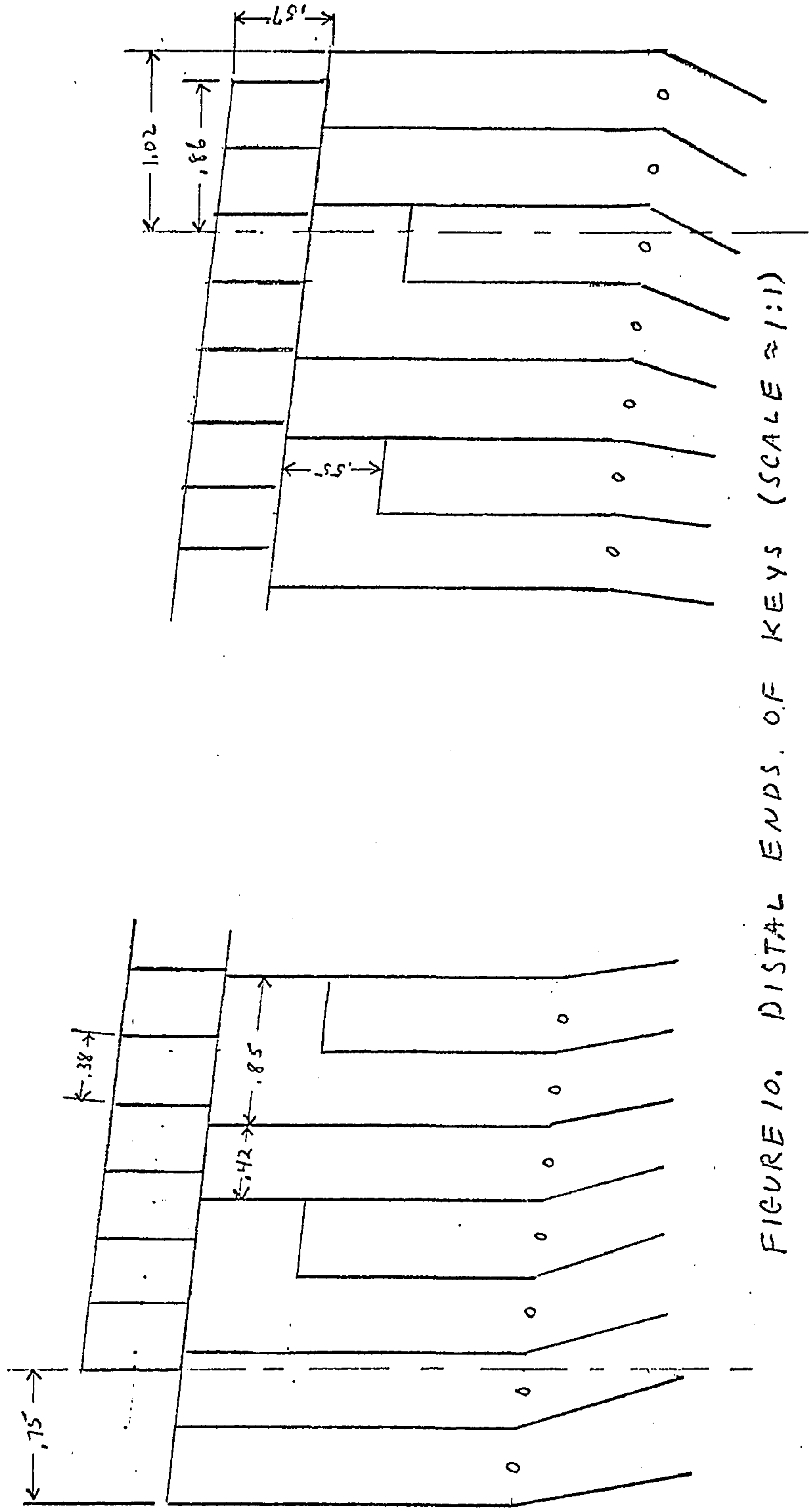


FIGURE 10. DISTAL ENDS OF KEYS (SCALE ≈ 1:1)

of the two frames of keys. This is the distance from the front of the template, *i.e.*, the edge of the nameboard, to the location of the jack guide. This distance at the right side, from the front to the back of the jack guide, is 6.38", at the left it is 9.60". This places the third row of jacks directly above the distal end of the first frame keys that extend beyond the second frame, the second row of jacks above the long keys of the second frame, and the first row of jacks above the short keys of the second frame.

The width of the template is 29.06, slightly larger than the width of the other two templates, 28.82. The register template shows that the first jack on the left is in the second row of jacks, the second is in the third row, and the third is again in the second. Comparing this to the distal ends of the frames in Figure 10, it would be impossible for the jacks to rest on the keys in such an arrangement. On the right side of the template, the first jack is shown to be in the third row of jacks. Figure 10 would indicate that the first jack on the right would have to be in the second row. Also the register template shows only 130 jacks for the 132 keys of the two frames of keys, 44 in the third row, 45 in the second, and 41 in the first.

The distance between two similar points on adjacent jacks in any row appears to be equal, about 0.61", except at one point about the center of the template, between the 22rd and 23rd rack of the second row counting from the left. The space between the first and second row jacks at this point is about 0.90", between adjacent jacks in the third row is 1.53". (See Plate VI, infra p. 68).

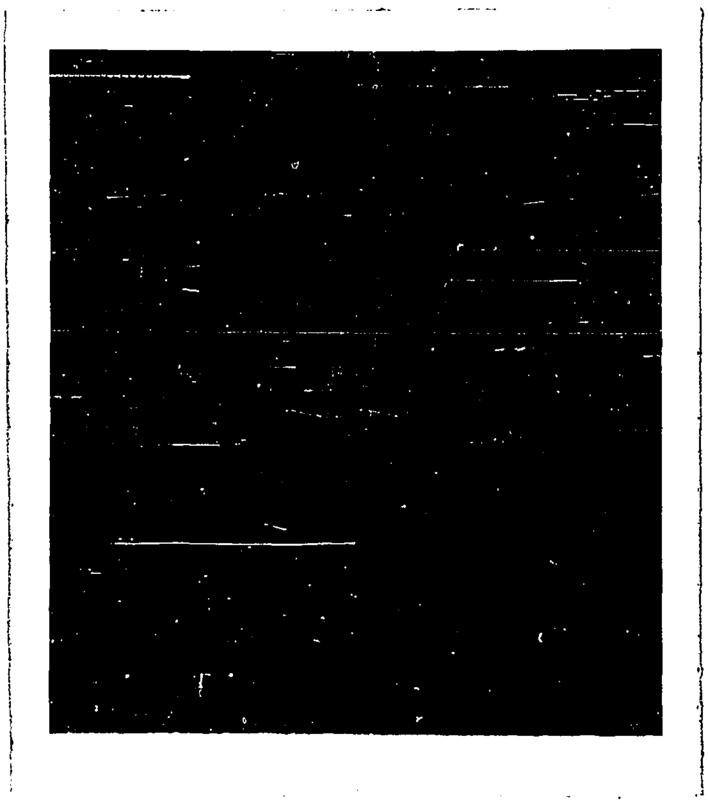


PLATE VI

DETAIL OF REGISTER TEMPLATE

(Reduction 1: 2.50)

If this wide spacing is reduced such that it is similar to the spacing of the rest of the jacks, the total width of the template compares very favorably with the two templates of the frames of the keys. However there is still one wide space in the third row where it appears that possibly one jack was omitted. Even if a jack were inserted in the third row there would be a total of only 131, still one short.

At the point of this wide spacing it can be observed that the horizontal and slanted lines of the template do not meet exactly. Two plates were probably spliced together at this point. But in order to adjust the splice so that the horizontal and slanted lines come together without any breaks, the template would have to be made somewhat wider, rather than smaller.

The tuning pins are also shown on the template. There are 132 shown. Vicentino shows the stringing sequence at the extreme left and right hand sides of the template. This sequence shows that the first or nearest row of tuning pins holds the strings that are to be sounded by the third row of jacks, the second row of pins, by the second row of jacks, and the third row of pins, by the first row of jacks. If we continue this stringing sequence toward the middle of the template from both sides to the point of the splice, and where the sequence of the placement of the tuning pins is broken, we find that there is no way in which one or two jacks could be added in any row in such a way as to continue the stringing sequence without a break (See Figure 11, infra p. 70).

In order to produce a working instrument, we are forced to make some adjustments in Vicentino's templates. First it will be recalled that

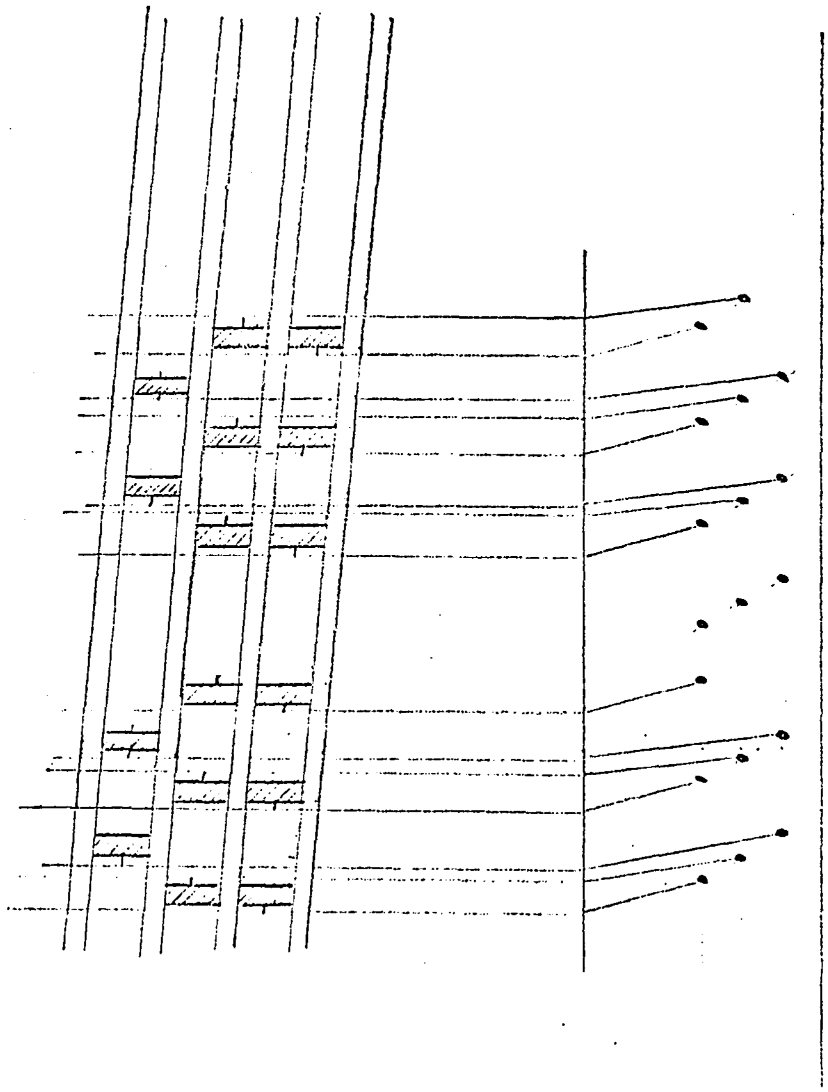
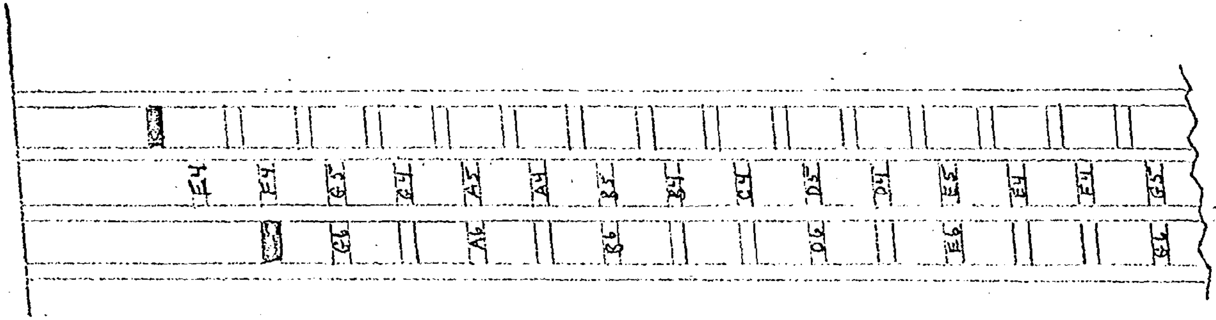


FIGURE 11. CENTER OF REGISTER TEMPLATE (NOT TO SCALE)

if the wide spacing on the template of the jacks is reduced such that it is similar to the spacing between the other jacks, then the widths of the three templates are very similar, i.e., slightly less than 29". Because of the similarity in widths, it appears that this was Vicentino's intention. Then let a jack be placed in the third row of jacks where the wide spacing occurs in that row. This would continue the sequence of the jacks. However as previously mentioned, there is still a total of only 131 jacks. The placement of the last jack presents a more complex problem.

Reviewing the information available at this time, it is known that the distal end of the first frame keys extends beyond those of the second frame and from the lengths of the keys shown on the templates, it appears that the first frame keys should govern jacks in the third row, and the second frame keys should govern jacks in the second row and some of those in the first row. Now there are exactly 45 jacks shown in the second row of jacks, and there are exactly 45 keys in the fourth and fifth order, i.e., on the second frame. It must be the case that the fourth and fifth order keys are related to the second row of jacks. Then the sixth order keys govern 18 of the keys in the first row of jacks. This would leave 23 jacks in the first row for the lower frame, plus the 45 in the third row. The sequence of the second frame is given in Figure 12 (infra p. 72). Only the left and right ends are shown since the sequence would repeat at the octave. There would be three practical positions for the missing jack, which will be governed by a key from the first frame, either at the extreme left in the third row of jacks, the



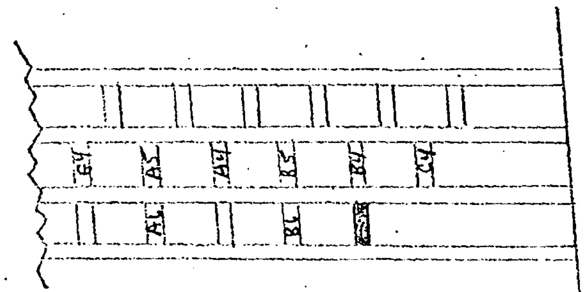


FIGURE 12. JACK SEQUENCE OF SECOND FRAME KEYS (NOT TO SCALE)

extreme left of the first row of jacks, or the extreme right of the first row of jacks. These three practical positions are shown in Figure 12 as the solid colored jacks. The resulting sequences of the relationship with the keys of the first frame are shown in Figures 13, 14, and 15, resp. (infra pp. 74-76). The arrow indicates the added jack.

It is noted that in Figure 15, where the 132nd jack was added at the extreme right hand side of the first row of jacks, all of the jacks of the first and second orders of keys are found in the third row exclusively, and the jacks for the "extra" row of keys, i.e., the third order, are to be found in the first row of jacks. This is the same arrangement found in the upper frame, i.e., the jacks for the fourth and fifth frame were found in the second row of jacks only and the "extra", or sixth order jacks were placed in the first row. Therefore this solution of placing the final jack in the first row at the right, if not the correct solution, is the most symmetrical.

We shall maintain Vicentino's equal spacing of the jacks of about 0.61" between centers measured on the horizontal. However this will necessitate some alterations in the distal ends of the keys of both frames so as to bring the distal ends below their respective jacks. In order to do this, the equal widths of the distal ends of the keys must be abandoned, because it is impossible to find equal widths, for 17 keys per octave in the second frame, 19 keys per octave in the first frame, and 36 jacks per octave, that will bring the proper keys and jacks together. Therefore in Figure 16 (infra p. 77), which illustrates the left half of the two frames, three widths have been

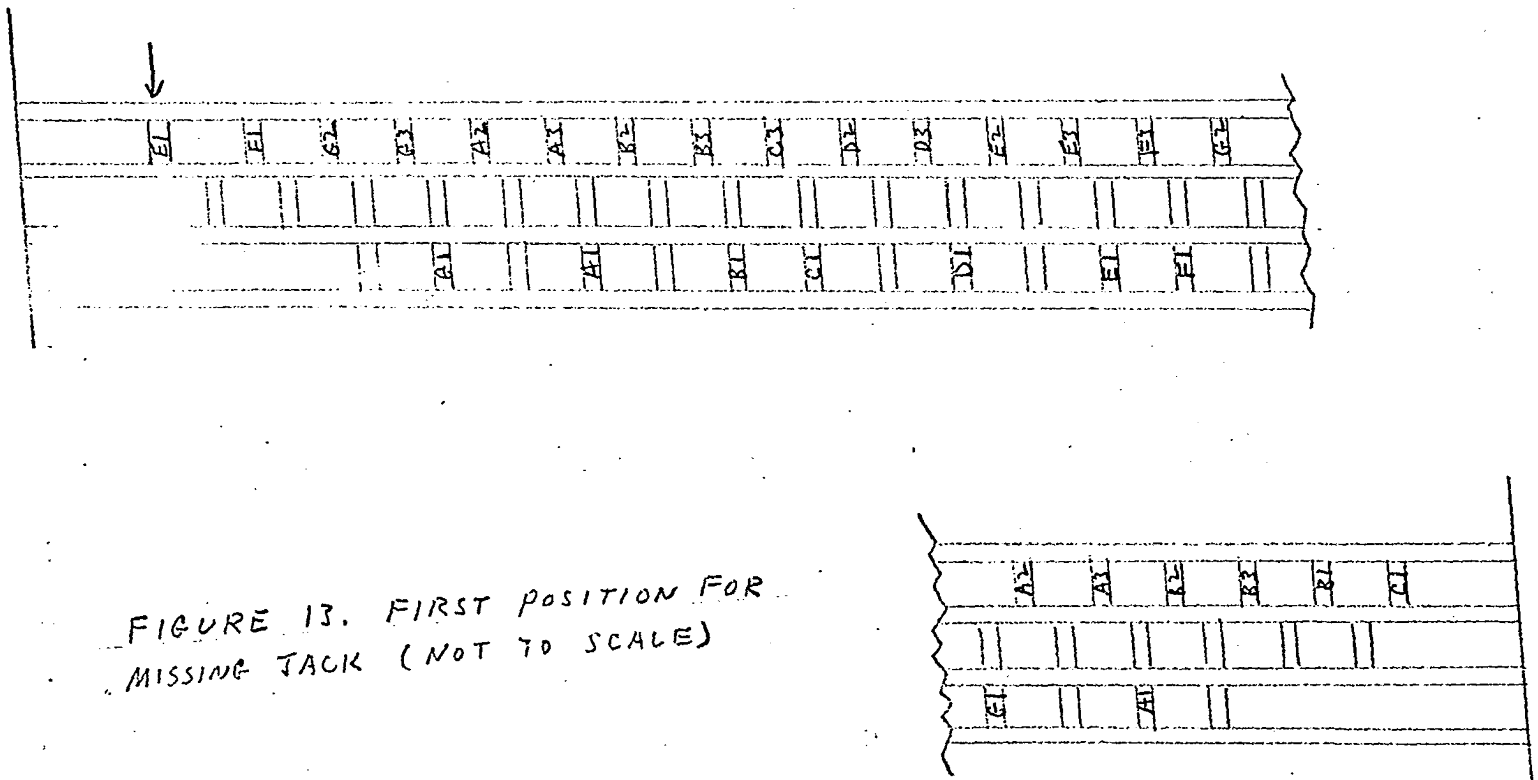
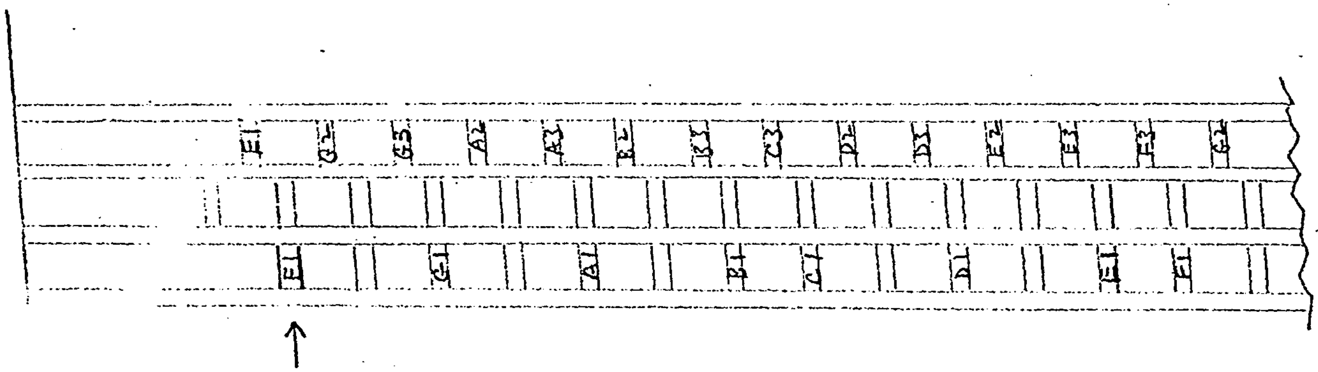


FIGURE 13. FIRST POSITION FOR MISSING JACK (NOT TO SCALE)



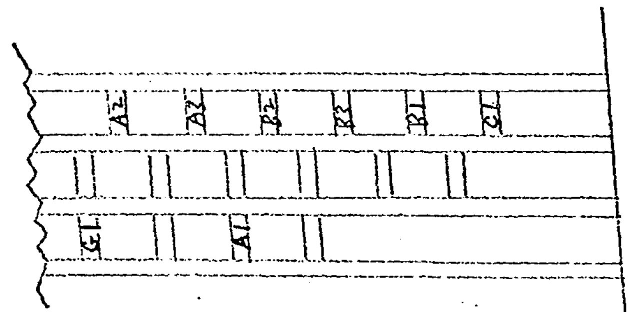
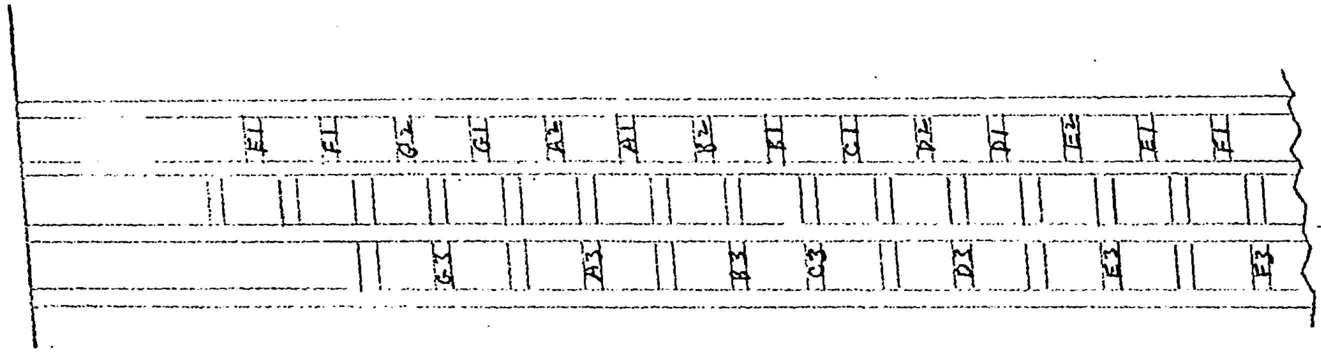


FIGURE 14. SECOND POSITION FOR MISSING JACK (NOT TO SCALE)



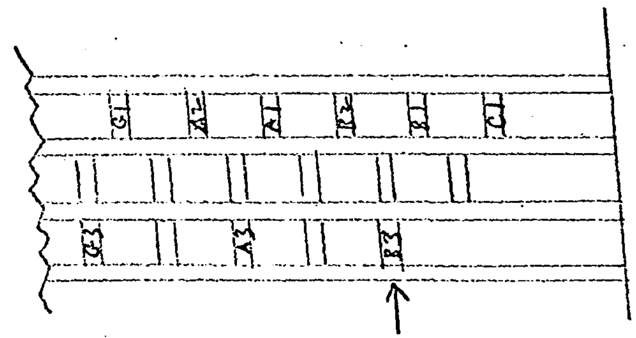


FIGURE 15. THIRD POSITION FOR MISSING JACK (NOT TO SCALE)

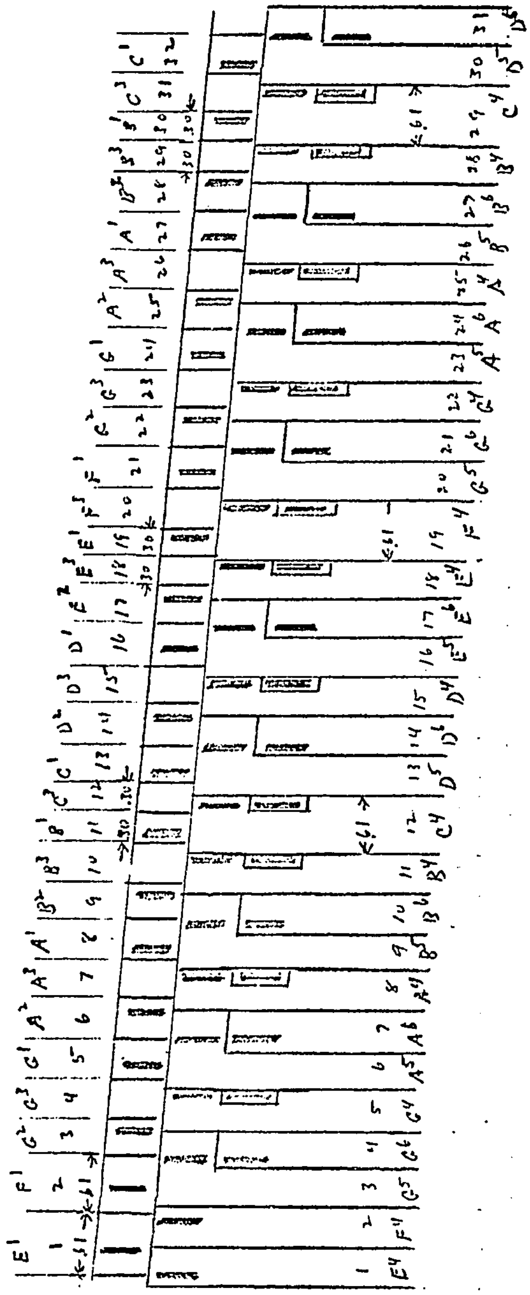


FIGURE 16. JACKS ON KEYS (SCALE 1:2)

adopted for the distal ends of the keys on the two frames. On the first frame, the widths of 0.41", 0.30", and 0.61" are used, on the second frame, 0.41" and 0.61". The sequence of the various widths can be seen on the drawing. The keys which are not dimensioned are 0.41". The placement of the jacks are shown as the solid rectangles on the keys.

The placement of the bridge is known at the right side of the instrument and it is also known that the distance from the bridge to the hitch pins is 4.16" on the right. This distance remains constant until about the middle of the instrument and then increases. The curve of the bridge would, in the treble, approximate an exponential curve, <u>i.e.</u>, the curve would be such that for each octave the string length would double. However in Italian instruments the bridge breaks this curve for the last 4 to 8 strings of the bass.[29] The bridge for the lower bass strings runs in a straight line about 30 degrees off the horizontal. There is no reason to assume that the Archicembalo would do otherwise. On some of the instruments of the period, the back of the body follows the same angle as that of the bridge, on others the back is built at 90 degrees to the left side of the instrument. There is no way of knowing which shape the Archicembalo was supposed to take, however, there would be little difference as far as the working elements of the instrument would be concerned. Figure 17 (infra p. 82) shows the back of the instrument following the same line as the bridge.

Vicentino does not mention the method of bracing the instrument or the ribs of the soundboard. It must be assumed that these items were

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[29]See Shortridge, <u>op</u>. <u>cit</u>., p. 101, Russell, <u>op</u>. <u>cit</u>., Plates 6, 7, and 13.

left entirely to the builder. For details concerning these items for Italian instruments, the reader is referred to Shortridge and Russell[30] for the construction of the body, and also to Shortridge and Hubbard[31] for the placement of the ribs of the soundboard.

The only major problem remaining in the completion of the top view is the placement of the nut. It will be recalled that there are two possible positions for the nut at the right side of the instrument, one taken from the template, and the other from the given lines. The only information we have to use in this problem is the sounding lengths of the strings. With as many variables present as there are in string lengths, at best we can only guess which location is correct, but perhaps we can find one position to be more probably than the other.

Since the three templates all have widths of some 28.80", it would seem that the 30.88" given for the width of the Archicembalo must include the outer case. The same is probable true for the length. The total length is given as 77.2". The sounding length of the longest string would be equal to the 77.2" minus the length of the keys outside the instrument, 5.10", the thickness of the nameboard, 0.16", the distance from the nameboard to the tuning pins, 0.76", the tuning pin to the nut, less than 2.48" on the left, the bridge to the hitch pin, greater than 4.16" on the left, the hitch pin to the back of the instrument, 0.25", and the thickness of the back of the inner and outer case, about 1.0". The sounding length of the longest string is about 63 to 64 inches. The

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[30]Shortridge, loc. cit., Russell, op. cit., Plate 16.

[31]Shortridge, loc. cit., Hubbard, op. cit., p. 8.

length of this lowest string for instruments with the same range averages about 12 times the length of the shortest.[32] This would be about 5.3" then for the short string, almost exactly between the two possible lengths, 4.71" and 6.00".

Shortridge[33] compares the string lengths of 20 instruments, most with the same apparent range as the Archicembalo, and finds that the average length for the C'' is 10.45". The average length for the string an octave above would be roughly one half of this or 5.22". This slightly favors the shorter dimension of 4.71", obtained when the nut is placed according to the lines, over the 6.00", obtained when the nut is placed as shown on the template (See Figure 7, supra p. 59).

Vicentino states that the strings should be "large and solid." The fact that he does use the adjective "large" would tend to support the shorter dimension since a large string would have to be shorter in length to produce a given pitch, other variables being equal.

Russell describes an instrument built in 1521 in Rome by Jerome of Bologna.[34] In time and location this instrument is about as close to the Archicembalo that could be found. The highest note of the instrument, D'' has a string length of 4 1/2" and the length of C'' is 10 1/2". The C''' of this instrument would be slightly greater than 4 1/2", perhaps the order of 4.75". The length given for the C'' compares

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[32] Shortridge, op. cit., p. 100, Russell, op. cit., Plates 5 and 7.

[33] Shortridge, op. cit., pp. 102-107.

[34] Russell, op. cit., Plates 5 and 6.

favorably with the average length given by Shortridge. And the length for C''' compares favorably to the highest string of the Archicembalo if the shorter dimension is used.

For the reasons given above, the author feels that the placement of the nut according to the lines, which yields a sounding string length of 4.71", is the correct location. The nut has been incorrectly placed on the register template.

The following drawings of the Archicembalo are presented showing the complete top view, Figure 17, the front view, Figure 18, and a projection from the right front corner, Figure 19. In Figures 17 and 19 five strings are shown. These strings are for low E first, small C first, C' first, C'' first, and C''' first. Figure 17 also shows the jacks for these strings and Figure 19 shows the jack rail in position.

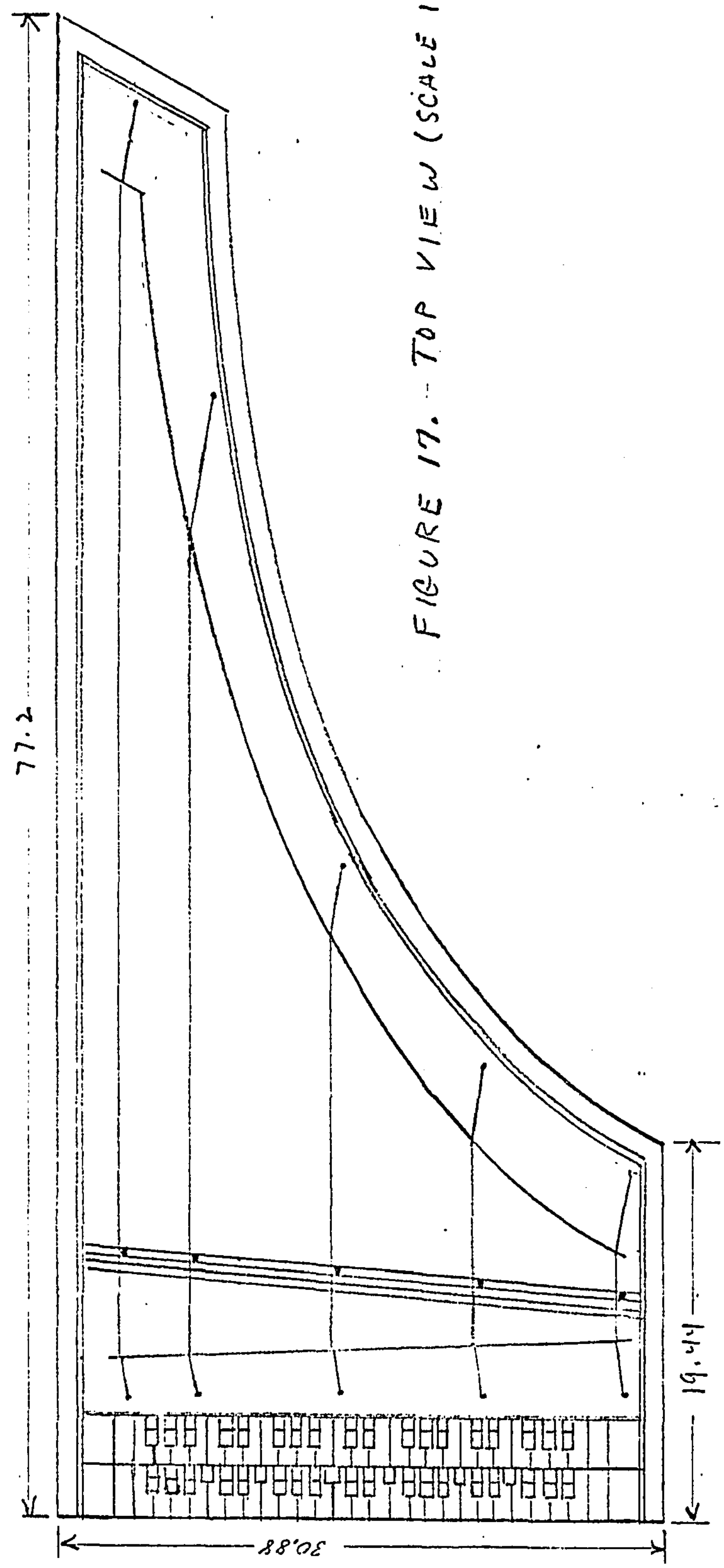

FIGURE 17. - TOP VIEW (SCALE 1:10)

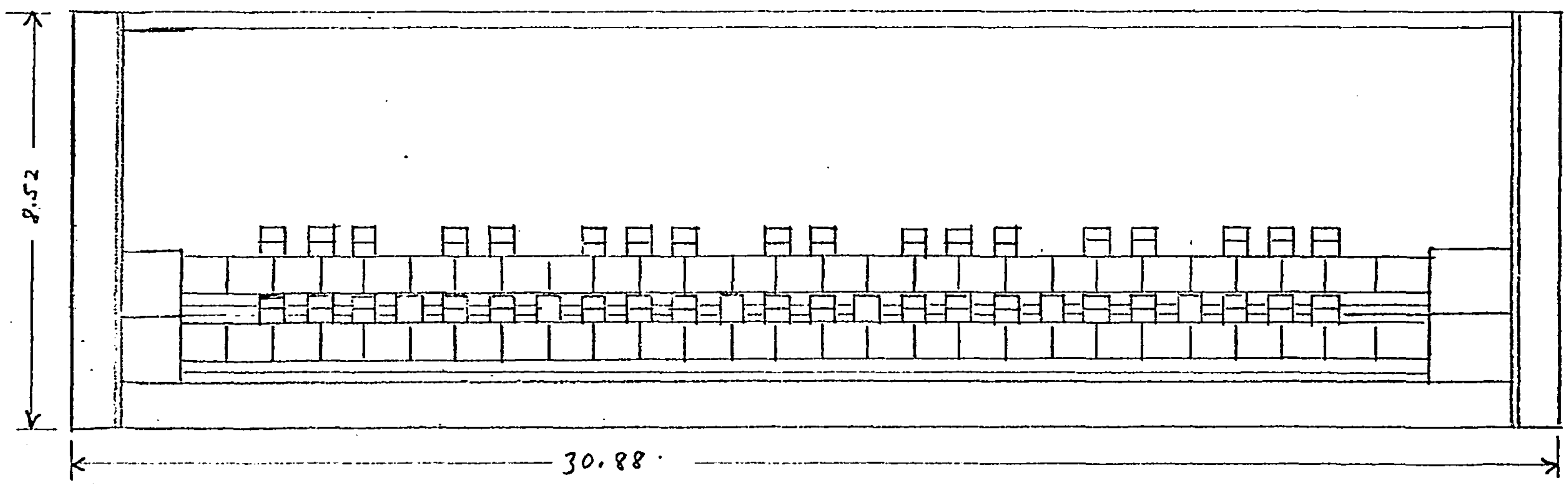

FIGURE 18. FRONT VIEW. (SCALE 1:4)

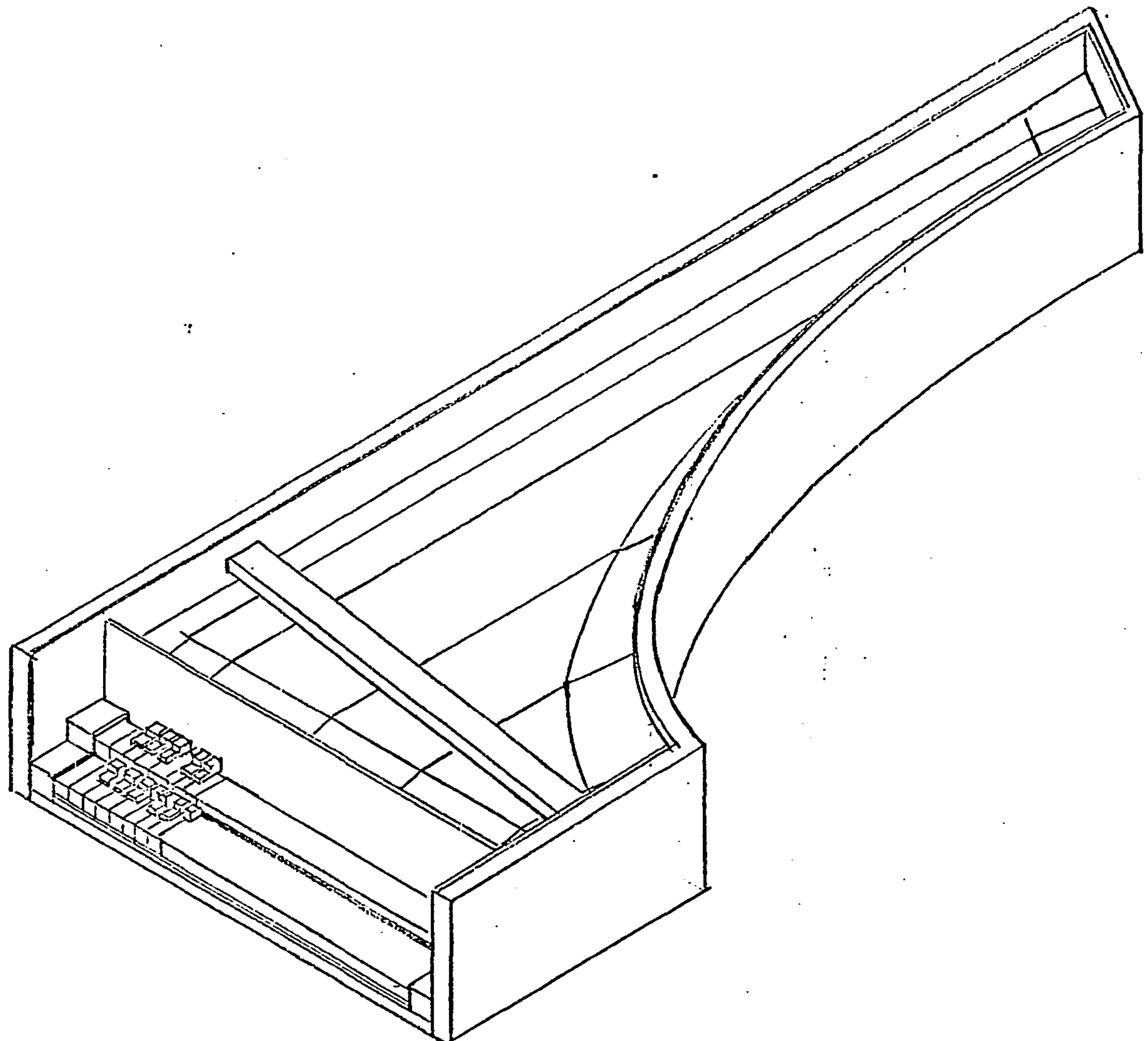

FIGURE 19. PROJECTION (SCALE 1:12)

CHAPTER IV

TUNING

We shall begin the discussion of the tuning of the Archicembalo with a brief description of two types of meantone tuning as defined by Barbour.[1] One quarter (1/4) meantone tuning, which Barbour considers the true meantone tuning, is based on the difference between a just major third (ratio of 5:4, or 386.3137 cents) and a Pythagorean major third (ratio of 81:64, or 407.8200 cents). This difference of 21.5063 cents (the syntonic or Didymic comma) is divided equally between each of the four fifths in an ascending series of fifths between C and E, *i.e.*, C to G, G to D, D to A, and A to E. Thus each fifth, instead of being a Pythagorean fifth or perfect fifth (ratio of 3:2, or 701.9550 cents), is lowered or flatted by 1/4 of the syntonic comma, 5.3766 cents, so that the value of each fifth in the series is 696.5784 cents.

In the one third 1/3 comma tuning, the difference between a just major sixth (5:3, 884.3933) and a Pythagorean major sixth (27:16, 905.8650) is divided between the three fifths, C to G, G to D, and D to A. This results in each fifth being lowered from a perfect fifth by 7.1572 cents, to 694.7978 cents.

In both the 1/4 comma and 1/3 comma meantone, the sequence of flat fifths is continued to E flat by descending fifths and to G sharp

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[1]J. Murray Barbour, *Tuning and Temperament* (East Lansing: Michigan State College Press, 1951), pp. 26-35.

by ascending fifths. In both systems there are flat fifths and therefore sharp fourths except for the wolf fifth, G sharp to E flat, which is 737.6376 cents in 1/4 comma and 757.2242 cents in 1/3 comma. The basic difference between the two systems is that 1/4 comma tuning will produce just major thirds and minor sixths, while 1/3 comma tuning will produce just minor thirds and major sixths.

It will be recalled that Vicentino speaks of two kinds of fifths, the perfect fifth and the common or altered fifth. The perfect fifth occurs in the second tuning, i.e., the tuning described in Chapter IV, between keys of the first and second frames. The perfect fifths occur in the first tuning between the first and sixth orders of keys and between the sixth and fourth. We shall assume that Vicentino's use of the term "perfect fifth" means a fifth with a ratio of 3:2. The common fifth, which occurs between notes of the same order, e.g., C to G, both in the first order, is a flat fifth. The first step in the solution of the tuning will be to find the limits of this flat fifth, i.e., in terms of cents values, the maximum and minimum values that this common fifth may have.

Then it will be shown that a 31 tone equal tempered scale, while it does satisfy the conditions for the first five orders of Vicentino's first tuning, does not produce the required perfect fifths between the first and sixth orders and the sixth and fourth orders. Then a set of values will be derived by beginning with the perfect fifths and finding a value for the common fifths to meet the requirements. There are two possibilities. It will be found that either the system has one large common fifth, between E dot and B dot, or we do not temper the descending fifths as required by Vicentino's instructions.

Now let us turn to the first tuning of the Archicembalo described by Vicentino in Chapter V (Vic. pp. 103v-104). It will be recalled that the ascending order of the 36 pitches within the octave is known (supra p. 25). It is also known, with the exception of the five pitches in the sixth order of the instrument, that the 31 keys are tuned in a series of common fifths, and that series is known (supra p. 21). If it is assumed that the 30 fifths in this series are equal, which seems likely since no inequality between the fifths is mentioned, then we can set limits on the size of the fifth in the series.

For an illustration of how such limits can be set, let us turn to something more familiar. Suppose that we have a series of 11 equal fifths, from E^b to $G^{\#}$, on the modern keyboard. We are told that if these pitches are placed within one octave, the ascending order will be C, $C^{\#}$, D, E^b, E, F, $F^{\#}$, G, $G^{\#}$, A, B^b, and B. This means that whatever the size of the fifths in the series, $F^{\#}$ must be lower than G, and G must be lower than $G^{\#}$. This can be represented as $F^{\#} < G < G^{\#}$.

First our series of fifths can be represented in part as G - D - A - E - B - $F^{\#}$ - $C^{\#}$ - $G^{\#}$, where the dash (-) indicates an ascending fifth. If a series of octaves on G were superimposed on this series of fifths, an octave of G would fall between D and A, another between E and B, a third between $F^{\#}$ and $C^{\#}$, and a fourth between $C^{\#}$ and $G^{\#}$. Therefore we can say that five fifths, i.e., G to $F^{\#}$, must be less than 3 octaves, and 7 fifths, i.e., G to $G^{\#}$, must be greater than 4 octaves. Knowing that an octave has the value of 1200 cents and representing the value of the fifth with X, we can convert the preceding statements into the following equations:

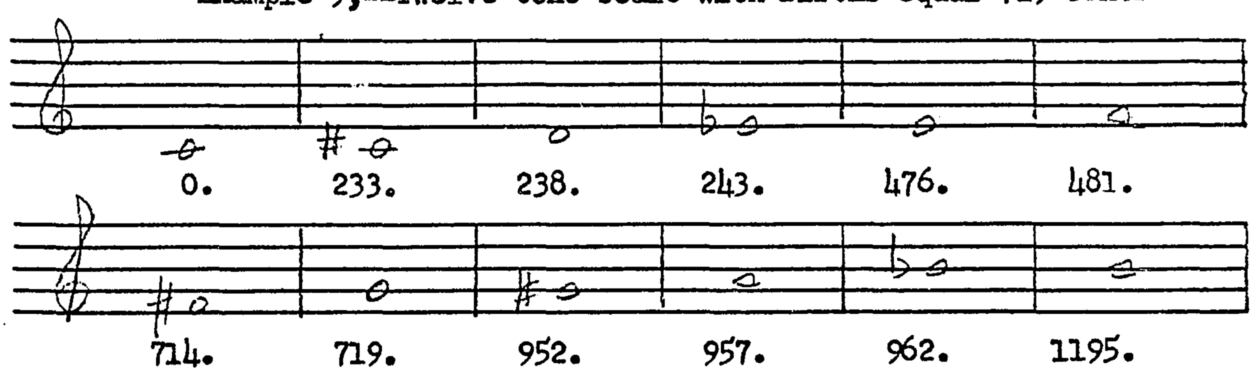
$$3(1200) > 5X; \text{ and } 7X > 4(1200).$$

$$685.7 < X < 720.0.$$

Then no matter what size of fifth is used in the series of fifths within these limits, an ascending scale would result.

For example, let X equal 719 cents. Then starting with C equal to 0 cents and computing ascending fifths through $G^{\#}$ and descending fifths through E^{b}, the scale shown in Example 9 is produced. As can be observed, this would not be the most satisfying 12 tone scale ever conceived, but within our given limitations it is a possibility. Of course if X equal 700 cents, the familiar 12 tone equal tempered scale would result.

Example 9,--Twelve tone scale with fifths equal 719 cents



Now applying this procedure to the 30 fifths in Vicentino's first tuning, we know that B sharp < C natural < C dot $(B^{\#} < C < C.)$. We also know that $B^{\#}$ and C. are found in the series of fifths above C, <u>i.e.</u>,

C - G - D - A - E - B - F# - C# - G# - D# - A# - E# - B# - Gb. - Db. - Ab. - Eb. - Bb. - F. - C.. Counting octaves, the seventh octave above C would fall between B# and Gb., and the eleventh octave above C would fall between F. and C.. Again let X be the value of the fifth in the series. Then since B# is the twelfth fifth in the series and C. is the nineteenth fifth:

$$12X < 7(1200), \text{ and } 11(1200) < 19X$$

$$694.7 < X < 700.0$$

In other words, whatever the value of Vicentino's fifths in this series, it must fall between 694.7 and 700.0 cents. But when we attempt to narrow the limits on the size of these fifths, we encounter some confusing and apparently contradictory information.

In Chapter LXIV (Vic. p. 146v), Vicentino states that the fifths are somewhat short so that the thirds and sixths can be in tune. Unfortunately he does not say whether he means the minor thirds and the major sixths, in which case he would be talking about 1/3 comma meantone tuning, or the major thirds and the minor sixths, in which case he would be talking about 1/4 comma meantone tuning. 1/3 comma meantone has fifths of 694.8 cents, and 1/4 comma meantone, 696.6 cents. Both of these values are within the limits found above.

Turning to the question of whether the fifth from E dot to B dot is similar to the other fifths in the series, Vicentino gives the common fifth[2] above E dot as B dot in Chapter XXIV (Vic. pp. 112v-113). In Chapter XVII (Vic. p. 109v), concerning G dot, he gives the major

[2]It will be recalled that this is Vicentino's term for the flat or short fifth in the series of 30 fifths.

third above G dot as B dot. In these two places and consistently throughout the book, Vicentino uses this B dot, which was tuned, as the reader will recall, by descending fifths, with pitches that were tuned by ascending fifths, to construct intervals and scales. These "enharmonic" uses of B dot suggest very strongly an equally divided scale having 31 dieses of equal value in the octave where the value of each fifth, including the fifth from E dot to B dot, would be 696.8 cents.

As Barbour points out[3] the fifths of an equal tempered 31 tone scale are very close to those of 1/4 comma tuning, 696.8 and 696.6 cents, respectively. The values Barbour gives for the first five orders of Vicentino's tuning are based on the fifth of the 1/4 comma tuning.[4] Barbour apparently used the 1/4 comma fifth and computed the values ascending from C through B flat dot and descending in fifths from C to F dot. This results in the fifth from F dot to B flat dot being larger than the other 30 fifths, 704 cents. It is noted that tuning the fourth order by descending in fifths and the fifth order ascending by fifths is not the sequence specified by Vicentino, which would be to ascend in fifths from C through E dot and descend from C to B dot. This also results in one large fifth, but it is from E dot to B dot. Table 2 (infra p. 91) shows the 31 notes included in the first five orders of the instrument and the cents values for these notes in a 31 tone equal tempered scale. In the third column Barbour's values will be found.

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[3]Barbour, Tuning . . ., pp. 117-119.

[4]Barbour, Equal Temperament: . . ., p. 69.

TABLE 2[5]

COMPARISON OF VALUES

Note	Equal Temperament	Barbour's Values	Fourth order Ascending
C	-0.0	0.	
C DOT	38.7	41.	34.
C SHRP[6]	77.4	76.	
D FLAT	116.1	117.	
D FLDT[7]	154.8	152.	
D	193.5	193.	
D DOT	232.3	234.	227.
D SHRP	271.0	269.	
E FLAT	309.7	310.	
E FLDT	348.4	345.	
E	387.1	386.	
E DOT	425.8	428.	421.
E SHRP	464.5	462.	
F	503.2	503.	
F DOT	541.9	545.	538.
F SHRP	580.6	579.	
G FLAT	619.4	621.	
G FLDT	658.1	655.	
G	696.8	697.	
G DOT	735.5	738.	731.
G SHRP	774.2	772.	
A FLAT	812.9	814.	
A FLDT	851.6	848.	
A	890.3	890.	
A DOT	929.0	931.	924.
A SHRP	967.7	966.	
B FLAT	1006.5	1007.	
B FLDT	1045.2	1041.	
B	1083.9	1083.	
B DOT	1122.6	1124.	
B SHRP	1161.3	1159.	
C	1200.0	1200.	

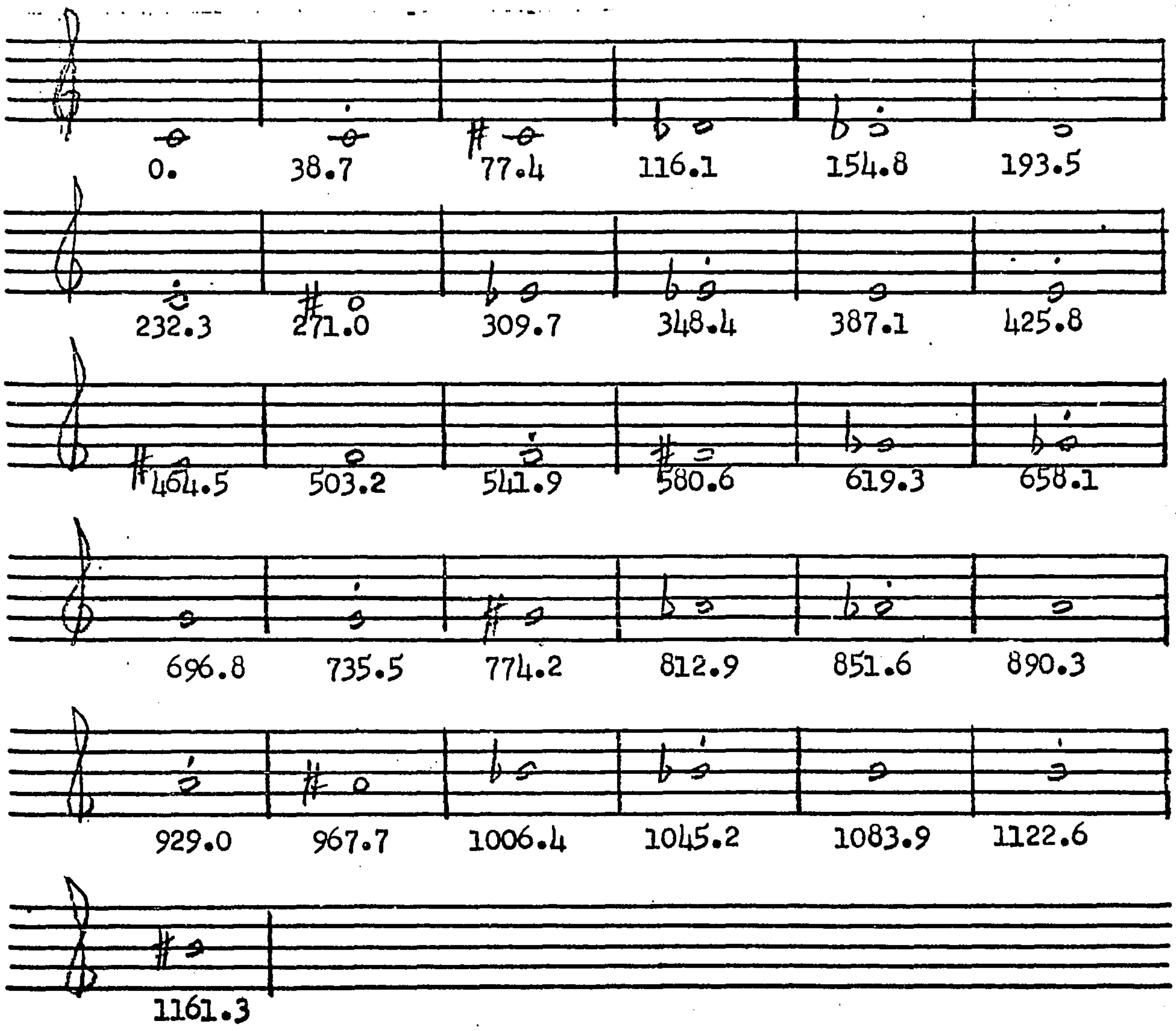
[5]This and many of the tables in this chapter were computed on an IBM 7090/94 located at the Numerical Computation Laboratory of The Ohio State University.

[6]C SHRP C SHARP.

[7]D FLDT D FLAT DOT.

The fourth column shows the changes from Barbour's values that are necessary in the fourth order notes if they are computed by ascending fifths from the fifth order. Example 10 shows Vicentino's notation for the first five orders of the instrument as it would be used for the 31 tone equal tempered scale.

Example 10,--Thirty-one tone equal tempered scale



However if either solution is accepted, then it is impossible to explain the relationships that are supposed to occur between the sixth order keys and those of the fourth and first orders. The first mention of the sixth order is in Chapter III (Vic. p. 101v), where Vicentino names all of the orders. "And in the sixth order one will speak of nothing except the sixth order or the order of the perfect fifth." As previously stated the sixth order is omitted from the chapter on the first tuning. It is mentioned in the second tuning, but because of the difference between the two tunings (supra p. 22), that is irrelevant here. Then in Chapter XIII, Vicentino discusses the consonant intervals above and below A sixth. Of the fifths above and below A sixth, Vicentino states (Vic. p. 108):

> One will find the fifth below on D sol re sixth tuned according to the use of the first order, and when the player will want the perfect fifth, he will touch the key of D sol re first with A la mi re sixth. ... Its A sixth common fifth ascending , as are in the other instruments, will be on E la mi sixth, and its perfect fifth will be on E la mi fourth, because it will have a comma more than the fifth which is used.

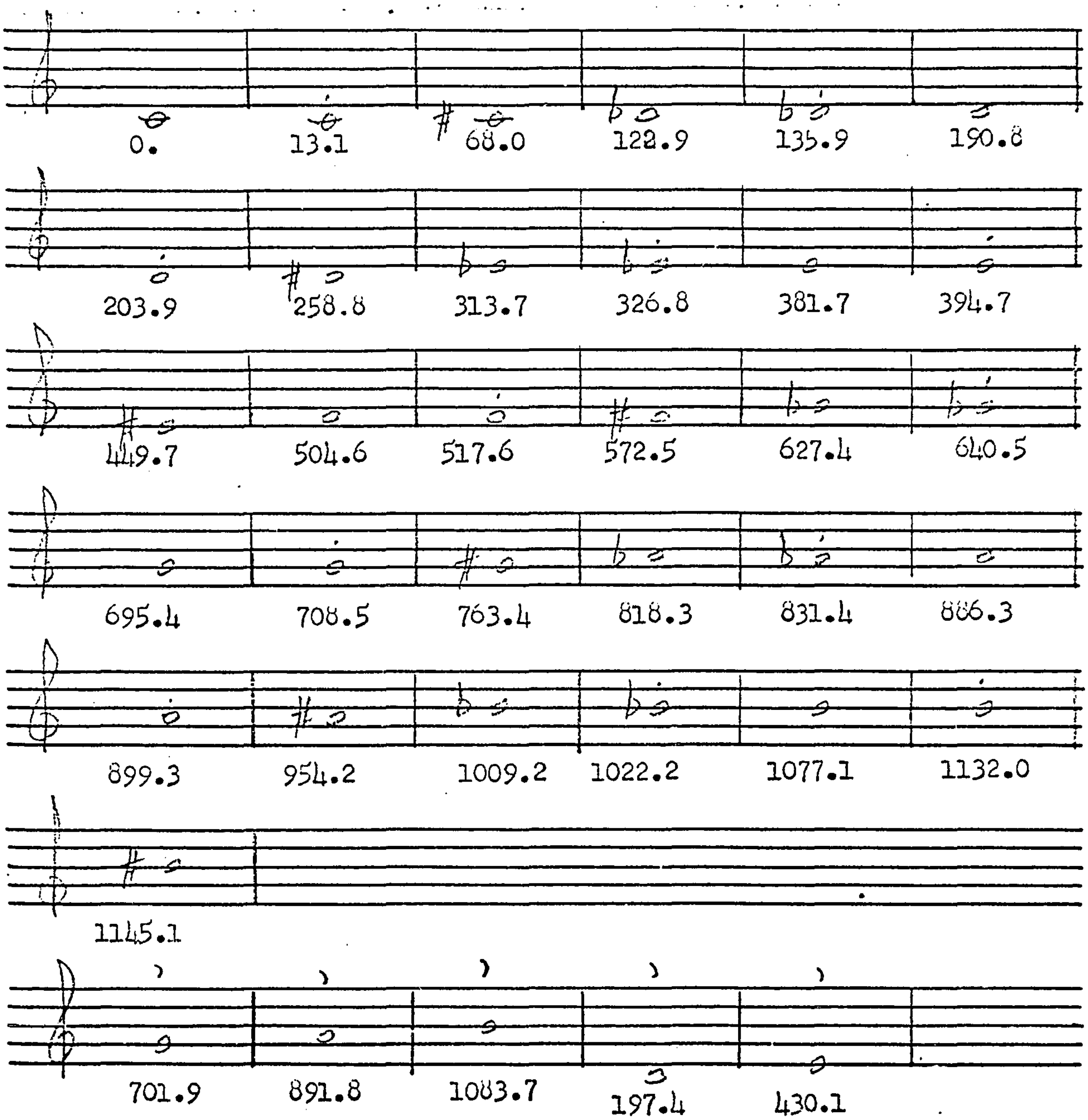
In other words, in the sixth order there are fifths the same size as the common fifths in the other five orders of the instrument, but if we descend from the sixth order to the first or ascend from the sixth to the fourth, we get a fifth that is larger than the common fifth by a comma. This seems to be confirmed in the notation. From D natural to A natural is a common fifth, and from A natural to A comma is a comma, thus from D natural to A comma is a common fifth plus a comma. Also from A comma to A dot is a comma, and from A dot to E dot is a common fifth, thus A comma to E dot is a common fifth plus a comma. This common fifth plus a comma should be equal to the distance from D natural to E dot plus an octave divided by 2. If we use the scale given above for an equal tempered 31 note scale, this would be 232.3 plus 1200 divided by 2 or 716.1 cents. This interval Vicentino calls a perfect fifth, but it certainly is not the interval usually thought of as a perfect fifth, i.e., an interval with a 3:2 ratio or 701.9 cents. Therefore at this point we must conclude that either Vicentino does not mean the same thing we mean and theorists of his time meant by "perfect fifth", or this is not an equally divided system.

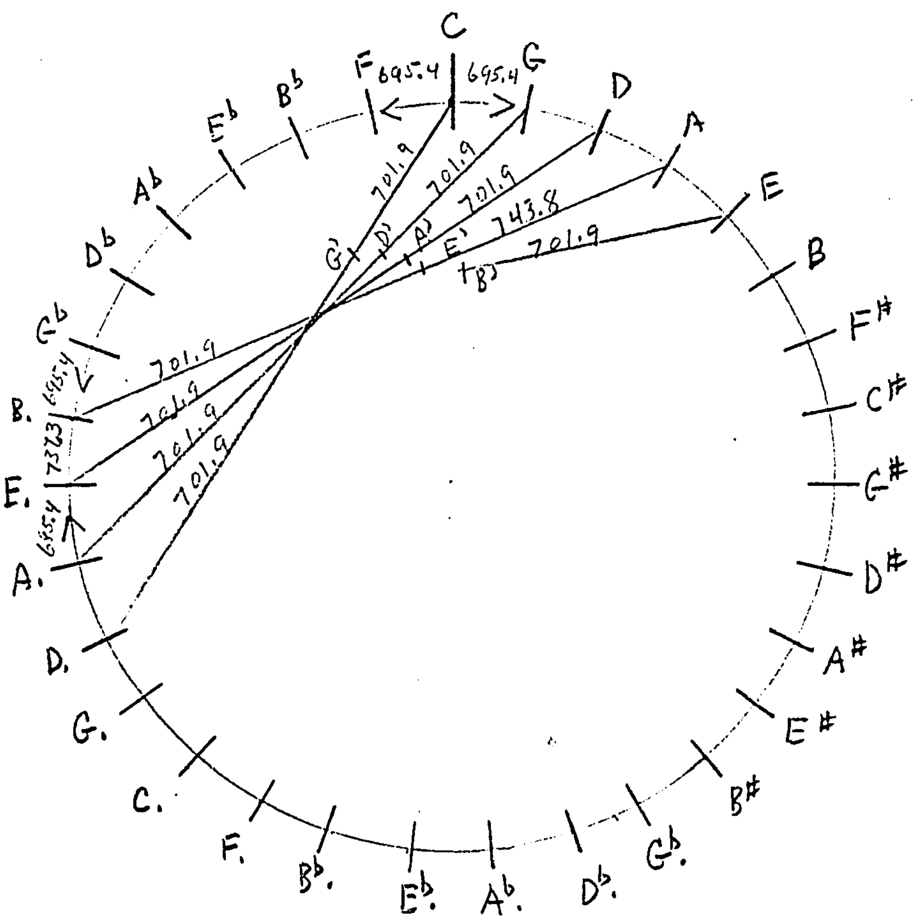
The above is the result obtained if the 31 note equal tempered scale is used but a new meaning has to be applied to "perfect fifth." An alternate possibility would be to keep the traditional meaning of "perfect fifth." If the interval from D natural to E dot is equal to two perfect fifths, then the size of the common fifths can be obtained as follows. The series of common fifths from D natural to E dot is D - A - E - B - F♯ - C♯ - G♯ - D♯ - A♯ - E♯ - B♯ - G♭. - D♭. - A♭. - E♭. - B♭. - F. - C. - G. - D. - A. - E., or 21 fifths extending more than 12 octaves. Two perfect fifths are 1403.9, plus eleven octaves would be 14603.9, divided by 21 is 695.4 cents. A common fifth of 695.4 cents is not only within the limits of the size of the possible common fifth, but is close to the fifth of 1/3 comma meantone tuning of 694.8 cents.

If the value of this common fifth is extended throughout the complete series of 30 fifths, from B dot ascending through E dot, the scale given in Example 11 (infra p. 96) results. The five sixth order notes are inserted in the scale as follows. G comma is given (Vic. p. 115v) as a perfect fifth above C and as a perfect fifth below D dot (Vic. p. 115). Thus it has the value 701.9 cents. A comma is given (Vic. p. 108 and p. 113v) as the perfect fifth above D and (Vic. p. 108 and p. 113) as the perfect fifth below E dot, and has the value 891.8. B comma is given (Vic. p. 111v) as the perfect fifth above E and has the value 1083.7. D comma is given (Vic. p. 108v) as the perfect fifth above G and has the value 197.4. This D comma is also the perfect fifth below A dot although Vicentino does not mention it. Finally E comma to B dot is given (Vic. p. 113 and p. 118) as a perfect fifth. A perfect fifth below B dot would be 430.1. Example 12 (infra p. 97) is a diagram showing the relationship of the common fifths (outer circle) and the perfect fifths (straight lines).

It is noted that with E comma the apparent sequence of the notation is broken. It is known from the notation that a note with a comma should fall between the natural and the dot, and this is the case with the remaining four sixth order notes. For example, G comma, 701.9, does fall between G natural, 695.4, and G dot, 708.5. However with the above value for E comma, E comma, 430.1, is greater than E dot, 394.7. Also we would expect, although Vicentino does not mention it, a perfect fifth between A and E comma, but this fifth is quite large, being 743.8. Also the fifth from E dot to B dot, which is called a common fifth (Vic. p. 113 and p. 118), is large, being 737.3.

Example 11,--Common fifths of 695.4 cents



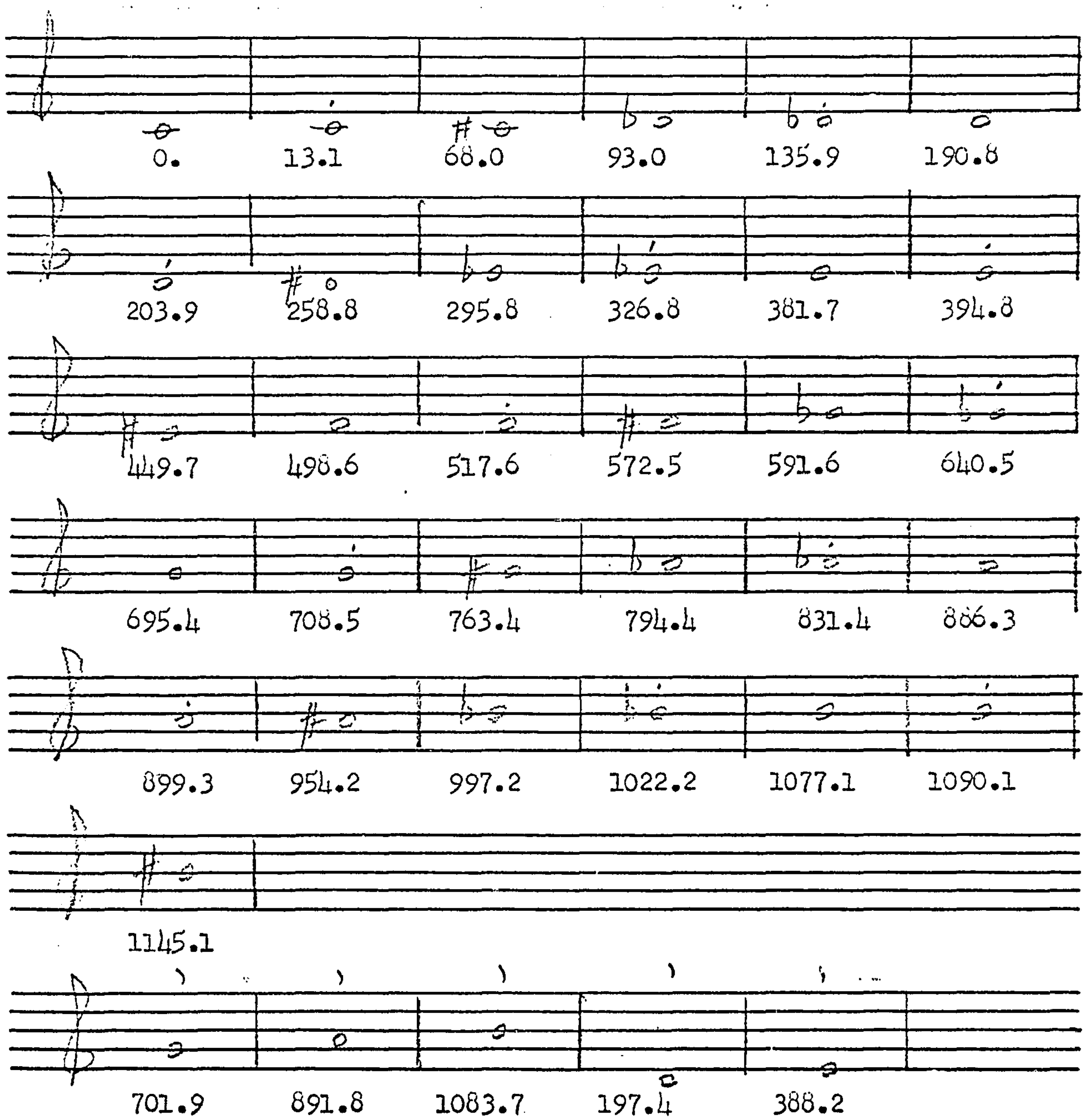


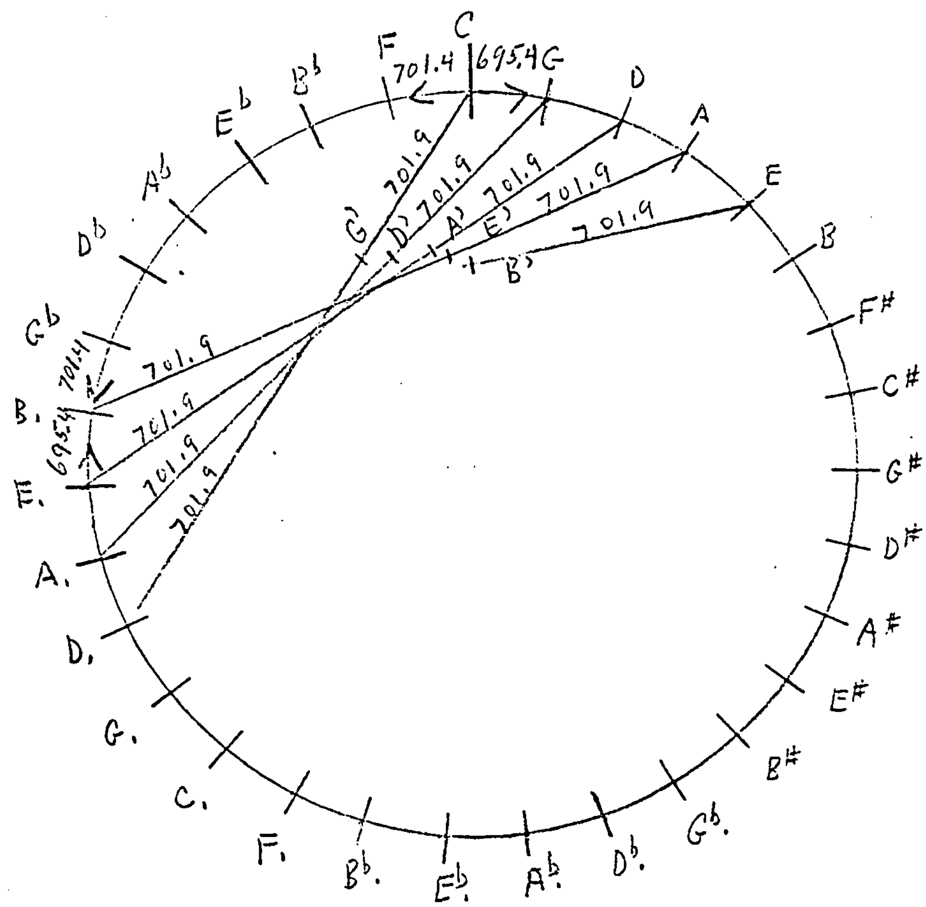
Example 12,--Common fifths and perfect fifths

There is a compromise solution that would allow the nine perfect fifths to exist as truly perfect fifths and at the same time allow the common fifth from E dot to B dot to be similar to the other common fifths. This possibility can be explored by starting with C natural, since it is the first pitch in the ascending series of common fifths that is also involved with the perfect fifths, and ascending in the series of common fifths, not just through E dot, but one more fifth to B dot. For these fifths the value of 695.4 is used as in the previous example and the result is that all the intervals that should be perfect fifths, are perfect fifths. For the remaining common fifths, i.e., the fifths descending from C natural to B dot, the distance could be divided equally. This will result in the smallest possible value for any one of the remaining fifths. These seven decending common fifths, i.e., C - F - B^b - E^b - A^b - D^b - G^b - B., are equal to the difference of 18 octaves, the total number of octaves required to complete a circle of 31 fifths, and 24 times 695.4, the distance already used by the 24 fifths ascending from C to B dot. Each descending fifth is equal to 701.4 cents. The scale which results is given in Example 13 (infra p. 99) and the diagram showing the relationship of the common and perfect fifths is given in Example 14 (infra p. 100).

While this solution does produce the perfect fifths in the required locations, an ascending pattern for all 36 pitches that agrees with the notation, and a closed system, the fact that two different sizes of common fifths were used presents a problem. The seven descending fifths that were used to close the circle are very nearly

Example 13,--Common fifths of 695.4 and 701.4 cents





Example 14,--Common fifths and 9 perfect fifths perfect fifths, the difference is less than 0.5 cents. But Vicentino states in Chapter V (Vic. p. 103v) that: "[the tuner] will play the key of high E la mi flat, in the second order, and he will temper its fifth which will be A la mi re third." In this compromise solution, this descending fifth from E flat to A flat would not sound tempered, especially in comparison to the ascending common fifths which have the value of 695.4 cents.

Neither of the three above solutions is completely satisfactory. In the first solution there are the fifths from the first order of keys to the sixth and the sixth order to the fourth, which are supposed to be perfect fifths but are quite sharp. In the second solution, there is one extremely large common fifth from E dot to B dot, and the large fifth from E comma to B dot which is supposed to be perfect. In the third solution, the use of two sizes of common fifths is not in accordance with Vicentino's instructions. But each of the above solutions have had one principle in common, namely they have been attempts to interpret sixteenth century tuning directions in twentieth century terms. Perhaps we could get a more accurate picture of the system of tuning if we tried to imagine ourselves in the place of the sixteenth century tuner who is sitting before the Archicembalo with a tuning hammer in his hand and the fifth chapter ov Vicentinos fifth book in front of him.

He would know exactly what Vicentino (Vic. p. 103v) meant by, "[he] tunes so that he will have the first and second keys in accordance with the usage of the other instruments, with the fifths and fourths somewhat altered ..." We do not. But we do know that our sixteenth century tuner did not have an electronic device or a frequency counter to help him with his tuning.

Many twentieth century tuners use a process of counting the beats that are produced by slightly out of tune fifths, i.e., the plusations produced by fifths which deviate slightly from the truly perfect fifth with a frequency ratio of 3:2. This beating fifth was known to the sixteenth century tuner, as references can be found concerning the tuning of fifths so that they beat. Arnold Schlick, in his Spiegel der Orgelmacher und Organisten of 1511, states:[8]

> Starting now with F in the manual and play its higher fifth, do not tune it quite high enough (as it would be if tuned pure), but let it beat somewhat lower, ... As the C is withdrawn, so tune its higher fifth exactly so, likewise the fifth above G, the D.

To our twentieth century way of thinking, this must mean three fifths that are equal, i.e., they are equal in terms of cents. But Schlick did not have cents values to measure the fifths, they were not developed until the late nineteenth century by Ellis,[9] nor in fact did he have the logarithms from which cents are derived. They were invented by Napier in 1614.[10] What then does Schlick mean by tuning the fifths exactly the same? We know that if fifths are to be the same, i.e., equal in cents, the speed of the beats would have to increase

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[8]Schlick, *op. cit.*, pp. 34, 35.

[9]Herman L. F. Helmholtz, On the Sensations of Tone, trans. Alexander J. Ellis (6th ed.; New York: Peter Smith, 1948), p. 431.

[10]Howard Eves, An Introduction to the History of Mathematics (New York: Holt, Rinehart, and Winston, 1961), p. 243.

logarithmically as we ascend the scale, but again this implies knowledge which was not available at the time. If Schlick had reasoned that the speed of these beats should increase for each of his three fifths, it would seem probable that he would have related that fact. But he uses the term, "exactly." This could mean then that the fifths were tuned so that they beat at "exactly" the same rate.

Let us examine carefully the differences that are produced when we first look at this series of fifths as three equal fifths and then as three fifths with equal beats. Schlick began the series with Great F. We do not know the frequency of that note, nor do we know the rate of beat between F and C. But all this is immaterial since we are only comparing the intervals to each other. For the sake of round numbers, let us suppose that F=100 cps. (cycles per second) and that the rate of the beating fifth is 1 cps. The relationship of the frequencies of the two members of a flat fifth, i.e., a fifth which is less than a 3:2 fifth, and the number of beats is given as $3f_0 - 2f_1 = k$,[11] where f_0 is the frequency in cycles per second of the lowest member of the fifth, f_1 is the frequency of the upper, and k is the frequency of the beats. Or solving for f_1, $f_1 = (3f_0 - k)/2$. Thus if the pitch, $F = f_0 = 100$ cps, the rate of beat = k = 1 cps, then $C = f_1 = 149.5$ cps. This can be converted into cents by the formula, $(\log f_1 - \log f_0)(1200/\log 2)$.[12] The fifth from F to C is 696.17 cents.

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[11]H. Smith, Modern Organ Tuning (London: William Reeves, n.d.), p. 73.

[12]Logarithms are to the base 10.

In a series of equal fifths, all the fifths have the same cents value. This in our series, F to C = 696.17, as does C to G, and G to D. Converting these values to cycles per second, if F = 100.00, then C = 149.50, G = 223.50, and D = 334.13. Then computing the value for the rate of beat in each of the fifths, if F to C equals 1 beat per second, C to G equals 1.50, and G to D equals 2.24.

On the other hand, if we have a series of fifths where the rate of beat is constant, namely equal to 1 in this problem, then if F = 100 cps, C = 149.50, G = 223.75, and D = 335.13. Then converting these to cents values, the fifth F to C = 696.2 cents, C to G = 698.0 and G to D = 699.3 cents. As can be seen, the higher the fifth in a series with a constant rate of beat, the larger the cents values, the limit being the perfect fifth.

As Schlick continues his tuning[13] to find A, he first says that the octave below D is tuned and then the A is found from that lower D. Now if the cents value of all these fifths were kept equal, the reason for going down an octave at this point might well be that the beats have become too fast to count. But the reason Schlick gives is that the pipes become too small. This is reasonable from the standpoint that larger pipes are easier to tune than smaller pipes. The interpretation of Schlick's tuning instructions in terms of a constant rate of beat, rather than a constant size of fifth, is not only possible, but appears to be probable.

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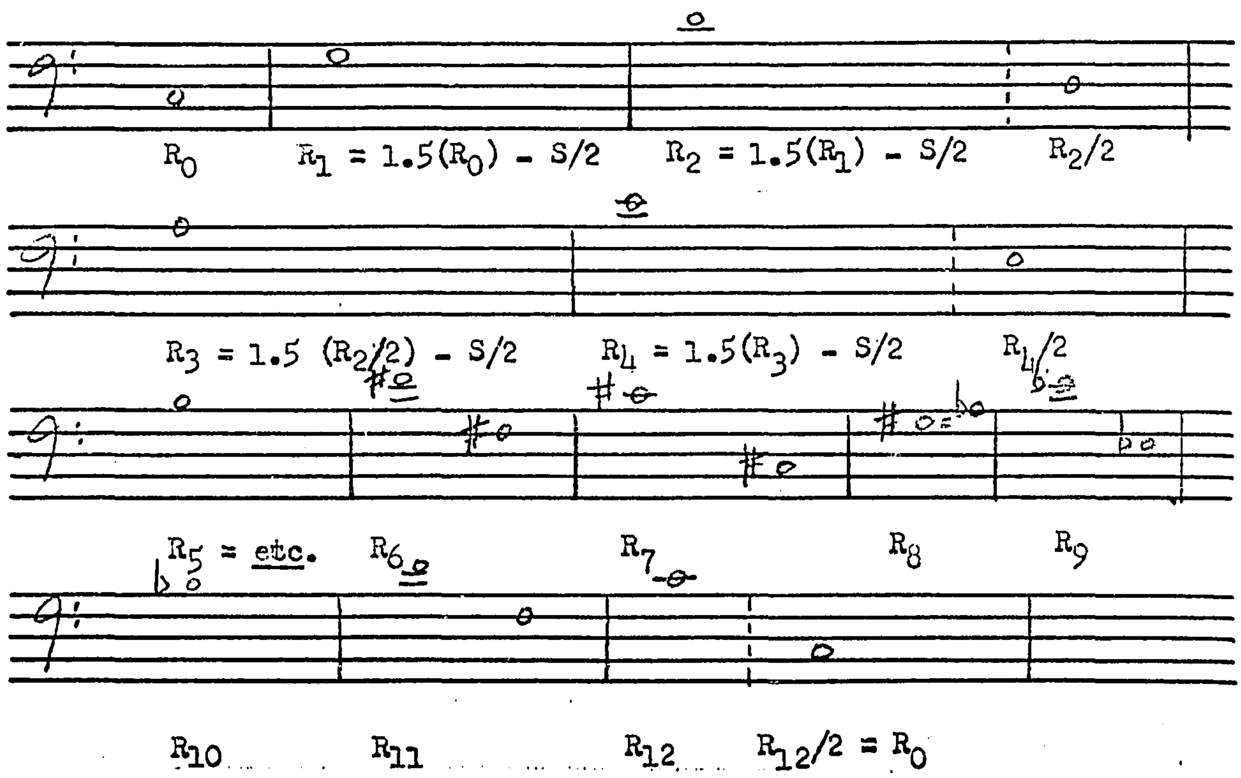
[13]Schlick, loc. cit.

Before returning to the scale of the Archicembalo, the formula of the constant rate of beat will be applied to the familiar 12 tone scale. The purpose is twofold. First it is a relatively simple illustration that shows the working of the formula itself and second it shows how this method of tuning could approximate the equal tempered scale. A sequence of ascending fifths and descending octaves is developed for the 12 tones of a scale, arranged so that the lowest member of each fifth is located within the octave above the starting C. This is shown in Example 15.

Example 15,--Formula for 12 tone scale



To begin, let R_0 equal 1.0, i.e., the ratio of C to C. Then the ratio of G, R_1, is 1.5 (R_0) - S/2, where S is the speed of the beats of the out of tune fifth. Then D, R_2, is 1.5 (R_1) - S/2. Before finding A, the lower octave of D is found, i.e., $R_2/2$. Then R_3, A, is equal to 1.5 ($R_2/2$)= S/2, etc. Finally the circle is complete to C, or R_{12} (Example 15). Since R_{12} is an octave above R_0, then $R_{12}/2 = R_0$. Now if the value of R_{12} can be found in terms of R_0 and S, then it will be possible to solve for S in terms of R_0.

The results of the above problem are given in Table 3 (infra p. 107). In the first column are found the names of the notes that were used in Example 15. In the second, the ratios of the notes as related to R_0 or 1.0 are shown. The ratio of the speed of the beats, again relative to R_0, is 0.0045598 (not shown in Table 3). It is important to note that these ratios, as are all interval ratios, are independent of time, i.e., the values are true for any octave, any pitch level, any transposition.

When the ratios are converted into cents values, column 3, it can be seen that tuning with a constant beat produces a very good approximation of true equal temperament, as no value is more than 0.7 cents from the true equal tempered value. If we commit ourselves to a specific time related value, then it is possible to compute the values of the speed of the beating fifths. If A = 220 cps, then the fifths beat at 0.5968 cps., which is shown in the fourth column. This is equal to about 35.8 beats per minute. The fifth column shows the rate for fifths of a mathematically correct equal tempered scale. This is

TABLE 3

TWELVE TONE SCALE WITH EQUALLY BEATING FIFTHS

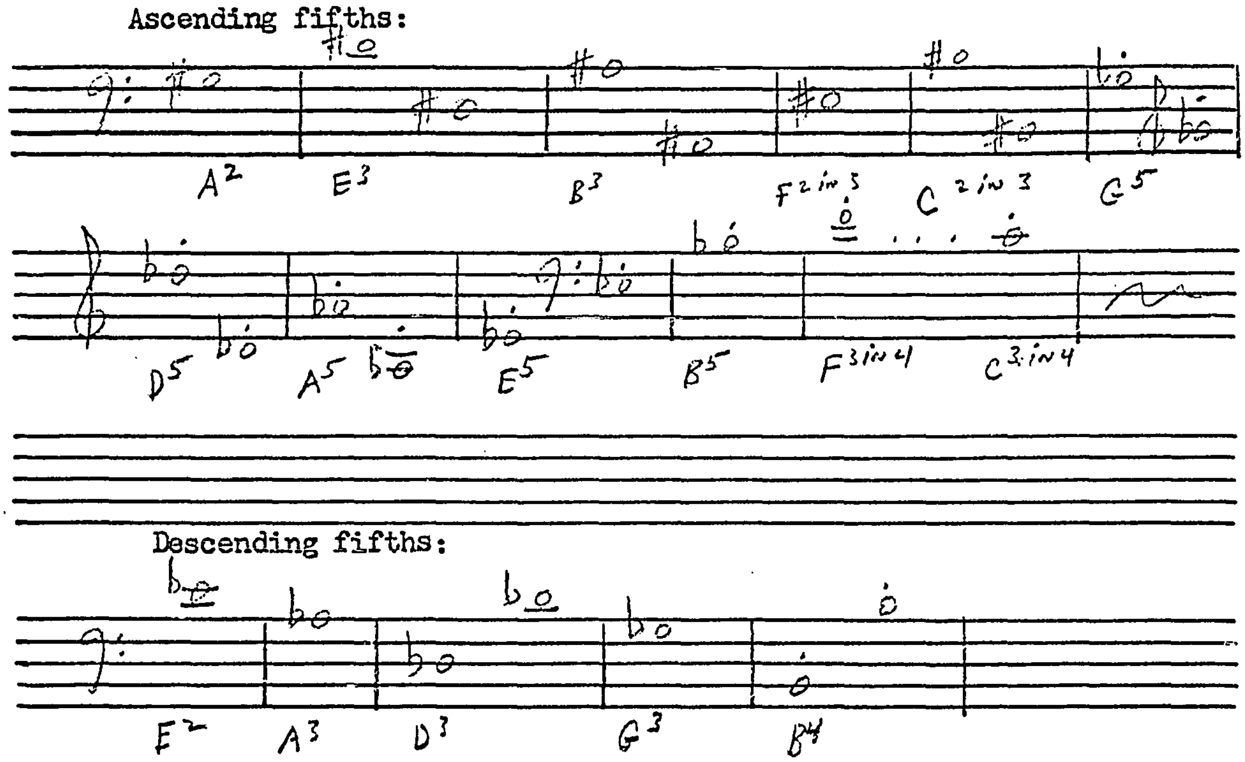
NOTE	RATIO	CENTS	5THS BEAT	E.T.5THS	C.P.S.
C	1.0000000	-0.000	0.5968	0.4431	130.879
C SHRP	1.0594839	100.034	0.5968	0.4695	138.664
D	1.1221501	199.519	0.5968	0.4974	146.866
E FLAT	1.1890694	299.800	0.5968	0.5270	155.624
E	1.2595689	399.516	0.5968	0.5583	164.851
F	1.3348532	500.017	0.5967	0.5915	174.704
F SHRP	1.4141651	599.941	0.5968	0.6267	185.084
G	1.4977201	699.322	0.5968	0.6640	196.020
G SHRP	1.5869459	799.503	0.5968	0.7034	207.697
A	1.6809452	899.127	0.5968	0.7453	220.000
B FLAT	1.7813242	999.540	0.5968	0.7896	233.137
B	1.8870734	1099.381	0.5968	0.8365	246.978
C	2.0000000	1200.000	1.1936	0.8863	261.757

based on the same value for the C as the series in Column 4. Notice that each fifth must beat at a slightly greater rate than the one lower. The last column shows the cycles per second for the original series.

Using the above sequence of fifths and octaves, the computer was asked to transpose the series until the rate of the beat was equal to 1 beat per second. This occurs when the beginning or lowest frequency in the series is equal to 219.306 cps. This is of course very close to A = 220 cps, or small A. In other words, if the series of fifths and octaves is transposed to small A, and then all the fifths are tuned so that they beat at 1 beat per second, i.e., 60 MM, then it should be possible to set a temperament that is within 1 cent of equal temperament. This has been done by the author and found to be a very quick and accurate method of setting a temperament.

Now returning to the Archicembalo, if we are going to consider fifths which beat at a constant rate it is first necessary to determine the octaves in which the tuning takes place. Unfortunately Vicentino furnishes only about half of this information. From Chapter V, we learn the exact sequence for the third order of keys, the fifth order, and parts of the second and fourth. The sequence is given below in Example 16, first with all the known ascending fifths and then all the descending fifths. From the same chapter we know that the fourth order is to be tuned like the first. Thus in the first order there is an octave break from great B to small B and first line C and first line F are included in the sequence.

Example 16,--Vicentino's tuning sequence



For additional information, the Schlick, Spiegel der Orgelmacher will be used as well as the Toscanello in musica of Aaron. The tuning described by Aaron in Chapter XLI is the 1/4 comma meantone tuning. Starting from C fa ut, he says:

> . . . take the octave above C fa ut and make it perfectly in tune. Then the major third above, which is E la mi, wanting it to be sonorus and just, . . . having this, take the fifth in between that is G sol re ut and make that a little short. Continue to the fifth above, which is d la sol re, and similarly tune it in the same manner as G sol re ut described above. From there tune D sol re, the octave to d la sol re, and then take the fifth above D sol re,

> forming in the place of A la mi re which must lack the same amount from E la mi as from D sol re so that they would be equal one from the other. . . since the fifths above this C fa ut, D sol re, and E la mi, which are G sol re ut, a la mi re, and b fa ♮ mi, are always less than perfect. For the second order and mode, you need the string above C sol fa ut which is perfectly tuned, to tune the fifth below, which needs to be the opposite of the others described above, that is it will be tempered and high. . . . . In this manner tune the semitone of b fa ♮ mi, below F fa ut, and then E la mi below b fa ♮ mi, which is of the same order and mode as F fa ut was with C sol fa ut. The third and final mode will tune the major semitones between their thirds, as is the semitone of C fa ut. Touching A re, match it together with E la mi, so that it rests in the middle of a major third with A re and a minor with E la mi. In the same manner from D sol re to A la mi re, the third in the middle, and the semitone of F fa ut. This completes the tuning . . . .[14]

The tuning sequence, which is shown in Example 17 (infra p. 111) involves taking an octave above small c and then finding e' as a just major third. Then tuning small g as a short or flat fifth to small c, tuning d' also as a small fifth, then down the octave to small d. Then the ninth, small d to e', is divided equally with small a. Then he apparently takes another octave down to small e and then a short fifth up to small b.

The second group involves a descending series of short fifths starting from c'' to f', to small B^b, to small E^b. The third group

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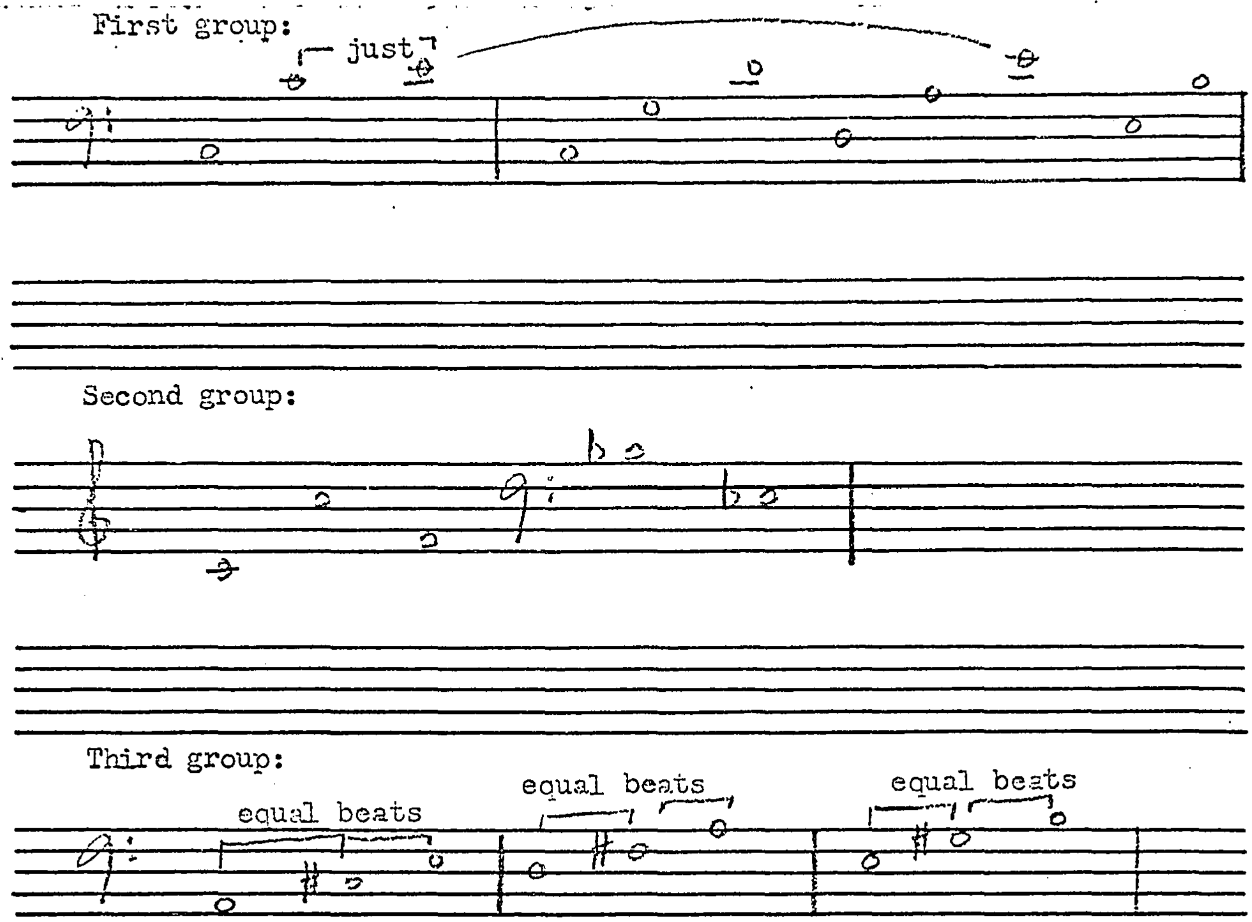
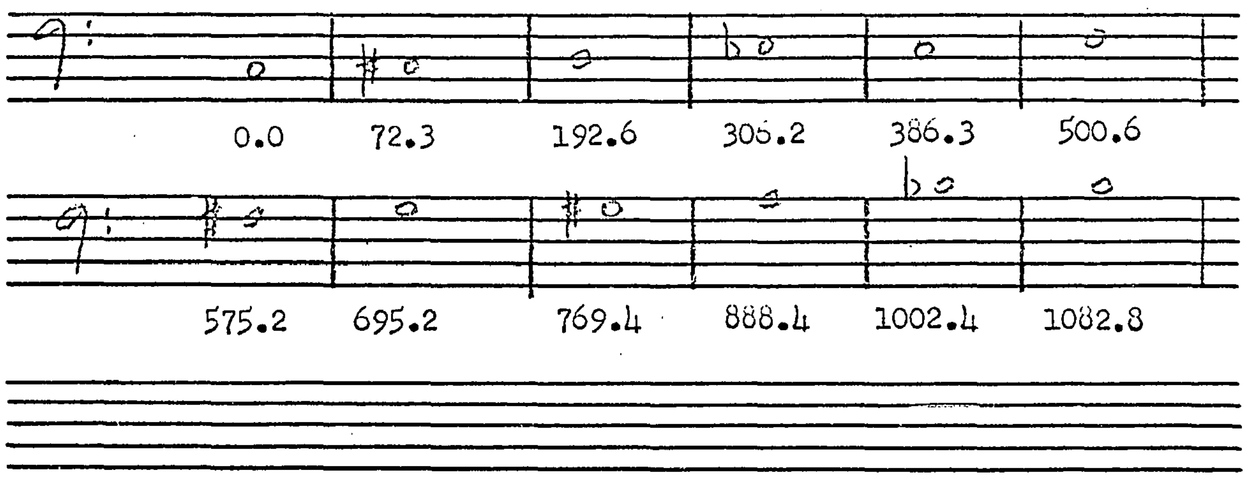
[14]Pietro Aaron, Toscanello in musica ... (Vinegia: 1529), Chapter XLI.

Example 17,--Aaron's tuning sequence



Example 18,--Values for Aaron's tuning



includes small c♯, f♯, and quite probably g♯.[15] These are to be tuned, not as Barbour suggests,[16] as just thirds above A, D and G, respectively, but so that they divide the flat fifth equally between the major third below and the minor third above. In order to be consistent, this must be interpreted as meaning that the major third and the minor third would beat at equal rates.

Aaron's tuning, as described above, interpreted with equally beating fifths, would produce the scale shown in Example 19 (supra p. 111). However of more importance at the moment is the fact that there are octave breaks on D, and E, and that for the descending series of fifths, first the c'' is tuned and all the fifths descend without any octave breaks.

Schlick's tuning directions[17] also show octave breaks on the same D and E. Now since these essentially different tunings both have octave breaks on these two notes, it is reasonable to assume that Vicentino's sequence would also have breaks on D and E. The system of Aaron and Schlick call for the tuning of small C and since we know that Vicentino's series involves c', it is reasonable that there is an octave break on C. Therefore using the same octaves found in both Aaron and Schlick all the first order notes can be located except for F. We do know that f' is included in the series, but it cannot be determined whether it is to be tuned a fifth below c'' or an octave above small f.

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[15]The final line quoted above should probably read, "and the semitone of G sol re ut, this completes the tuning."

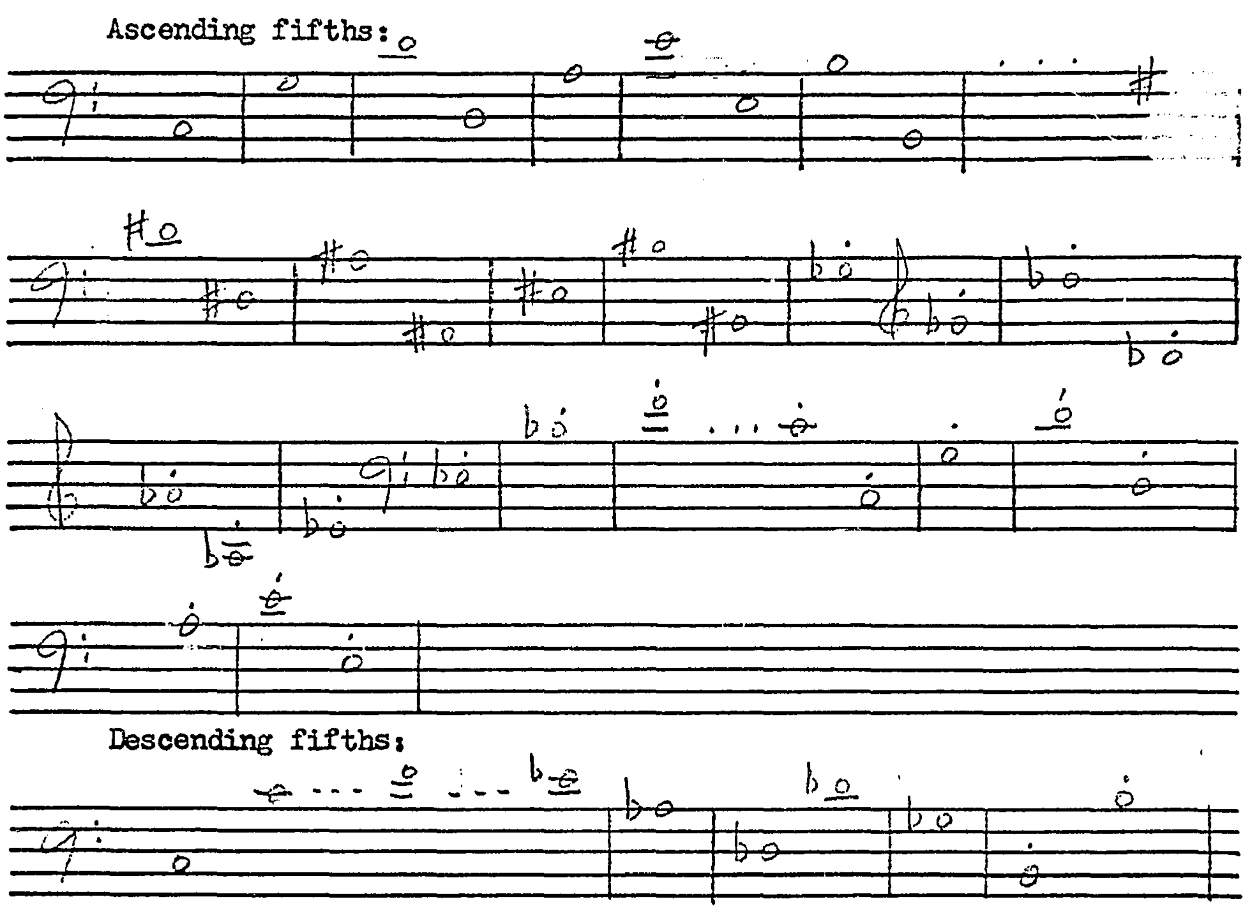
[16]Barbour, <u>Tuning . . .</u>, p. 26.

[17]Schlick, <u>loc</u>. <u>cit</u>.

The sequence of pitches is almost complete, as shown in Example 19 (infra p. 114). In the ascending sequence there is a gap between B and G sharp. We know that there is an octave break on B dot, and therefore such a break would probably occur on B. Then there still must be another octave break between B and G sharp, either on F sharp or C sharp. A break on F sharp would place this part of the sequence lower than any other, so that the break on C sharp seems more likely. In the descending fifths a gap occurs between F and E flat. Since there is an octave break on E flat dot in the fifth order between E flat dot and F dot rather than on B flat dot, it seems likely that the same would occur in the second order. Except for the octave breaks around F and F dot, which are still in question, the sequence of fifths and octaves shown in Example 20 (infra p. 115) is offered as the most probable succession of the sequence. The brackets indicate the portion of the sequence not given by Vicentino.

It is noted that Vicentino explicitly names the octave in which the tuning of certain fifths is to occur, except for those which were apparently common knowledge. There must be a reason for this concern with the octaves. If fifths are tuned so that they are equal in terms of cent values, the octave at which tuning takes place makes no difference. But if the fifths are to be tuned so that there is an equality in the rate of beating, then there is a difference, namely the higher fifths will be larger, i.e., they will have a greater cent value and more nearly approach a perfect fifth.

Example 19,--Continuation of Vicentino's sequence



As Barbour reports,[18] Christian Hughens, in his <u>Novus cyclus Harmonicus</u> showed that Aaron's 1/4 comma tuning was very similar to a division of the scale into 31 equal parts, and this is exactly the number of notes involved in Vicentino's first five orders of keys. However the major thirds, are a little sharp, sharp enough that if A equals 440 cps, the third c' to e' would beat about 35 beats per minute, whereas in Aaron's tuning, this c' to e' is just and would not beat.

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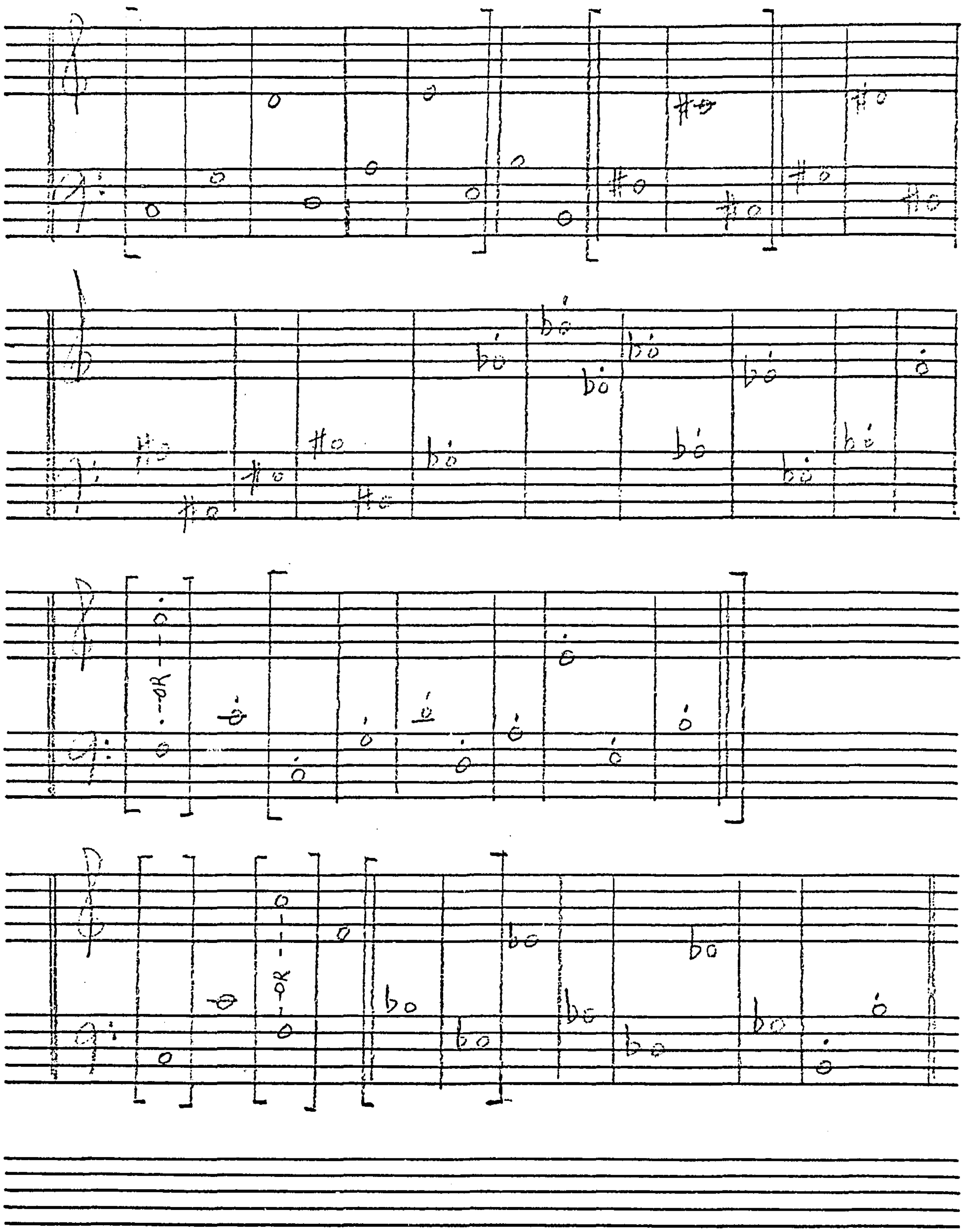
[18]Barbour, <u>Equal temperament: . . .</u>, p. 67.

Example 20,--A complete sequence



It is also known, or easily computed, that if a fifth, with a value such that four fifths equal a just major third (using cents values), is used in a series of 31 fifths, the final pitch is just short of 18 octaves.[19]

Most of the sequence of Vicentino's fifths are contained within a range of an octave and a sixth, except for an excursion into the upper range which reaches a high point on D flat dot. The fifths around this D flat dot in the series would be larger than the majority of fifths, i.e., if these fifths are tuned such that they are equal to the lower fifths in terms of the number of beats per time unit. It is beginning to appear that Vicentino, by carefully arranging the octaves, stretched some of the fifths, just enough, so that using Aaron's mean-tone tuning in the first two orders, the system does close on B 4.

However because we began with the assumption that the tuning was accomplished entirely with fifths and octaves, we shall first compute the values for such a tuning. The problem is constructed as before in the case of the 12 tone scale, letting small c equal the value 1. Then each ascending fifth in the series is based on the one before with a constant rate for the beating of the flat fifth. There are two possibilities, however, since the exact octave sequence around F and F dot are still in question. In the first case, which shall be refered to as Version 1, small F was tuned below c' as was small F dot tuned below c' dot. In the second case, Version 2, these fifths were tuned

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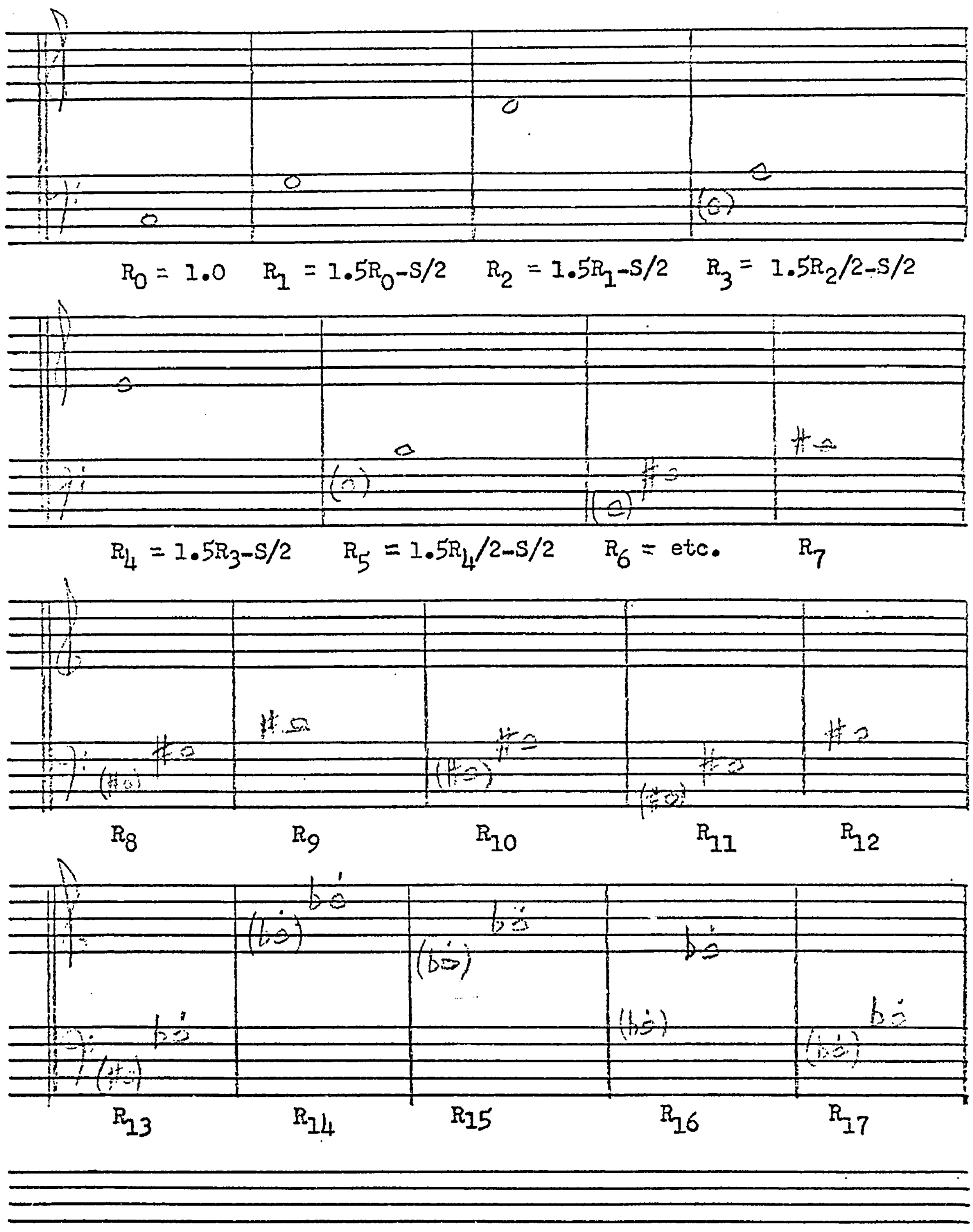
[19]Recall the values Barbour gives for the scale with one fifth larger than the others (See Table 2, supra p. 91).

an octave higher. The complete sequence of Version 1 is shown in Example 21 (infra pp. 118-119). Again R_n indicates the ratio of the pitch, and S indicates the speed of the beats. Example 22 (infra p. 120) shows where Version 2 differs from Version 1.

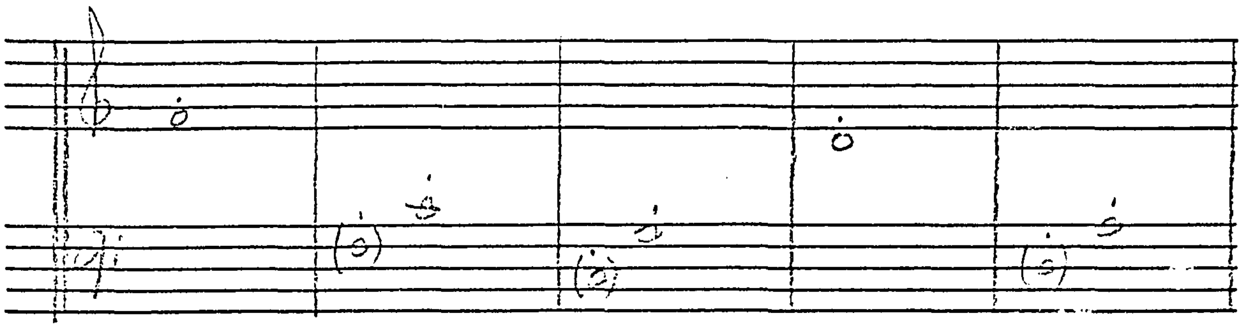
In order to follow Vicentino's directions more closely, although because of the circular nature of the relationships in these two versions of the problem it does not really make any difference, the fifths from C ascending through B dot were computed in one group, and then descending fifths were computed, starting again with C and going to B dot. The final equation then reads: find an S such that R_{24}, the B dot obtained in the ascending series, is equal to two times R_{31}, the octave above the B dot obtained in the descending series. After the original values were obtained, the octaves were transposed where necessary to produce an ascending scale from small c. The results are given in Tables 4, Version 1, and 5, Version 2 (infra pp. 121-124).

The first column in the tables gives the note name, the second and third give the ratio and cents value relative to small c, and the fourth gives the cps. based on small a equals 220 cps. The fifth column shows the cents value of each successive diesis in the scale. The sixth and seventh columns give the cents value and the beats per minute respectively of the fifth above each note. Some variation is noted in column seven, the number of beats per minute, which was to be the constant quantity. It is observed that whenever variation occurs, the number is either one half or two times the basic value for the speed of the beats. This is caused by the fact that some of the fifths were tuned in octaves other than the small octave and then

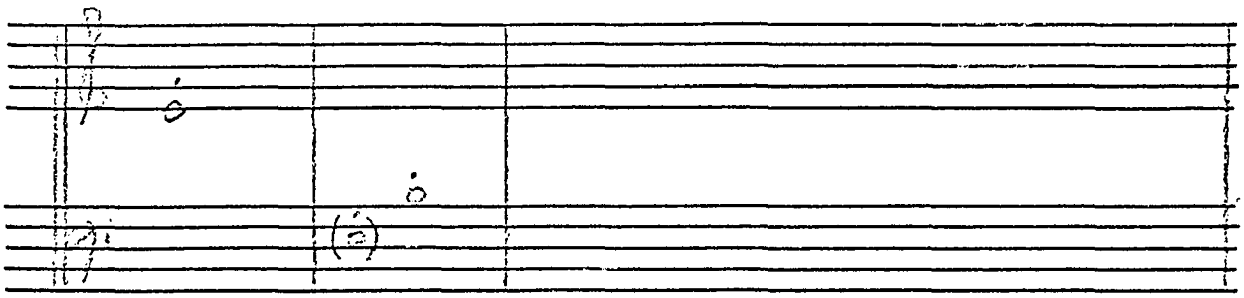
Example 21,--Sequence for Version 1



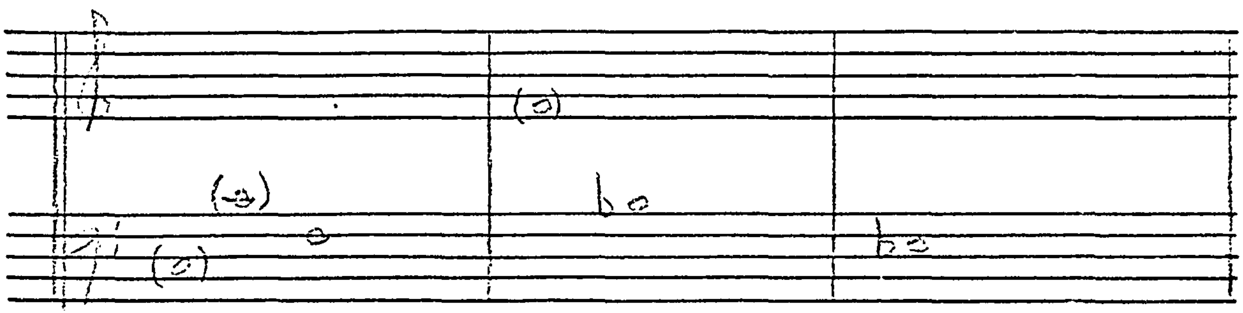
Example 21,--Continued



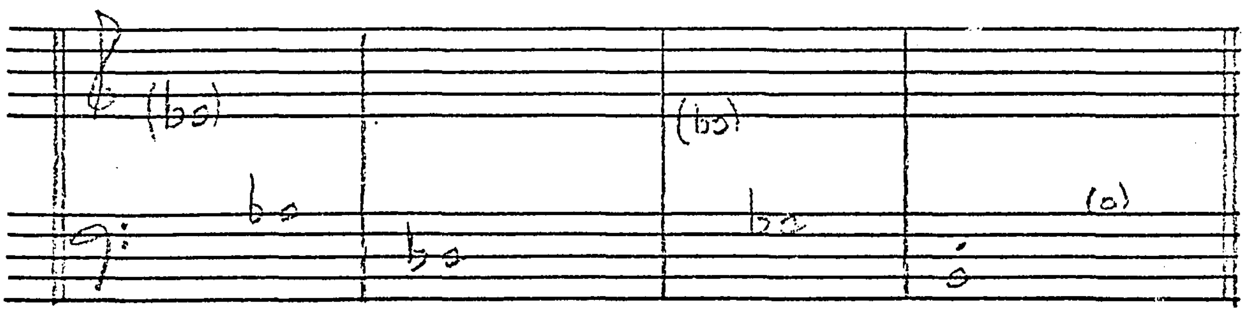
R_{18} R_{19} R_{20} R_{21} R_{22}



R_{23} R_{24}

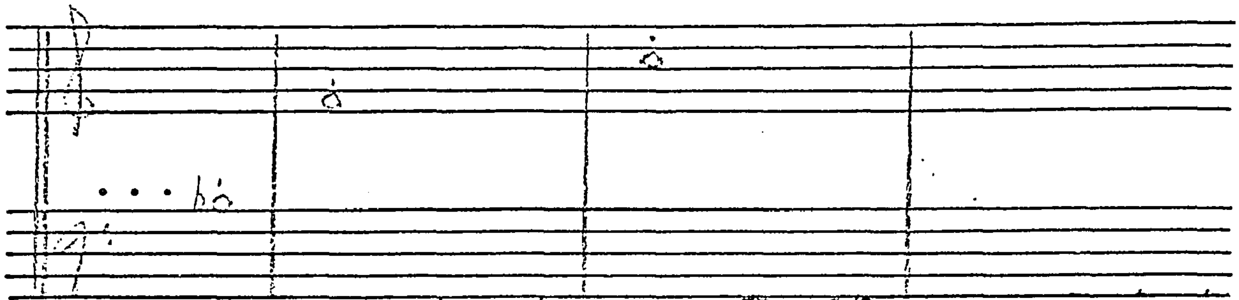


$R_{25} = 2(2R_0)/3 + S/3$ $R_{26} = 4R_{25}/3 + S/3$ $R_{27} = 2R_{26}/3 + S/3$

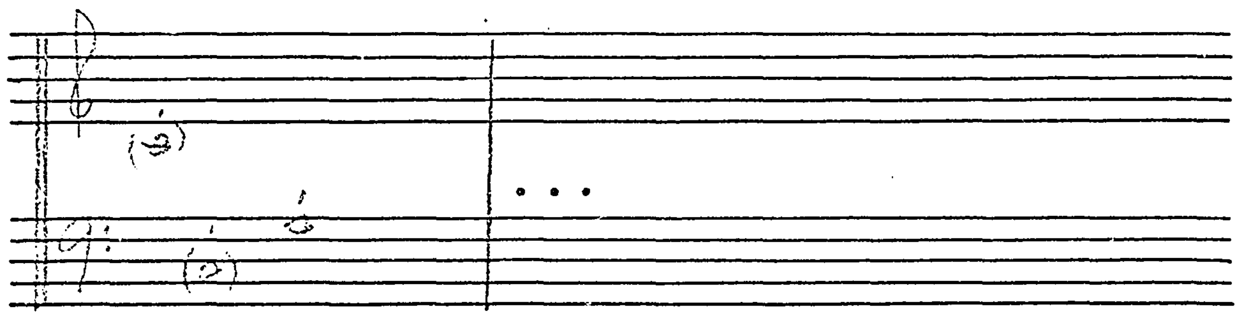


R_{28} = etc. R_{29} R_{30} R_{31}

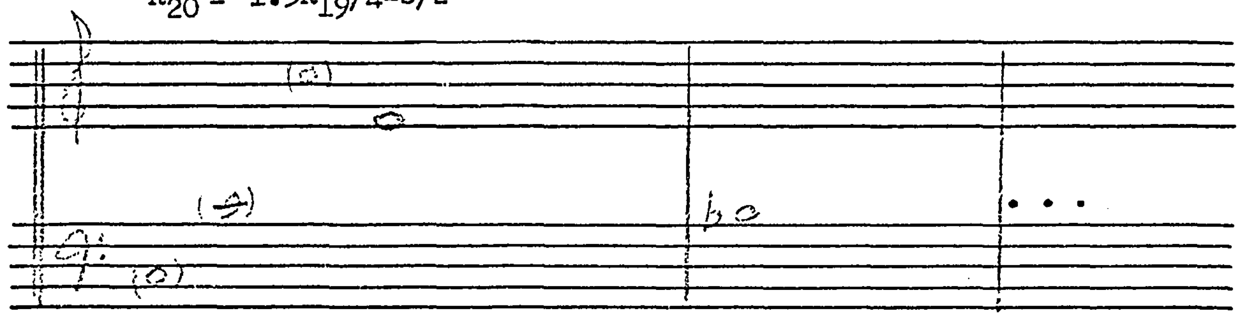
Example 22,--Sequence for Version 2



$R_{18} = 1.5R_{17} - S/2$ $R_{19} = 1.5R_{18} - S/2$



$R_{20} = 1.5R_{19}/4 - S/2$



$R_{25} = 4(2R_0)/3 + S/3$ $R_{26} = 2R_{25} + S/3$

TABLE 4

VALUES FOR VERSION 1

Note	Ratio	CPS.	Cents	Dieses
C	1.0000000	131.670	-0.000	-0.000
C DOT	1.0227825	134.670	38.999	38.999
C SHRP	1.0443823	137.514	75.180	36.181
D FLAT	1.0687111	140.717	115.046	39.866
D FLCT	1.0907811	143.623	150.434	35.388
D	1.1177570	147.175	192.728	42.294
D DOT	1.1433873	150.550	231.977	39.249
D SHRP	1.1676872	153.750	268.385	36.407
E FLAT	1.1950570	157.353	308.495	40.111
E FLCT	1.2220586	160.909	347.176	38.681
E	1.2502337	164.619	386.637	39.461
E DOT	1.2790678	168.415	426.111	39.474
E SHRP	1.3035079	171.633	458.879	32.768
F	1.3371962	176.069	503.053	44.174
F DOT	1.3675729	180.069	541.941	38.888
F SHRP	1.3963727	183.861	578.021	36.080
G FLAT	1.4288110	188.132	617.778	39.757
G FLCT	1.4563062	191.752	650.776	32.998
G	1.4942056	196.742	695.254	44.478
G DOT	1.5283794	201.242	734.403	39.149
G SHRP	1.5607791	205.508	770.720	36.316
A FLAT	1.5972722	210.313	810.732	40.013
A FLCT	1.6332744	215.054	849.321	38.588
A	1.6708412	220.000	888.689	39.369
A DOT	1.7092866	225.062	928.073	39.384
A SHRP	1.7457364	229.861	964.603	36.530
B FLAT	1.7867911	235.267	1004.845	40.242
B FLCT	1.8272935	240.600	1043.650	38.805
B	1.8695561	246.165	1083.235	39.585
B DOT	1.9128073	251.860	1122.830	39.595
B SHRP	1.9494675	256.687	1155.696	32.866
C	2.0000000	263.340	1200.000	44.304

TABLE 4--Continued

Fifths				Major thirds			
From	-To	Cents	B/Min.	From	-To	Cents	B/Min.
C	-G	695.254	91.55	C	-E	386.637	7.38
C DOT	-G DOT	695.404	91.55	C DOT	-E DOT	387.112	18.63
C SHRP	-G SHRP	695.540	91.55	C SHRP	-E SHRP	383.699	-62.25
D FLAT	-A FLAT	695.686	91.55	D FLAT	-F	388.007	41.31
D FLDT	-A FLDT	698.887	45.78	D FLDT	-F DOT	391.507	129.46
D	-A	695.962	91.55	D	-F SHRP	385.293	-26.03
D DOT	-A DOT	696.096	91.55	D DOT	-G FLAT	385.801	-1[illegible].37
D SHRP	-A SHRP	696.218	91.55	D SHRP	-G FLDT	382.392	-104.37
E FLAT	-B FLAT	696.350	91.55	E FLAT	-G	386.759	12.15
E FLDT	-B FLDT	696.474	91.55	E FLDT	-G DOT	387.227	25.47
E	-B	696.598	91.55	E	-G SHRP	384.082	-63.61
E DOT	-B DOT	696.719	91.55	E DOT	-A FLAT	384.621	-49.38
E SHRP	-B SHRP	696.817	91.55	E SHRP	-A FLDT	390.441	122.91
F	-C	696.947	91.55	F	-A	385.636	-20.67
F DOT	-C DOT	697.058	91.55	F DOT	-A DOT	386.132	-5.67
F SHRP	-C SHRP	697.159	91.55	F SHRP	-A SHRP	386.582	8.55
G FLAT	-D FLAT	697.268	91.55	G FLAT	-B FLAT	387.067	24.57
G FLDT	-D FLDT	699.657	45.78	G FLDT	-B FLDT	392.874	218.39
G	-D	697.473	91.55	G	-B	387.981	56.85
G DOT	-D DOT	697.574	91.55	G DOT	-B DOT	388.427	73.73
G SHRP	-D SHRP	697.665	91.55	G SHRP	-B SHRP	384.976	-47.61
A FLAT	-E FLAT	697.763	91.55	A FLAT	-C	389.268	107.75
A FLDT	-E FLDT	697.855	91.55	A FLDT	-C DOT	389.679	125.52
A	-E	697.948	91.55	A	-C SHRP	386.490	6.74
A DOT	-E DOT	698.038	91.55	A DOT	-D FLAT	386.973	25.72
A SHRP	-E SHRP	694.276	183.11	A SHRP	-D FLDT	385.831	-19.22
B FLAT	-F	698.208	91.55	B FLAT	-D	387.883	64.00
B FLDT	-F DOT	698.291	91.55	B FLDT	-D DOT	388.327	83.99
B	-F SHRP	694.786	183.11	B	-D SHRP	385.150	-49.64
B DOT	-G FLAT	694.948	183.11	B DOT	-E FLAT	385.666	-28.28
B SHRP	-G FLDT	695.080	183.11	B SHRP	-E FLDT	391.480	230.15
C	-G	695.254	183.11	C	-E	386.637	14.77

TABLE 5

VALUES FOR VERSION 2

Note	Ratio	CPS.	Cents	Dieses
C	1.0000000	131.711	-0.000	-0.000
C DOT	1.0224306	134.665	38.403	38.403
C SHRP	1.0436558	137.461	73.975	35.572
D FLAT	1.0676085	140.616	113.259	39.284
D FLDT	1.0892478	143.466	147.999	34.739
D	1.1175330	147.191	192.381	44.382
D DOT	1.1427674	150.515	231.038	38.657
D SHRP	1.1666458	153.660	266.840	35.802
E FLAT	1.1935926	157.209	306.373	39.533
E FLDT	1.2201769	160.711	344.508	38.136
E	1.2497576	164.607	385.978	41.470
E DOT	1.2781463	168.346	424.864	38.886
E SHRP	1.3020227	171.490	456.905	32.042
F	1.3353245	175.877	500.629	43.723
F DOT	1.3652320	179.816	538.975	38.347
F SHRP	1.3955235	183.806	576.968	37.992
G FLAT	1.4274604	188.012	616.141	39.173
G FLDT	1.4543217	191.550	648.416	32.275
G	1.4940264	196.779	695.047	46.631
G DOT	1.5276722	201.211	733.602	38.555
G SHRP	1.5595101	205.404	769.311	35.709
A FLAT	1.5954391	210.137	808.744	39.433
A FLDT	1.6308849	214.805	846.786	38.042
A	1.6703259	220.000	888.155	41.370
A DOT	1.7081774	224.985	926.949	38.794
A SHRP	1.7439950	229.703	962.875	35.926
B FLAT	1.7844152	235.027	1002.542	39.667
B FLDT	1.8242917	240.279	1040.804	38.262
B	1.8686628	246.123	1082.407	41.604
B DOT	1.9112456	251.732	1121.416	39.008
B SHRP	1.9470604	256.449	1153.557	32.141
C	2.0000000	263.422	1200.000	46.443

TABLE 5--Continued

Fifths				Major thirds			
From	-To	Cents	B/Min.	From	-To	Cents	B/Min.
C	-G	695.047	94.41	C	-E	385.978	-7.66
C DOT	-G DOT	695.199	94.41	C DOT	-E DOT	386.460	3.42
C SHRP	-G SHRP	695.336	94.41	C SHRP	-E SHRP	382.930	-80.51
D FLAT	-A FLAT	695.485	94.42	D FLAT	-F	387.369	25.73
D FLDT	-A FLDT	698.787	47.21	D FLDT	-F DOT	390.977	116.08
D	-A	695.775	94.41	D	-F SHRP	384.587	-44.03
D DOT	-A DOT	695.911	94.41	D DOT	-G FLAT	385.103	-31.57
D SHRP	-A SHRP	696.035	94.41	D SHRP	-G FLDT	381.576	-125.98
E FLAT	-B FLAT	696.169	94.42	E FLAT	-G	388.674	64.35
E FLDT	-B FLDT	696.295	94.41	E FLDT	-G DOT	389.094	77.48
E	-B	696.430	94.41	E	-G SHRP	383.334	-84.93
E DOT	-B DOT	696.552	94.42	E DOT	-A FLAT	383.881	-70.93
E SHRP	-B SHRP	696.652	94.41	E SHRP	-A FLDT	389.881	106.10
F	-C	699.371	47.21	F	-A	387.527	36.99
F DOT	-C DOT	699.428	47.21	F DOT	-A DOT	387.974	51.76
F SHRP	-C SHRP	697.007	94.41	F SHRP	-A SHRP	385.907	-12.94
G FLAT	-D FLAT	697.118	94.41	G FLAT	-B FLAT	386.401	2.84
G FLDT	-D FLDT	699.583	47.21	G FLDT	-B FLDT	392.388	201.98
G	-D	697.334	94.41	G	-B	387.361	35.71
G DOT	-D DOT	697.436	94.41	G DOT	-B DOT	387.814	52.32
G SHRP	-D SHRP	697.528	94.41	G SHRP	-B SHRP	384.246	-73.57
A FLAT	-E FLAT	697.628	94.41	A FLAT	-C	391.256	180.22
A FLDT	-E FLDT	697.722	94.41	A FLDT	-C DOT	391.617	197.72
A	-E	697.822	94.41	A	-C SHRP	385.820	-18.83
A DOT	-E DOT	697.914	94.41	A DOT	-D FLAT	386.310	-0.15
A SHRP	-E SHRP	694.030	188.83	A SHRP	-D FLDT	385.124	-47.36
B FLAT	-F	698.087	94.42	B FLAT	-D	389.839	143.73
B FLDT	-F DOT	698.172	94.41	B FLDT	-D DOT	390.234	163.43
B	-F SHRP	694.560	188.83	B	-D SHRP	384.432	-80.19
B DOT	-G FLAT	694.725	188.84	B DOT	-E FLAT	384.957	-59.17
B SHRP	-G FLDT	694.858	188.83	B SHRP	-E FLDT	390.951	206.36
C	-G	695.047	188.83	C	-E	385.978	-15.33

transposed. The eighth and ninth columns give the cents value and beats per minute of the major thirds. In the ninth column a positive number indicates that the third is sharp, relative to a just major third, a negative number indicates that the third is flat.

One word about the accuracy of the values given in these and later tables. In order to test the accuracy of the values obtained, parts of these problems were done again in double precision, a method of programing that will produce accuracy to 16 significant digits for one operation. In comparing the results of the double precision, it was found that, on all the values compared, the first six significant digits are accurate, and the seventh is accurate about 80% of the time. Anything beyond seven significant digits in any table should be ignored.

Some interesting observations can be made from the results. First is the variation in the size of the dieses, ranging from 32.768 cents to 44.478 in Version 1, and 32.042 to 46.631 in Version 2. These scales are not equal tempered in the usual sense. The size of the diesis in the 31 tone equal tempered scale, it will be recalled, is 38.710 cents. The sizes of the fifths also vary somewhat, from 694.276 to 699.657 in Version 1 and 694.030 to 699.583 in Version 2. The speed of the beat for the fifths in Version 1 is 91.55 beats per minute and in Version 2 is 94.41.

The most significant result is found in the major thirds. While the size of the thirds do vary, it is noted that the third from C to E is very close to a just third in both cases. Version 1 produced a C to E third of 386.637 cents, which is slightly sharp to a just major third of 386.314. The C to E third in Version 2 is slightly flat, being 385.978. In the first line octave, which is where Aaron tuned his just major third of C to E, this third in Version 1 would beat sharp at a rate of 14.77 beats per minute, this third in Version 2 would beat flat at 15.33 beats per minute, both on the order of 1 beat every 4 seconds. In either case this is nearer to a just third than the 31 tone equal tempered third of 387.097 cents, which for the same two notes would beat sharp at a rate of 35.71 beats per minute or more than two times as fast as the thirds in Versions 1 and 2.[20]

The above results can only add weight to the theory that the tuning was done with equal beats and the possibility that Vicentino tuned the first two orders in the 1/4 comma meantone tuning of Aaron. Thus for a third solution to the problem, Version 3, we shall begin the tuning with Aaron's instructions, which will tune the first two orders of keys, and then continue with Vicentino.

Recalling Example 17 (supra p.111) Aaron's tuning begins with small c, which is $R_0 = 1.0$. Then e' is tuned as a just major third, thus $e' = R_4 = 2.50$. Then the four fifths, C to G, G to D, D to A, and A to E, are tuned such that the speed of the beat is constant. Then the fifth E to B is also tuned as a "common" fifth. For the descending

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[20]The same third in the 12 tone equal temperament beats slightly more than 10 times per second, or about 601 beats per minute.

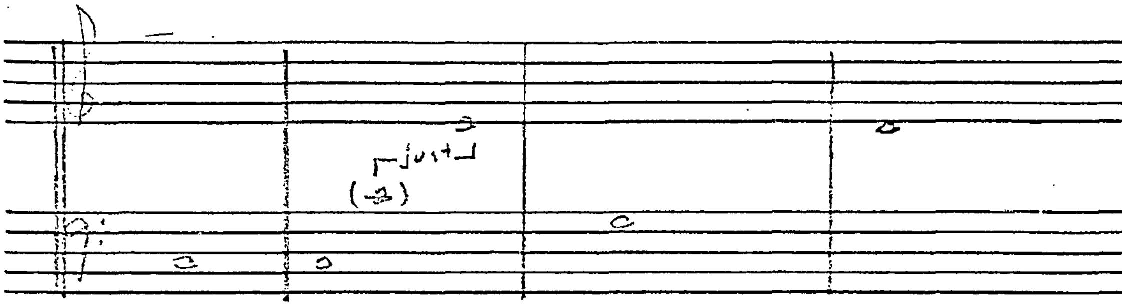
fifths, first c'' is tuned and then the fifths c'' to f', f' to small B^b, and small B^b to small E^b are tuned as common fifths. For the remaining three notes in the second order, $C^{\#}$, $F^{\#}$, and $G^{\#}$, the error of the common fifth is divided between the major and minor thirds (for the formula, see Appendix VII).

We continue from $G^{\#}$ with common fifths in the sequence given by Vicentino until C dot is reached. At that point Vicentino states that the fourth order is tuned like the first. This must mean that we tune another just major third from C dot to E dot, as C to E was tuned in the first order and then finding four equal fifths for C dot to G dot, G dot to D dot, D dot to A dot, and A dot to E dot. Then we have one more fifth to B dot to complete the fourth order. This fifth would be flatted the same amount as the four just tuned.

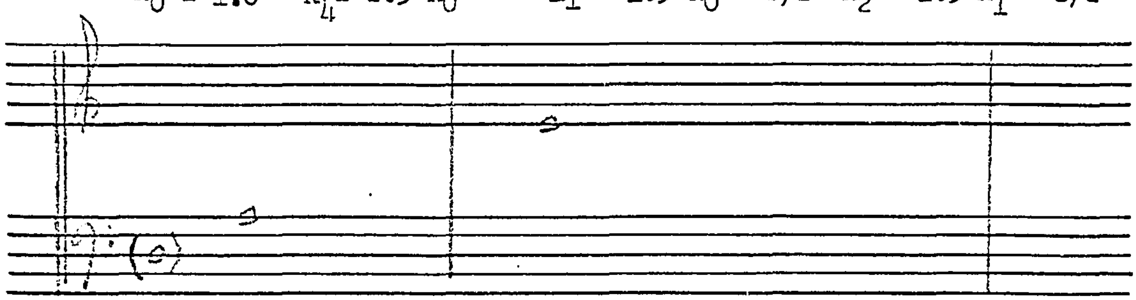
The remaining fifths of the descending series, starting from E flat, would all be tuned short such that they would beat as all the rest of the common fifths. This series ends on B dot, and thus we have a pitch which is approached from both the ascending and descending fifths and this permits a check point on the tuning.

The problem was constructed as shown in Example 23 (infra pp. 128-130), and the results are given in Table 6 (infra pp.131-132). The original and independent values obtained for B dot are 1.912002 (1122.101 cents) for B dot obtained in the ascending series (R_{24}) and 0.9553297, raised an octave, 1.910659 (1120.885 cents) for B dot obtained in the descending series (R_{31}), a difference of only 1.216 cents. After tuning some 31 intervals, it would seem quite reasonable

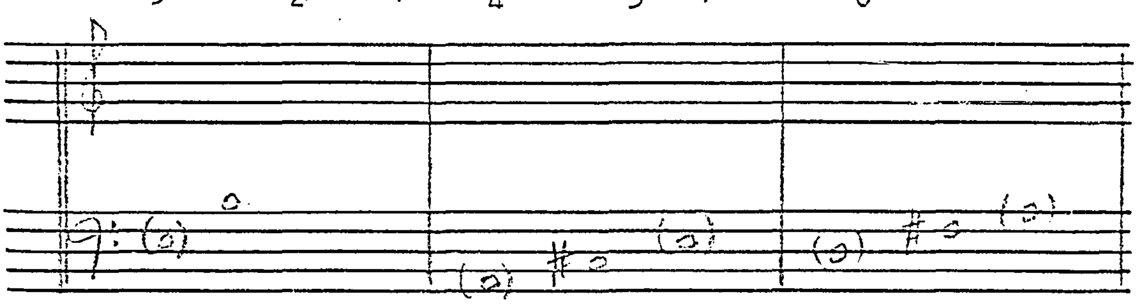
Example 23,--Sequence for Version 3



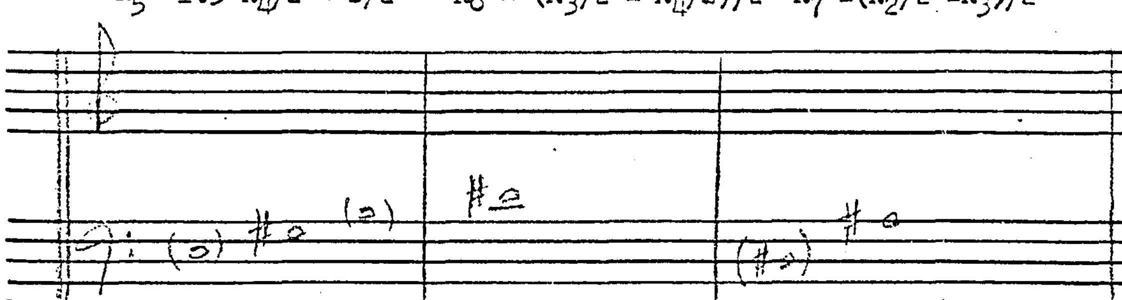
$R_0 = 1.0$ $R_4 = 2.5\ R_0$ $R_1 = 1.5\ R_0 - S/2$ $R_2 = 1.5\ R_1 - S/2$



$R_3 = 1.5\ R_2/2 - S/2$ $R_4 = 1.5\ R_3 - S/2 = 2.5\ R_0$

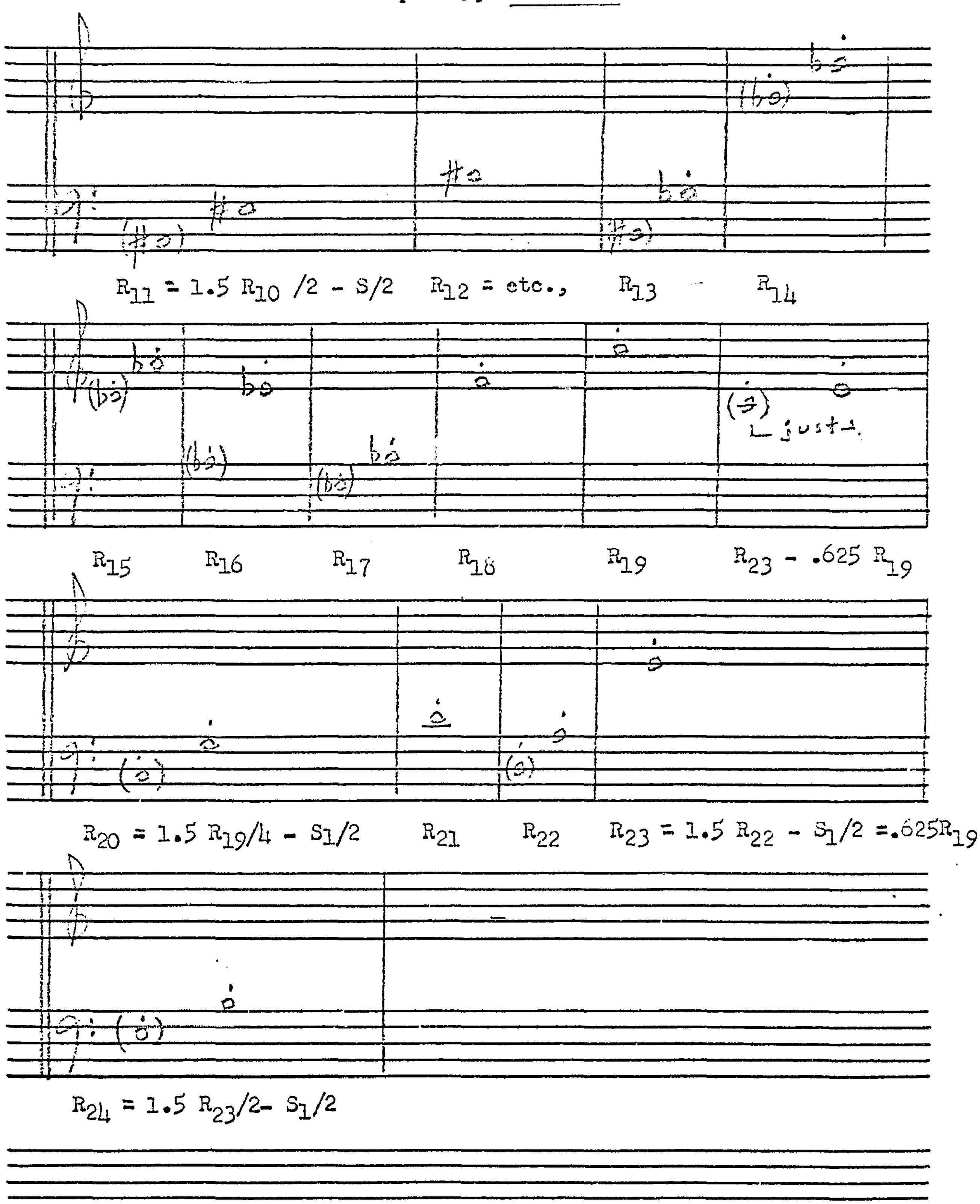


$R_5 = 1.5\ R_4/2 - S/2$ $R_6 = (R_3/2 - R_4/2)/2$ $R_7 = (R_2/2 - R_3)/2$

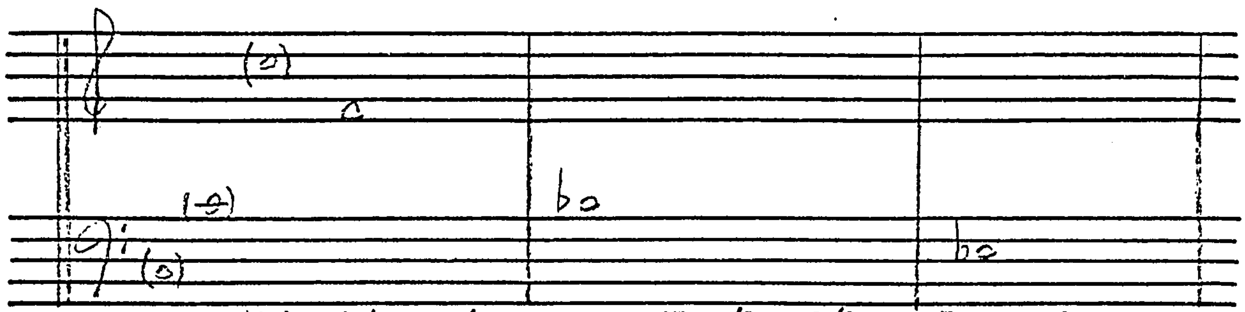


$R_8 = (R_4/2 - R_5)/2$ $R_9 = 1.5\ R_8 - S/2$ $R_{10} = 1.5\ R_9/2 - S/2$

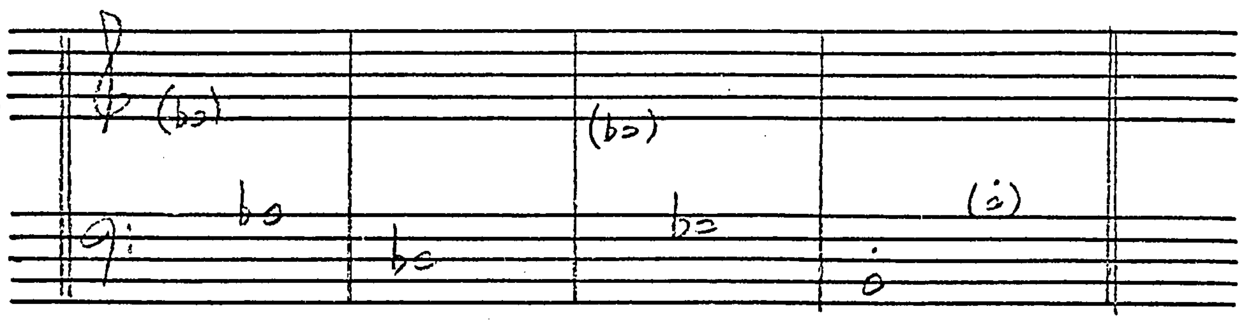
Example 23,--Continued



Example 23,--Continued



$R_{25} = 4(2R_0)/3 + S/3$ $R_{26} = 2R_{24}/3 + S/3$ R_{27} = etc.



R_{28} R_{29} R_{30} R_{31}

to allow this much error. Thus the values given in Table 6 are for the mean ratio of the two values computed for the B dot. The values for S and S_1, in terms of R_0, 0.0117647 and 0.0120346.

Version 3 has about the same variation in the sizes of the dieses, fifths, and major thirds that were found in Versions 1 and 2, however it does have several advantages. First the first two orders were tuned with the 1/4 comma meantone, a system of tuning that was known at the time. Second, this system of tuning with a constant beat ratio is such that it could be done by ear, since it would be very possible to set the speed of the beating fifth with the first four fifths that are tuned to the just major third. However when

TABLE 6

VALUES FOR VERSION 3

Note	Ratio	CPS.	Cents	Dieses
C	1.0000000	131.690	-0.000	-0.000
C DOT	1.0229436	134.712	39.272	39.272
C SHRP	1.0426470	137.306	72.301	33.029
D FLAT	1.0673929	140.565	112.910	40.609
D FLDT	1.0896031	143.490	148.563	35.654
D	1.1176471	147.183	192.558	43.994
D DOT	1.1432899	150.560	231.830	39.272
D SHRP	1.1667279	153.647	266.962	35.132
E FLAT	1.1934641	157.167	306.186	39.224
E FLDT	1.2206565	160.748	345.189	39.002
E	1.2500000	164.613	386.314	41.125
E DOT	1.2786795	168.389	425.586	39.272
E SHRP	1.3022748	171.497	457.241	31.655
F	1.3352941	175.845	500.589	43.348
F DOT	1.3653856	179.874	539.804	39.215
F SHRP	1.3941176	183.592	575.223	35.419
G FLAT	1.4271121	187.937	615.718	40.496
G FLDT	1.4547650	191.578	648.943	33.225
G	1.4941176	196.761	695.152	46.209
G DOT	1.5283981	201.275	734.424	39.272
G SHRP	1.5595588	205.379	769.366	34.941
A FLAT	1.5952070	210.073	808.492	39.127
A FLDT	1.6314635	214.848	847.400	38.908
A	1.6705882	220.000	888.427	41.027
A DOT	1.7089175	225.048	927.699	39.272
A SHRP	1.7442095	229.695	963.088	35.389
B FLAT	1.7843138	234.977	1002.443	39.355
B FLDT	1.8251023	240.348	1041.573	39.130
B	1.8691176	246.144	1082.829	41.256
B DOT	1.9113306	251.703	1121.493	38.664
B SHRP	1.9475298	256.470	1153.974	32.482
C	2.0000000	263.380	1200.000	46.026

TABLE 6--Continued

Fifths				Major thirds			
From	-To	Cents	B/Min.	From	-To	Cents	B/Min.
C	-G	695.152	92.96	C	-E	386.314	-0.00
C DOT	-G DOT	695.152	95.09	C DOT	-E DOT	386.314	0.00
C SHRP	-G SHRP	697.064	69.72	C SHRP	-E SHRP	384.940	-32.68
D FLAT	-A FLAT	695.583	92.96	D FLAT	-F	387.679	33.28
D FLDT	-A FLDT	698.837	46.48	D FLDT	-F DOT	391.241	122.68
D	-A	695.870	92.96	D	-F SHRP	382.665	-92.96
D DOT	-A DOT	695.870	95.09	D DOT	-G FLAT	383.889	-63.22
D SHRP	-A SHRP	696.126	92.96	D SHRP	-G FLDT	381.982	-115.20
E FLAT	-B FLAT	696.257	92.96	E FLAT	-G	388.966	72.30
E FLDT	-B FLDT	696.384	92.96	E FLDT	-G DOT	389.236	81.46
E	-B	696.515	92.96	E	-G SHRP	383.052	-92.96
E DOT	-B DOT	695.907	105.70	E DOT	-A FLAT	382.907	-99.32
E SHRP	-B SHRP	696.734	92.96	E SHRP	-A FLDT	390.159	114.41
F	-C	699.411	46.48	F	-A	387.838	46.48
F DOT	-C DOT	699.468	46.48	F DOT	-A DOT	387.895	49.32
F SHRP	-C SHRP	697.078	92.96	F SHRP	-A SHRP	387.865	49.38
G FLAT	-D FLAT	697.191	92.96	G FLAT	-B FLAT	386.725	13.39
G FLDT	-D FLDT	699.620	46.48	G FLDT	-B FLDT	392.630	210.05
G	-D	697.405	92.96	G	-B	387.676	46.48
G DOT	-D DOT	697.405	95.09	G DOT	-B DOT	387.068	26.33
G SHRP	-D SHRP	697.596	92.96	G SHRP	-B SHRP	384.609	-60.64
A FLAT	-E FLAT	697.694	92.96	A FLAT	-C	391.508	189.36
A FLDT	-E FLDT	697.789	92.96	A FLDT	-C DOT	391.872	207.26
A	-E	697.886	92.96	A	-C SHRP	383.874	-92.96
A DOT	-E DOT	697.886	95.09	A DOT	-D FLAT	385.210	-43.01
A SHRP	-E SHRP	694.153	185.92	A SHRP	-D FLDT	385.475	-33.36
B FLAT	-F	698.146	92.96	B FLAT	-D	390.114	154.93
B FLDT	-F DOT	698.231	92.96	B FLDT	-D DOT	390.257	164.41
B	-F SHRP	692.394	244.01	B	-D SHRP	384.133	-92.96
B DOT	-G FLAT	694.226	201.83	B DOT	-E FLAT	384.693	-70.64
B SHRP	-G FLDT	694.969	185.92	B SHRP	-E FLDT	391.214	218.10
C	-G	695.152	185.92	C	-E	386.314	-0.00

the question concerning the tuning of the five notes in the sixth order is asked, the difficulties return.

The four ninths that are to be divided into two perfect fifths, i.e., C to D dot, D to E dot, G to A dot, and A to B dot, are again much too large. The cents values for these four ninths are given below in Table 7 followed by the amount that they exceed a Pythagorean ninth, i.e., two perfect fifths.

TABLE 7

VALUES OF LARGE NINTHS

Ninth	Cents	Excess
C to D dot	1431.829	27.919
D to E dot	1433.028	29.118
G to A dot	1432.547	28.638
A to B dot	1433.065	29.155

The value for the same ninths of the 31 tone equal tempered scale is 1432.258, 28.348, very close to the above values.

Thus we are left with one of the following possibilities:

1. One of the fifths involved is a perfect fifth and the other is a sharp fifth.
2. Neither of the so-called perfect fifths is a perfect fifth in the usual meaning of the term, i.e., a 3:2 frequency ratio, but the large ninth is somehow divided into two sharp fifths.
3. Vicentino does not tune the first two orders with 1/4 comma meantone, but some other kind of tuning.

An argument can be set up for each case. However before this is done, we shall discuss additional information that can be found in the fifth book. This additional information, while it is not conclusive, will help with the decision concerning the three above possibilities. First in Vicentino's second tuning, Chapter VI, we find evidence that his use of the term "perfect fifth" does mean a truly perfect fifth. Also in this chapter we find the phrase "major and minor thirds of the ancients" which are to be available in this second tuning. In Chapters LX through LXIII, Vicentino discusses the various intervals found in the instrument and gives ratios for these intervals. However we will find that the ratios are self-contradictory. Finally in the last chapter, Chapter LXVI, Vicentino gives the division of the fingerboard of the lute. The divisions can be measured and the cents values computed for the intervals, but the result is of no direct help to the tuning problem as will be seen.

Let us begin by turning to Chapter VI (Vic. p. 104), the chapter on the second tuning. Vicentino says that after the first three orders of keys are tuned as before, i.e., in the manner described in Chapter V:

> The player touches the key of C fa ut first, or another of the first order which to him will turn out to be more convenient, and above this C fa ut tunes G sol re ut in the fourth order, with the perfect fifth and so will continue, key to key, end to end, with the perfect fifths of the white keys of the fourth order above the white keys of the first order.

Then the fifth order is tuned similarly above the second and the sixth above the third. Vicentino seems to be saying that the entire fourth order is tuned above the first for the entire length of the keyboard rather than one octave of the fourth order is tuned above the first and the remaining notes of the fourth are tuned in octaves from those already tuned.

Let us assume for the moment that Vicentino, by the term "perfect fifth" does not mean the 3:2 perfect fifth, but a fifth that is tuned sharp. If we tune two fifths, e.g., small c to small g, and c' to g', such that the two fifths beat sharp at an equal rate, then it can be shown (See Appendix VIII) that the small g to g' octave is out of tune. Out of tune octaves are a very slim possibility if not impossible in light of the statement in Chapter VI, "And then he plays the fourth, fifth, and sixth orders as he does the first second and third." So here we seem to have one good argument for making the perfect fifths perfect, at least in the second tuning.

While we are concerned with the second tuning, let us examine another statement found a little later in Chapter VI, (Vic. p. 104v).

> ..when the player will play in the first order and not moving the fingers of the hand when reaching the octave, he will be able to move the middle fingers that will play the third and fifth. In the same orders that he will play the perfect fifth, in that he will find also the major third, more perfectly tuned than those which we use. In this method he will have the perfect fifth and major and minor thirds that the ancients used.

If Vicentino is speaking of Pythagorean thirds, then a triad C 1, E 4, G 4, i.e., c, e comma, g comma, should have the values of such a triad in Pythagorean tuning. C 1 to G 4 is a perfect fifth, but if we take a perfect fifth above A 1, working with the values of Version 3, then E comma is 390.382 cents, sharp, but a long way from being a Pythagorean major third of the ratio 81/64 = 1.265625 = 407.820 cents. The other two major thirds formed from the first order ascending to the fourth are also not large enough to be Pythagorean major thirds, namely F to A comma, 393.924, and G to B comma, 393.144. On the other hand, the minor thirds E to G comma, A to C comma, and D to F comma are too large to be Pythagorean minor thirds of 294.135 cents, being 315.641, 314.117, and 311.840, resp. The minor third B to D comma is not included here since Vicentino is speaking of triads only where the third and the fifth are found in the same order, namely the fourth, and the fifth of the B triad would be in the fifth order. Apparently the "thirds of the ancients" does not mean Pythagorean thirds, at least not with the values of Version 3.

The interval ratios which Vicentino gives in Chapters LX through LXIII are found in Table 8 below. (Vic. pp. 143v-145v).

TABLE 8

INTERVAL RATIOS

Interval	Ratio	Cents
Minor semitone or major diesis	20:21	84.467
Major semitone	13:14	128.298
Small whole tone	12:13	138.572
Minor whole tone	9:10	182.404
Major whole tone	8:9	203.910
Major accidental tone	7:8	231.174
Minor third	5:6	315.641
Related of minor third	4½:5½	347.403
Major third	4:5	386.314
Related of major third	3½:4½	435.085
Fourth	3:4	498.045
Fifth	2:3	701.955

It can be seen that these ratios do not supply the answer to the tuning questions. For example the minor semitone plus the major semitone do not equal either of the ratios given for the major or minor tone, _i.e._, (21/20) (14/13) = 147/130 $\neq$ 10/9 or 9/8. In terms of cents values, the same is 84.5 + 128.3 = 212.8 $\neq$ 182.4 or 203.9. In addition Vicentino states that no difference is heard between the minor whole tone and the major whole tone (Vic. p. 143v) and that no fourth or fifth is "just" so that the thirds and sixths will be in tune (Vic. p. 145v). Therefore these ratios are only approximate.

At the end of Chapter LXVI (Vic. p. 146v), Vicentino gives the division of the fingerboard of the lute (See Plate VII, infra p. 138). The lines are not straight, thus exact measurements cannot be taken. If measurements that represent the mean of the maximum and minimum lengths of the frets are taken, the ratios which result do not make any sense. These ratios and the corresponding cents values, given in Table 9 (infra p. 139), do not approximate the values given by his interval ratios (Table 8, supra p. 136). If Vicentino is not trying to be accurate in this drawing, but only trying to represent the divisions of the lute, we would have to conclude that the picture "looks" more like it would produce the divisions of Version 3, which are fairly evenly spaced, rather than some unequal division of the scale.

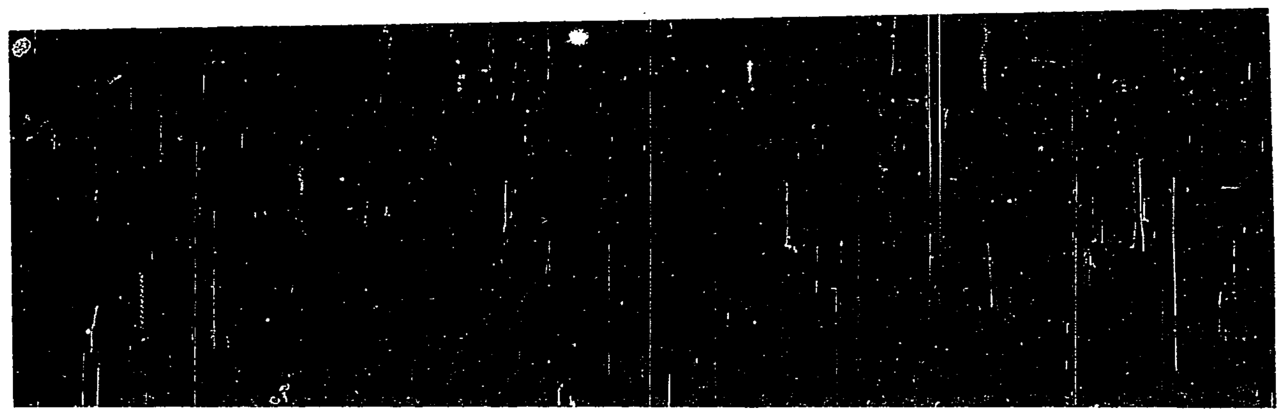


PLATE VII

FINGERBOARD OF LUTE (Actual Size)

TABLE 9

LUTE RATIOS

Ratio	Cents
1.0000	0.0
1.0280	47.8
1.0580	97.6
1.0836	139.0
1.1136	186.3
1.1383	224.3
1.1684	269.4
1.1990	314.2
1.2315	360.5
1.2597	399.7
1.2820	443.5
1.3021	457.0
1.3190	479.3
1.3333	498.0

Returning now to the first tuning, let us examine the three possibilities previously stated in connection with the large ninths. The first possibility states that one of the perfect fifths in the large ninth is perfect, the other sharp. If this is the case it would be more likely that the fifth from the first order ascending to the sixth would be the perfect fifth, since we have one fifth, E to B comma, which has no upper fifth as the others do, i.e., a fifth above B comma. The values for the five sixth order notes are given in Table 10.

TABLE 10

VALUE OF FIVE PERFECT FIFTHS ABOVE FIRST ORDER

Note	Cents	Size of the comma		Fifth above in the fourth order		
		Below	Above	To	Cents	Rate fifths beat sharp per second
G comma	701.955	6.802	32.469	D dot	729.874	9.6344
A comma	894.513	6.085	33.187	E dot	731.073	11.2340
B comma	1088.269	5.440	33.224	. .	. . .	. . .
D comma	197.108	4.550	34.722	A dot	730.592	14.7678
E comma	390.382	4.069	35.203	B dot	731.110	16.8136

The second column in Table 10 gives the cents value of each note relative to small c. The third and fourth show the size of the comma below and above, e.g., G to G comma is 6.802, and G comma to G dot is 32.469. The four fifths from the sixth order to the fourth, shown in the last three columns are quite sharp, the largest being the fifth from E comma to B dot. This fifth is 731.110 cents and in the first line octave would beat at a rate of 16.8136 beats per second. This is certainly not a perfect fifth in the usual sense. But as previously mentioned, this case can be supported. First the five sixth order pitches would be easy to tune as perfect fifths above the first order. Also, where Vicentino is speaking of the A sixth (Vic. p. 108), he says that A comma down to D is a perfect fifth, and then "its A la mi re sixth perfect fifth will be on E la mi fourth, because it will have a comma more than the fifth which is used." Rephrasing this statement, it could be interpreted as the fifth from A 6 to E 4 is called a perfect fifth, not necessarily because it is a perfect fifth in the usual sense, but because it is a comma more than the common fifth.

However we are still uneasy about calling two intervals by the same name, particularly "perfect fifth", when they have two distinct values and values that would be so easily audible. But if these values are used for the sixth order notes, they are the same values that were discussed for their fourth order counterparts in the second tuning. On the other hand it will be recalled that for these values there is the problem of the "thirds of the ancients". All things considered, it appears that there are too many unsatisfied conditions to conclude without question that the lower fifth of the large ninths is perfect, the upper being sharp.

Now let us consider the second possibility, that dividing the error would require tuning the two fifths so that the rate of beat in each is identical. This would work for four of the five sixth order notes, namely G comma, A comma, D comma, and E comma, because there is a fifth above and below. However Vicentino never mentions a fifth above B comma, only that B comma is the perfect fifth above E. By counting dieses, it can be determined that if there were such a fifth it should be G flat, and this is the value that we have used for computing the value of B comma. The intervals are shown in Example 24 (infra p. 142) and the values obtained are given in Table 11.

The most striking value is that of E comma, being 407.934 cents above C, almost exactly a Pythagorean major third. The other two major thirds are close to the Pythagorean third; F to A comma is 411.453, G to B comma is 408.458. However the two minor thirds, E to G comma and B to D comma, are much too large to be Pythagorean minor thirds, being 332.447 and 331.511 resp.

Example 24,--Ninths divided with equal beats

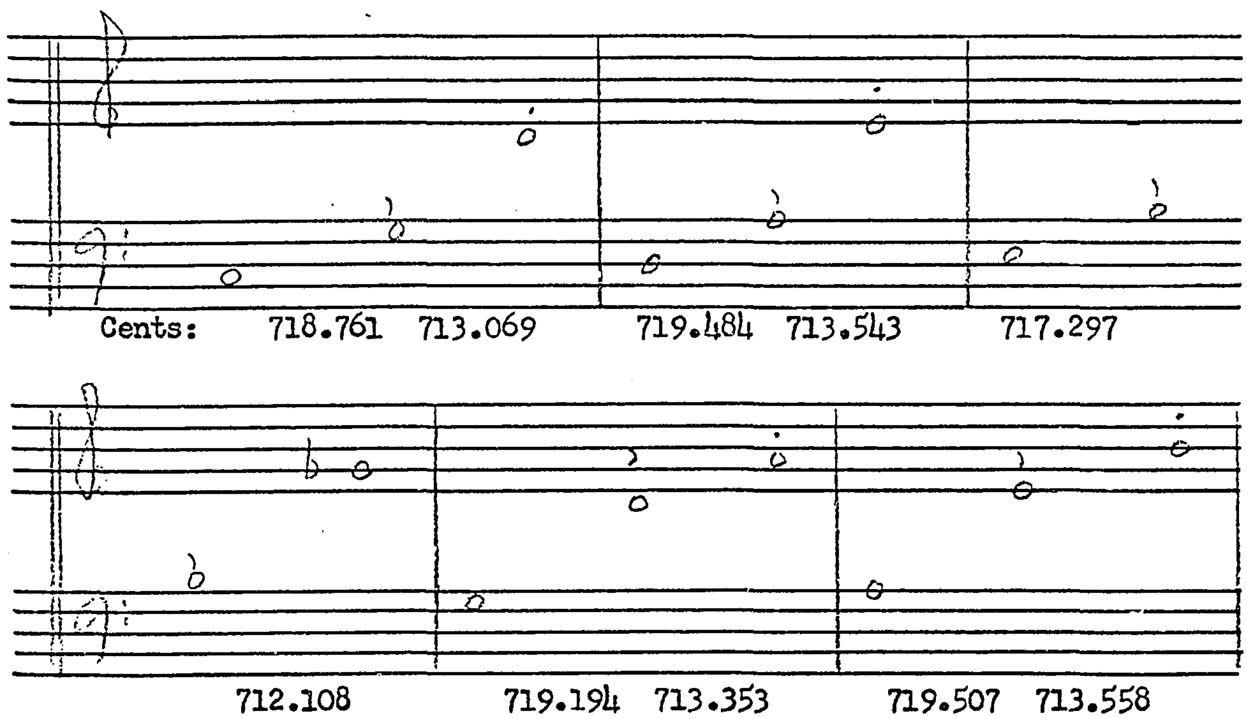


TABLE 11

VALUE OF FIVE SIXTH ORDER NOTES

Note	Cents	Rate fifths beat sharp per sec.	Size of the comma	
			Below	Above
G comma	718.761	3.8538	23.608	15.664
A comma	912.042	4.4936	23.615	15.657
B comma	1103.610	4.3957	20.782	17.483
D comma	214.346	5.9071	21.789	17.483
E comma	407.934	6.7254	21.621	17.651

Finally suppose that Vicentino does not use 1/4 comma tuning in the first two orders. In order to consider this possibility, a fourth solution based on the compromise solution offered earlier in this chapter (supra p. 98) is considered. The difference between the previous solution and the present one, which shall be called Version 4, is that the former was solved in terms of cent values, the latter in terms of equally beating fifths. Version 4, using the same sequence of fifths and octaves of Version 1 (See Example 21, supra p. 118), solves first for a constant rate of beat for the ascending fifths from small C to D dot, such that C to D dot is a Pythagorean ninth., i.e., a ratio of 9/4. Then A dot is a Pythagorean ninth above G, B dot is a Pythagorean ninth above A, and E dot is a Pythagorean ninth above D. Then the descending fifths are considered and a second constant rate of the beating fifth is found such that the descending series results in the same B dot found above. The values obtained for the scale, dieses, fifths, and major thirds are given in Table 12 (infra p. 144).

The ratio of the beat found for the ascending series of fifths in terms of C, is 0.0144544; for the descending series is 0.000631884. In addition to having all of the perfect fifths in their proper position between the first and sixth order and between the sixth and fourth order, we find that the value of the major sixth from C to A is 884.416 cents, extremely close to the value of a just major sixth, 884.359. This suggests that the tuning of the ascending series of fifths could be accomplished with 1/3 comma tuning. There

TABLE 12

VALUES FOR VERSION 4

Note	Ratio	CPS.	Cents	Dieses
C	1.0000000	131.996	-0.000	-0.000
C DOT	1.0080303	133.056	13.847	13.847
C SHRP	1.0385740	137.087	65.525	51.678
D FLAT	1.0543274	139.167	91.588	26.063
D FLDT	1.0785240	142.360	130.870	39.282
D	1.1159660	147.303	189.952	59.082
D DOT	1.1250001	148.495	203.910	13.958
D SHRP	1.1593618	153.031	255.997	52.087
E FLAT	1.1857234	156.510	294.921	38.924
E FLDT	1.2070156	159.321	325.733	30.812
E	1.2464277	164.523	381.359	55.626
E DOT	1.2554617	165.715	393.862	12.503
E SHRP	1.2916344	170.490	443.037	49.176
F	1.3335439	176.022	498.318	55.281
F DOT	1.3488586	178.043	518.087	19.769
F SHRP	1.3895835	183.419	569.583	51.496
G FLAT	1.4059804	185.583	589.892	20.309
G FLDT	1.4404411	190.132	631.813	41.921
G	1.4927728	197.039	693.593	61.781
G DOT	1.5048183	198.629	707.507	13.914
G SHRP	1.5506338	204.677	759.430	51.923
A FLAT	1.5811751	208.708	793.197	33.767
A FLDT	1.6141724	213.064	828.954	35.757
A	1.6667217	220.000	884.416	55.462
A DOT	1.6793694	221.669	897.503	13.088
A SHRP	1.7318155	228.592	950.742	53.239
B FLAT	1.7782691	234.724	996.568	45.826
B FLDT	1.8032963	238.027	1020.764	24.195
B	1.8624143	245.831	1076.609	55.845
B DOT	1.8750619	247.500	1088.326	11.717
B SHRP	1.9302244	254.781	1138.522	50.196
C	2.0000000	263.991	1200.000	61.478

TABLE 12--Continued

Fifths				Major thirds			
From	-To	Cents	B/Min.	From	-To	Cents	B/Min.
C	-G	693.593	114.48	C	-E	381.359	-113.17
C DOT	-G DOT	693.660	114.48	C DOT	-E DOT	380.015	-144.97
C SHRP	-G SHRP	693.905	114.48	C SHRP	-E SHRP	377.512	-208.55
D FLAT	-A FLAT	701.609	5.01	D FLAT	-F	406.731	495.29
D FLDT	-A FLDT	698.084	57.24	D FLDT	-F DOT	387.217	22.29
D	-A	694.464	114.48	D	-F SHRP	379.631	-170.24
D DOT	-A DOT	693.593	128.79	D DOT	-G FLAT	385.982	-8.54
D SHRP	-A SHRP	694.745	114.48	D SHRP	-G FLDT	375.816	-277.54
E FLAT	-B FLAT	701.647	5.01	E FLAT	-G	398.673	336.38
E FLDT	-B FLDT	695.030	114.48	E FLDT	-G DOT	381.774	-125.17
E	-B	695.250	114.48	E	-G SHRP	378.071	-234.45
E DOT	-B DOT	694.464	128.79	E DOT	-A FLAT	399.335	375.33
E SHRP	-B SHRP	695.485	114.48	E SHRP	-A FLDT	385.916	-11.74
F	-C	701.682	5.00	F	-A	386.098	-6.59
F DOT	-C DOT	695.760	114.48	F DOT	-A DOT	379.417	-212.37
F SHRP	-C SHRP	695.942	114.48	F SHRP	-A SHRP	381.159	-163.59
G FLAT	-D FLAT	701.696	5.00	G FLAT	-B FLAT	406.677	658.72
G FLDT	-D FLDT	699.057	57.24	G FLDT	-B FLDT	388.951	86.96
G	-D	696.358	114.48	G	-B	383.015	-112.51
G DOT	-D DOT	696.403	114.48	G DOT	-B DOT	380.819	-188.84
G SHRP	-D SHRP	696.567	114.48	G SHRP	-B SHRP	379.093	-255.58
A FLAT	-E FLAT	701.724	5.00	A FLAT	-C	406.803	745.44
A FLDT	-E FLDT	696.780	114.48	A FLDT	-C DOT	384.893	-52.42
A	-E	696.943	114.48	A	-C SHRP	381.109	-198.12
A DOT	-E DOT	696.358	128.78	A DOT	-D FLAT	394.084	299.15
A SHRP	-E SHRP	692.295	228.95	A SHRP	-D FLDT	380.128	-244.60
B FLAT	-F	701.750	5.00	B FLAT	-D	393.383	288.14
B FLDT	-F DOT	697.323	114.48	B FLDT	-D DOT	383.146	-130.52
B	-F SHRP	692.974	228.95	B	-D SHRP	379.388	-294.43
B DOT	-G FLAT	701.566	10.01	B DOT	-E FLAT	406.595	874.95
B SHRP	-G FLDT	693.290	228.95	B SHRP	-E FLDT	387.211	39.63
C	-G	693.593	228.95	C	-E	381.359	-226.33

is a possibility that Vicentino could have known 1/3 comma tuning since both Zarlino and Salinas describe it.[21]

However the values of the descending fifths presents a problem in light of Vicentino's instruction. This value results in fifths which beat at a rate of 5 times per minute. On a harpsichord, an instrument with a relatively rapid decay rate, it is doubtful if the difference between a truly perfect fifth and a fifth which beat 1 time in 12 seconds could be detected. Vicentino states that the descending fifths should be tempered.

The great variation in the sizes of intervals is also noted. As pointed out in the first chapter, Vicentino uses the 31 pitches in the first five orders enharmonically, which would suggest something close to an equal division. But we find in Version 4, the following variations in some of the intervals (Table 13).

TABLE 13

RANGE OF INTERVAL SIZE

Interval	Smallest		Largest		Range
	Cents	Notes	Cents	Notes	
Diesis	11.717	B -B.	61.781	G^b -G	50.064
Min. Sem.	61.678	E -E#	111.674	B^b -C	49.996
Maj. Sem.	91.573	F -G^b	127.617	G -G#	36.044
Tone	187.040	D# -E#	203.432	B^b -C	16.392
Min. 3rd.	294.878	F -A^b	318.929	G^b.-A#	24.051
Maj. 3rd.	375.816	D# -G^b	406.803	A^b -C	30.988
Fifth	692.295	A# -E#	701.750	B^b -F	9.455

[21]See Arthur Daniels, "Microtonality and Mean-tone Temperament in the Harmonic System of Francisco Salinas", Journal of Music Theory, Vol. 9, Spring, 1965, pp. 2-51 and Winter 65, pp. 234-280, esp. 252-254.

It seems very unlikely that, in a scale that is used enharmonically, there would be such variation in the values for similar intervals. The variation is so great that some minor semitones exceed the size of some major semitones by some 20 cents. There is also a great variation in the sizes of the major and minor third, which are the basic consonant intervals. This amount of variation is audible, particularly in triadic combinations.

It is appropriate now to bring together all the information that has been presented and attempt an evaluation. The following points have been established.

1. Version 3 is based on 1/4 comma tuning which was known at least 30 years before 1555. Version 4 is based on 1/3 comma tuning which could have possibly been known to Vicentino which had not been reported in print, so far as is known, until after 1555.[22]
2. The tuning in both cases could have been accomplished by ear with the beating fifths. Version 3 is based on fifths which are set to beat in such a manner that a just major third is reached after four such fifths. In Version 4, a just major sixth is reached after three such fifths.
3. In the basic 31 pitches, *i.e.*, the first five orders, the range of values for similar intervals in Version 3

[22] Daniels, *op. cit.*, p. 252 ff. First in print by Zarlino, *Dimonstrationi harmoniche* (1571), but Salinas is credited with invention earlier.

is small enough to permit the "enharmonic" use of these pitches which Vicentino employs in the course of the book. However in Version 4, the range for some is quite great, especially in the sizes of the major and minor semitones where the smallest major semitone is less than the greatest minor semitone by some 20 cents, and in the sizes of the major and minor thirds which deviate considerably from the just intervals. A comparison is given in Table 14.

TABLE 14

COMPARISON OF INTERVAL RANGES

	Version 3			Version 4		
Interval	Smallest	Largest	Range	Smallest	Largest	Range
Min. Sem.	70.927	85.481	14.554	61.678	111.674	50.064
Maj. Sem.	109.139	120.422	11.283	91.573	127.617	49.996
Tone	188.909	198.457	9.548	187.040	203.432	16.392
Min. 3rd.	303.755	314.145	10.390	294.878	318.929	24.051
Maj. 3rd.	382.665	392.629	9.964	375.816	406.803	30.988
Fifth	692.394	699.920	6.220	692.295	701.750	9.455

4. In Version 3, the fifths that exist between the first and sixth orders and the sixth and fourth orders, both of which are to be perfect fifths, cannot both be perfect fifths in the usual meaning of perfect fifths. Yet in the second tuning there is evidence that Vicentino does mean an interval of 3:2 when he speaks of perfect fifth. In Version 4 these fifths between the first and sixth orders and the sixth and fourth do exist as perfect fifths.

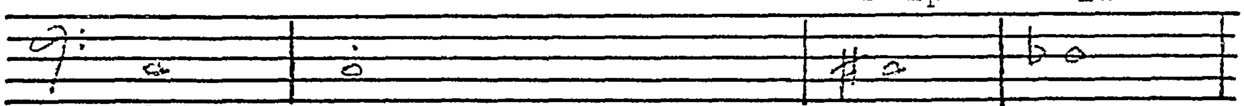
5. In Version 3 all of the 31 common fifths have very nearly the same rate of beat. In Version 4, there are basically two sizes of fifths both called common fifths by Vicentino. The common fifths in the series descending from C to B dot in Version 4 are almost perfect, and Vicentino states that these common fifths are tempered as are the common fifths in the ascending series.
6. In neither version 3 or 4 is there a satisfactory explanation of the thirds from the first to the sixth order that are to be "thirds of the ancients".
7. The interval ratios given by Vicentino in Chapter LX through LXIV do not agree with either version, in fact they do not agree with themselves.
8. The drawing of the lute looks as though it would produce a scale where the adjacent dieses would be more or less equal, i.e., such as the dieses in Version 3, rather than the unequal dieses found in Version 4, even though the actual measurements do not produce a scale anywhere near either version.

The total picture seems to be in favor of Version 3, i.e., the version which tunes the first two orders of keys according to Aaron's quarter comma tuning. The one item which is still in question is the tuning of the sixth order notes in the first tuning. Because of the definite meaning of perfect fifth in the second tuning, it seems to the writer that the most probable tuning of these five pitches is a perfect fifth above the first order, even though these tunings do not produce the perfect fifths from the sixth to the fourth order. The complete scales for both of Vicentino's tunings, i.e., the first tuning described in Chapter V and the second tuning described in Chapter VI, are found in the following example, Example 25. The examples show the name of the pitch, the notation on the staff for the small octave, the name of the key on the instrument, and the cents value for the pitch relative to small C natural.

Example 25,--First and second tuning

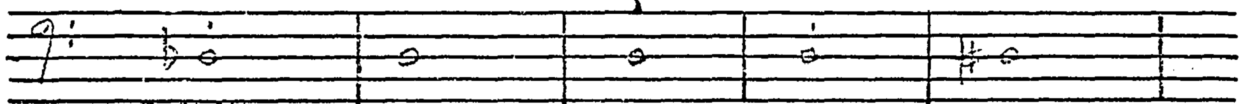
First Tuning:

C	C dot	C sharp	D flat



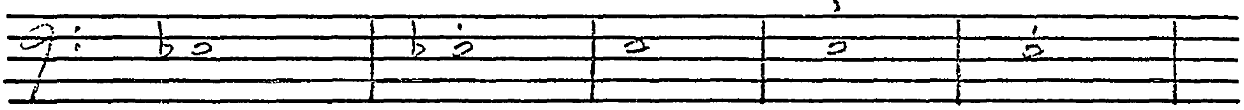
C first	C third in the fourth order	D second	D third
0.0	39.272	72.301	112.910

D flat dot	D	D comma	D dot	D sharp



D fifth	D first	D sixth	D fourth	E third
148.563	192.558	197.018	231.830	266.962

E flat	E flat dot	E	E comma	E dot

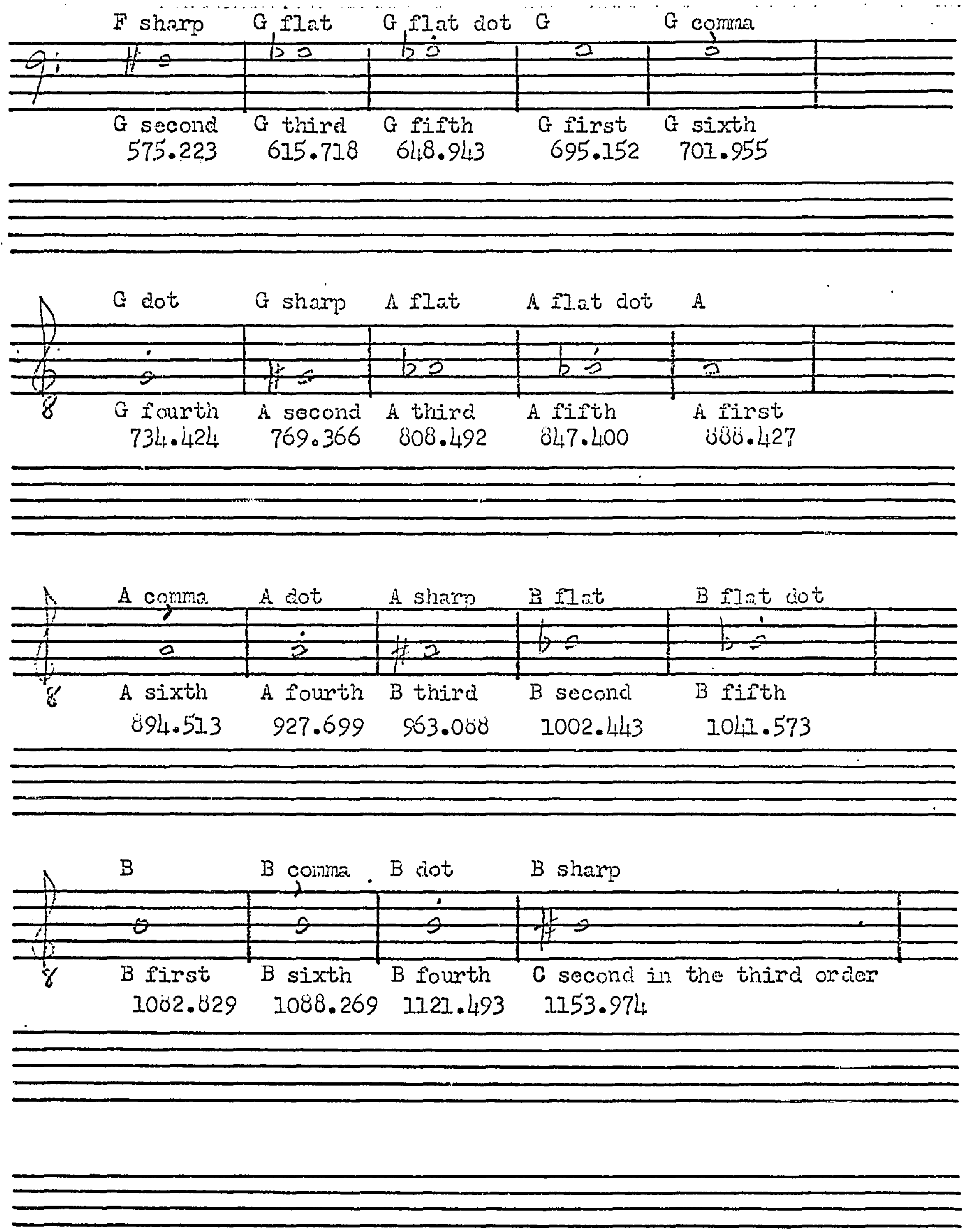


E second	E fifth	E first	E sixth	E fourth
306.186	345.189	386.314	390.382	425.586

E sharp	F	F dot

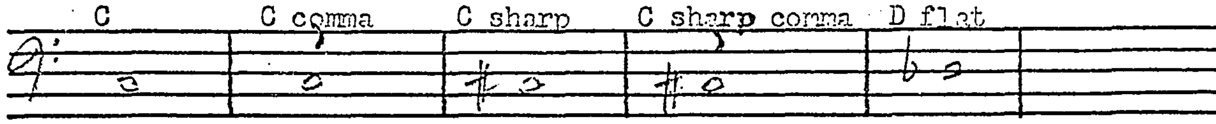
F second inthe third	F first	F third in the fourth
457.241	500.589	539.804

Example 25,—Continued



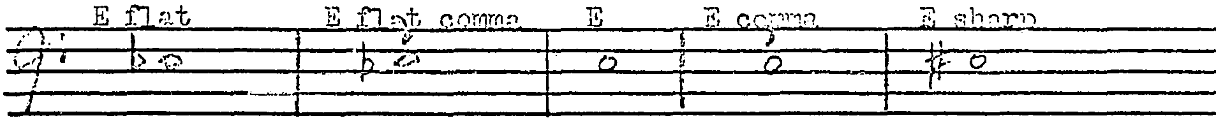
Example 25,--Continued

Second tuning:

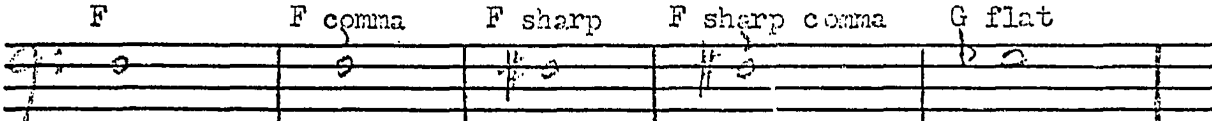


C	C comma	C sharp	C sharp comma	D flat
C first	C third in 4	D second	D fifth	D third
0.0	2.544	72.301	77.178	112.910

D flat comma	D	D comma	D sharp	D sharp comma
D sixth	D first	D fourth	E third	E sixth
117.673	192.558	197.108	266.962	271.321

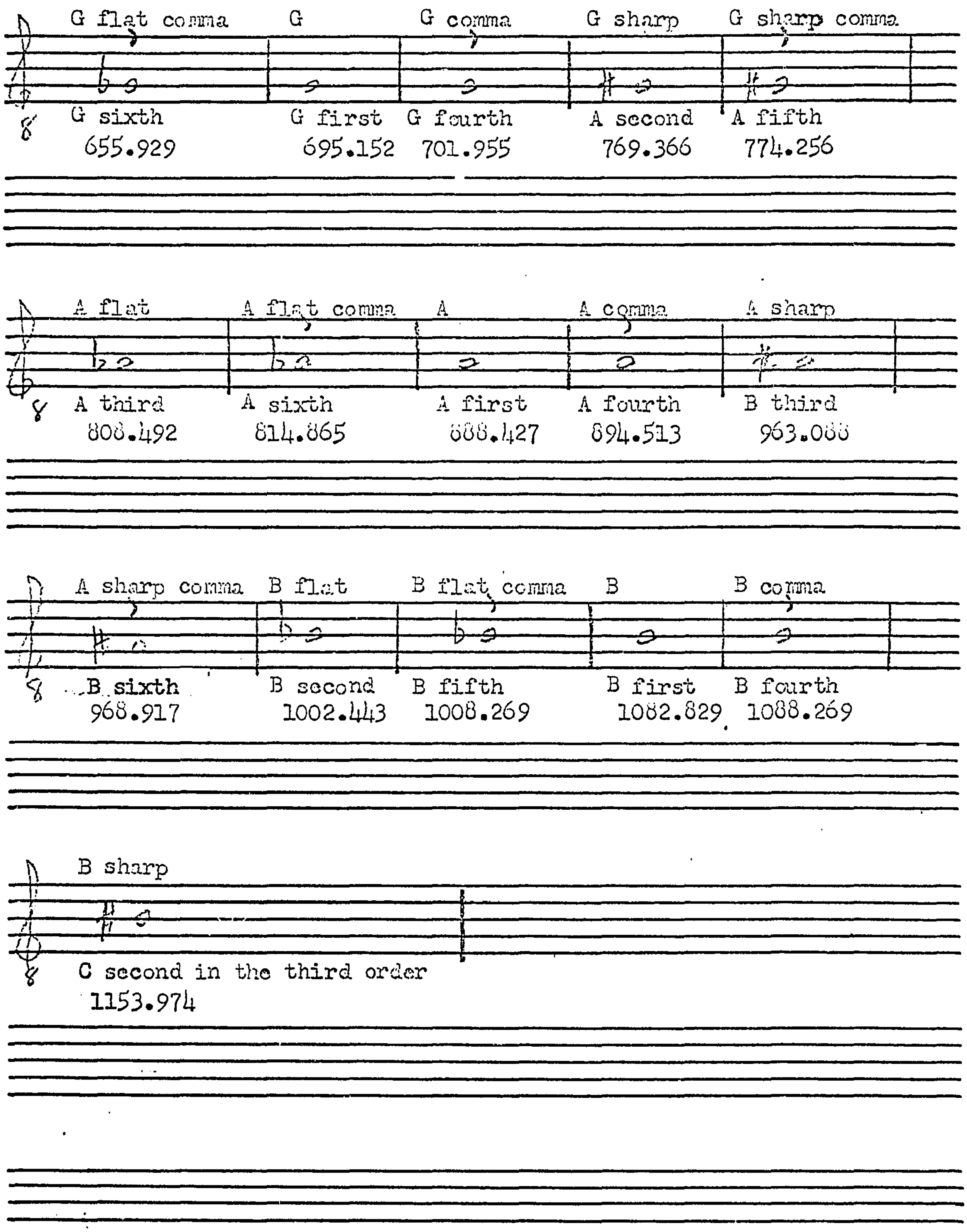


E flat	E flat comma	E	E comma	E sharp
E second	E fifth	E first	E fourth	F second in third
306.186	310.447	386.314	390.382	457.241



F	F comma	F sharp	F sharp comma	G flat
F first	F fourth	G second	G fifth	G third
500.589	504.398	575.223	584.784	615.718

Example 25,--Continued



APPENDIX I

HEIGHT OF KEYS ABOVE FRAME

"b" (See Figure 3, supra p. 41) = 0.19".

"d" $>$ 0.19.

1. Let "e" = 0.97.

   Then: $a = 0.97$.

   Since: $a + b + c + d = 1.42$;

   $c < 0.07$,

   which is too small for the thickness of the frame.

2. Let "f" = 0.97.

   Then: $a = 0.97 - 0.19 = 0.78$

   And: $c < 0.26$,

   which is reasonable.

3. Let "g" = 0.97.

   Then: $a + b + c = 0.97$.

   $a + c = 0.78$.

   And: $d = 1.42 - 0.97 = 0.45$.

   Although "d" must be greater than 0.19, 0.45 appears too great and 0.78 must be divided between the thickness of the key and the thickness of the frame, "a" and "c", respectively.

APPENDIX II

MAXIMUM LIMIT OF "x"

The total height of the instrument is 8.52 (See Figure 20, infra p. 157). The height from the top to the soundboard is "x". The height of the bridge is 0.40. The height from the string to the top of the jack, i.e., the portion of the jack containing the damper would not be greater than 1.25 (supra p. 44). The length of the long jack is 4.32. Thus the distance from the top of the instrument to the upper surface of the distal end of the first frame keys is:

$$x - 0.04 - 1.25 + 4.32; \text{ or } 2.67 + x.$$

1. The thickness of the frame, "a" (Figure 5, supra p. 47, and Figure 20), equals 0.20, but the minimum for "x" is 1.59. Thus "a" ≠ "x".
2. The top of the bottom boards to the upper surface of the first order keys, "b" (Figure 5), is 1.17. Thus "b" ≠ "x" for the same reason.
3. If "c" = "x", then:

   $$8.52 = (2.67 + x) + x + 0.97.$$

   $$x = 2.44.$$
4. If "d" = "x", then:

   $$8.52 = (2.67 + x) + x.$$

   $$x = 2.92.$$
5. If "e" = "x", then this has been established as 1.81 (supra p. 47).

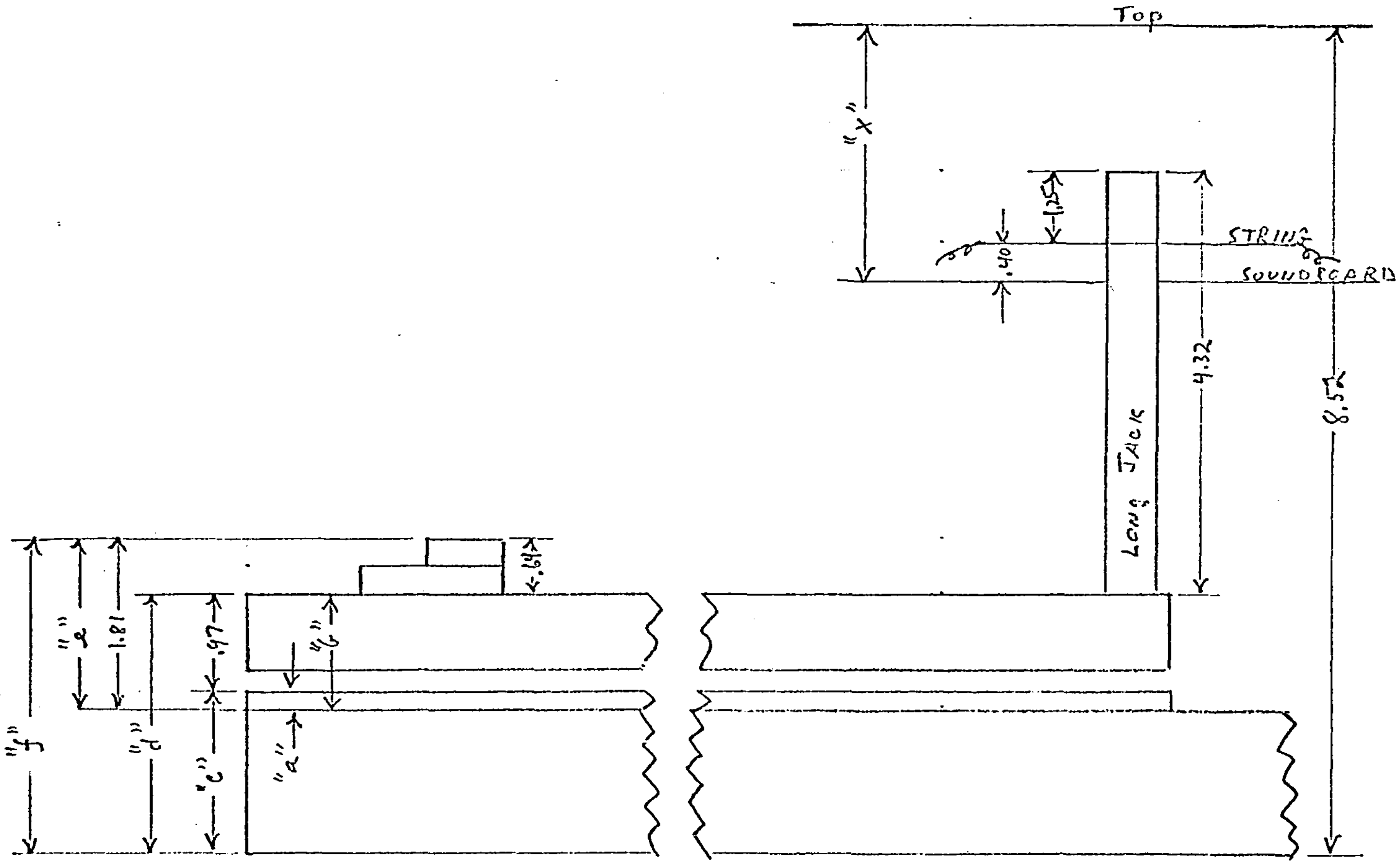


FIGURE 20. LIMITS FOR MISSING DIMENSION (SCALE ≈ 1:2)

6. If "f" = "x", then:

$$8.52 = (2.67 + x) + x - 0.64.$$

$$x = 3.25.$$

APPENDIX III

"c" EQUALS "x"

The height of the long jack is 4.32 (See Figure 21, infra p. 160). The distance from the upper key to the lower is 1.42; the clearance from upper keys to jack guide, minimum of 0.06; action, 0.19; height of jack guide, 1.42; height of bridge, 0.40; and the amount of jack extending above the string, minimum, 0.25, maximum, 1.25, represented in Figure 21 with the letter "z". Distance from the lower frame of keys to the top of the jack guide, "w", is:

$w \geqq 1.42 + 0.06 + 0.19 + 1.42.$

$w \geqq 3.09.$

Also: $w - 4.32 - z - 0.40.$

Thus: $2.67 \leqq w \leqq 3.67.$

But: $w \geqq 3.09.$

Therefore: $0.25 \leqq z \leqq 0.81.$

The total height is 8.52.

Therefore: $8.52 = x - 0.40 - z + 4.32 + 0.97 + x.$

$x = 1.82 + z/2.$

Thus: $1.95 \leqq x \leqq 2.23.$

The thickness of the bottom board, "y" is $x - 0.21$.

Then: $1.74 \leqq y \leqq 2.02.$

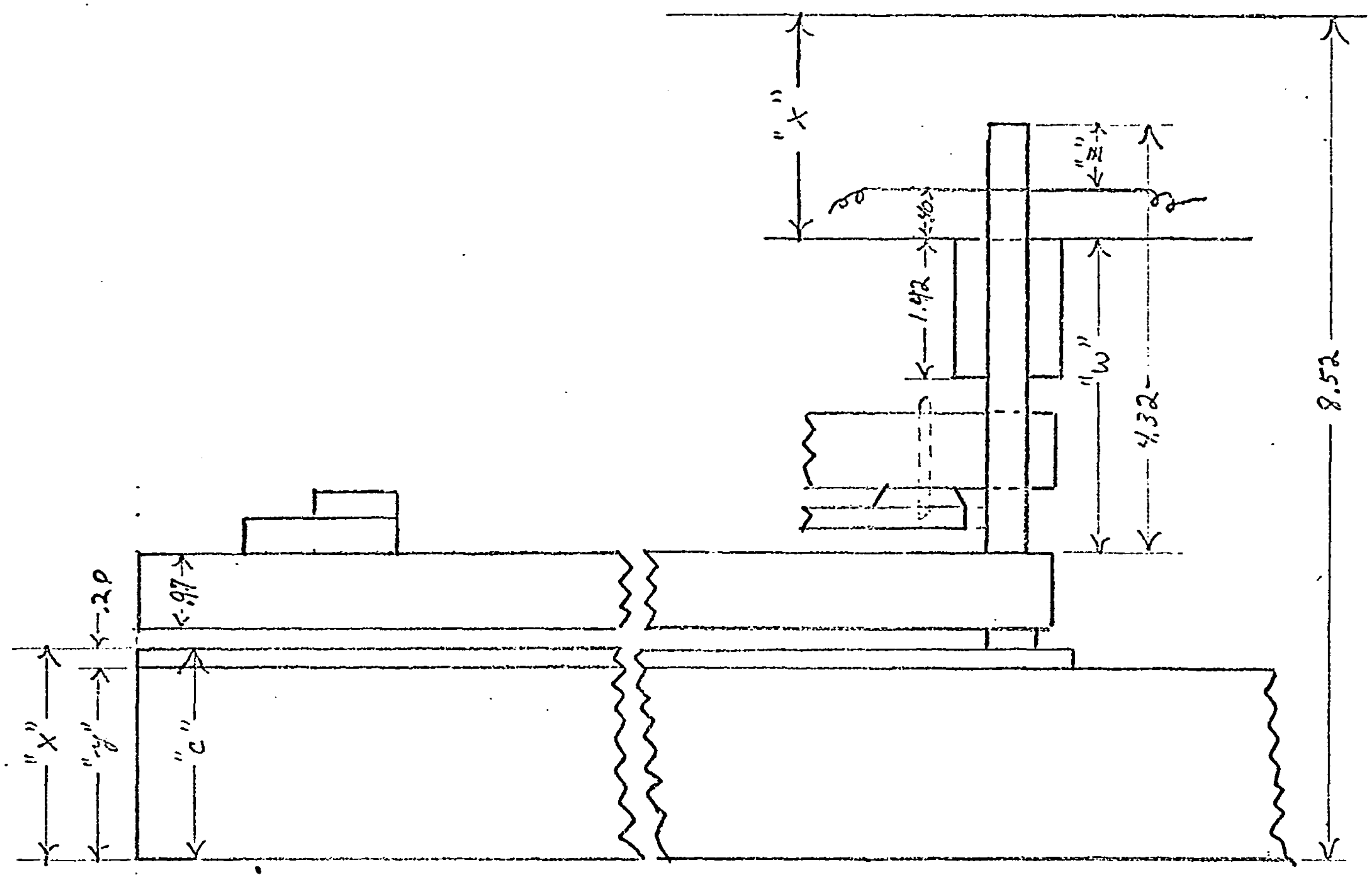


FIGURE 21. "C" EQUAL X (SCALE 1:2)

APPENDIX IV

"d" EQUALS "x"

$1.59 \leqq x \leqq 2.92.$

Substitute: $x = 2.71$

Therefore the bottom boards (See Figure 22, infra p. 162) are:

$2.71 - (0.97 + 0.21) = 1.53.$

The height from the soundboard to the top of the second frame keys is:

$8.52 - (2(2.71) + 1.42) = 1.68.$

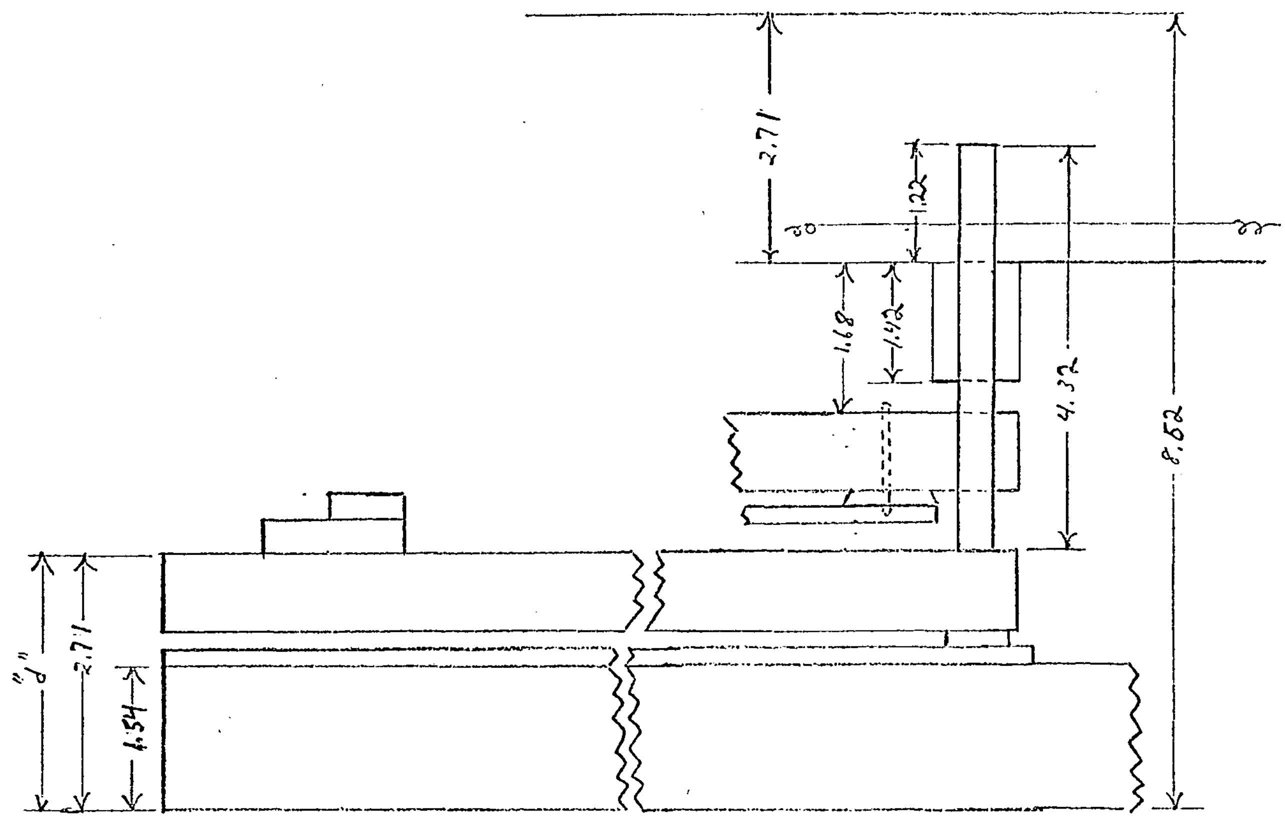


FIGURE 22. "d" EQUAL X (SCALE≈1:2)

APPENDIX V

"e" EQUALS "x"

"e" = "x" = 1.81.

The thickness of the bottom boards (See Figure 23, infra p. 164, letter "y") is:

$$8.52 = 1.81 - 0.40 - z + 4.32 - 0.64 + 1.81 + y.$$

$$y = 1.62 + z.$$

Since: $0.25 \leqq z \leqq 0.81.$

$$1.87 \leqq y \leqq 2.43.$$

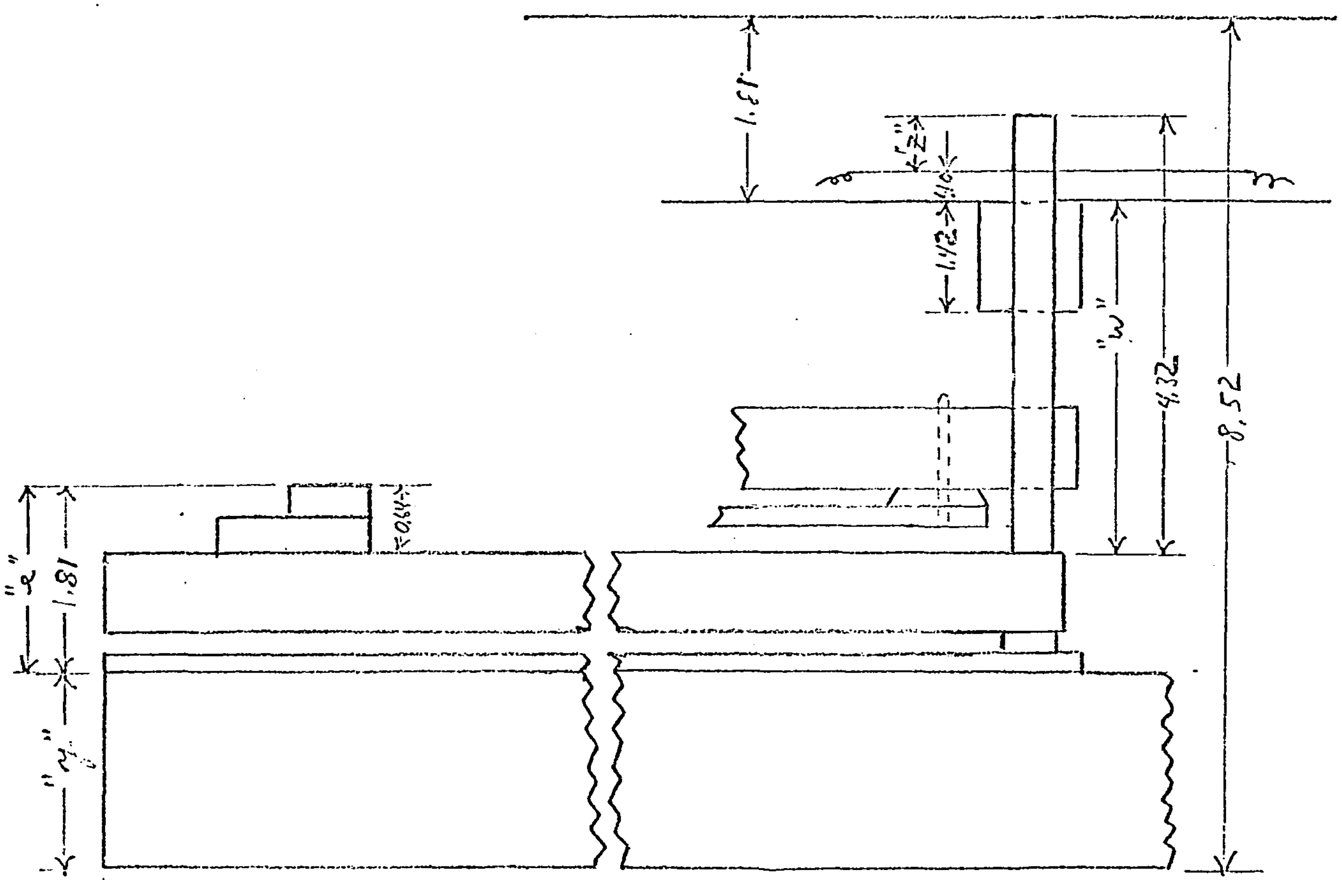


FIGURE 23. "e" EQUAL X (SCALE≈1:2)

APPENDIX VI

"f" EQUALS "x"

$1.59 \leqq x \leqq 3.25.$

Substitute: $x = 2.71$

The thickness of the bottom boards (See Figure 24, infra p. 166) is:

$2.71 - 1.81 = 0.90.$

The height of the jack above the string (See Figure 24, letter "z")

is:

$8.52 = 2.71 - 0.40 - z + 4.32 - 0.64 + 2.71$

$z = 0.18.$

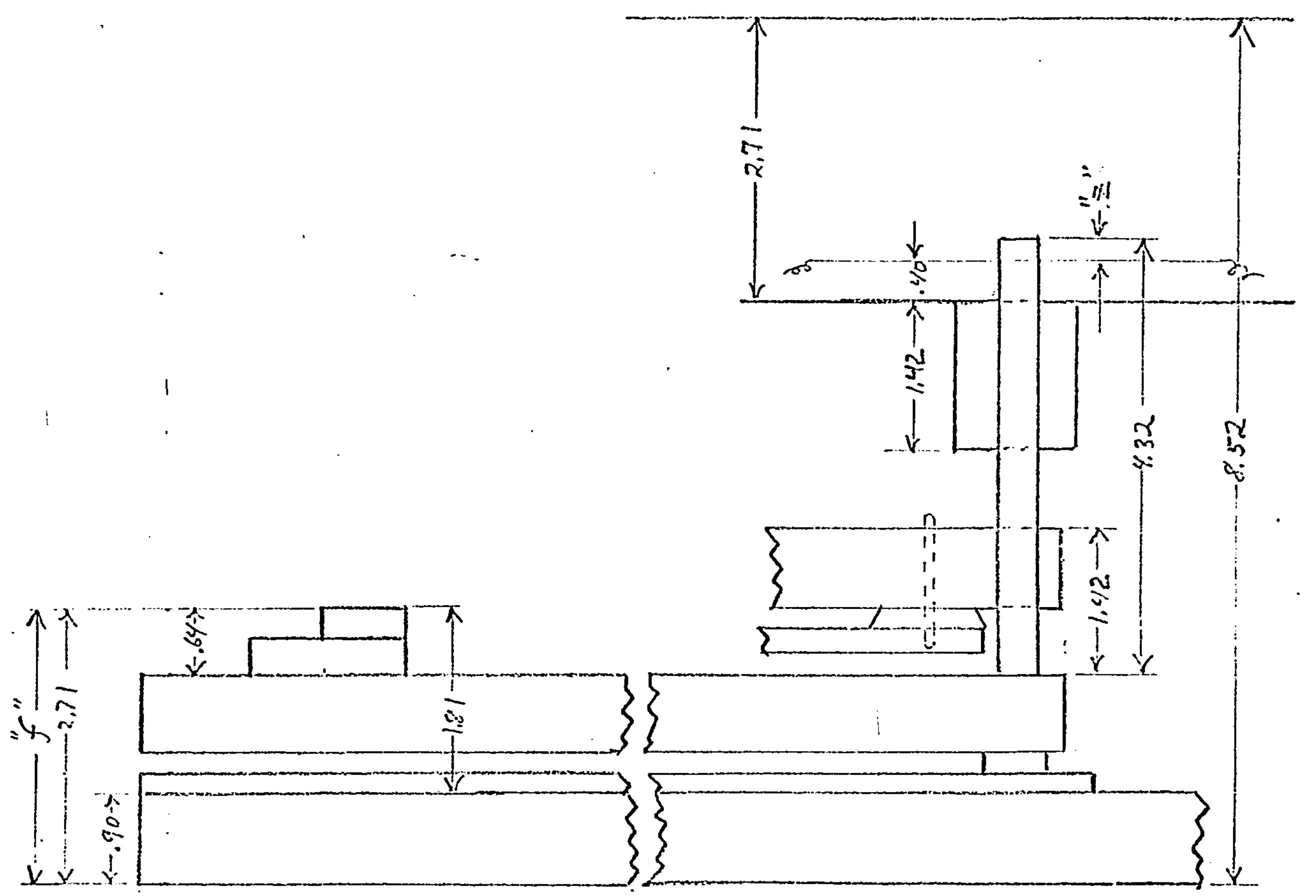


FIGURE 2.4. "f" EQUAL X (SCALE≈1:2)

APPENDIX VII

BEATS OF MAJOR AND MINOR THIRDS

If X_0 is the ratio of the lower note, X_1 the ratio of the fifth above X_0, X_2 is the ratio of the major third above X_0, and Y the speed of the beats of the major thirds, then we can solve for X_2. The speed of the beats of a flat major third is:

$$Y = 5X_0 - 4X_2.$$

The minor third is:

$$Y = 6X_2 - 5X_1.$$

Y is a constant, thus:

$$5X_0 - 4X_2 = 6X_2 - 5X_1$$

$$X_2 = (X_0 - X_1)/2.$$

APPENDIX VIII

PERFECT FIFTHS

The formula for a sharp fifth is $2(R_1) - 3(R_0) = S$, where R_0 is the frequency of the lower member of the fifth, R_1, the frequency of the upper, and S, the speed of the beats. Let R_0 = small c; R_1 = small g; R_2 = c'; R_3 = g'; and S is the constant speed of the beats.

Then: $2(R_1) - 3(R_0) = S$

and: $2(R_3) - 3(R_2) = S$

The octave from small c to c' is $2R_0 - R_2$.

Substituting for R_2:

$$2(R_3) - 6(R_0) = S.$$

or: $R_3 = 3(R_0) + S/2.$

and also: $2(R_1) = 3(R_0) + S.$

But the octave small g to g', $R_3 = 2(R_1)$, if and only if:

$$3(R_0) + S/2 = 3(R_0) + S.$$

i.e., $S/2 = S$.

Therefore for the small g to g' octave to be in tune, $S = 0$, *i.e.*, the fifths must be perfect 3:2.

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